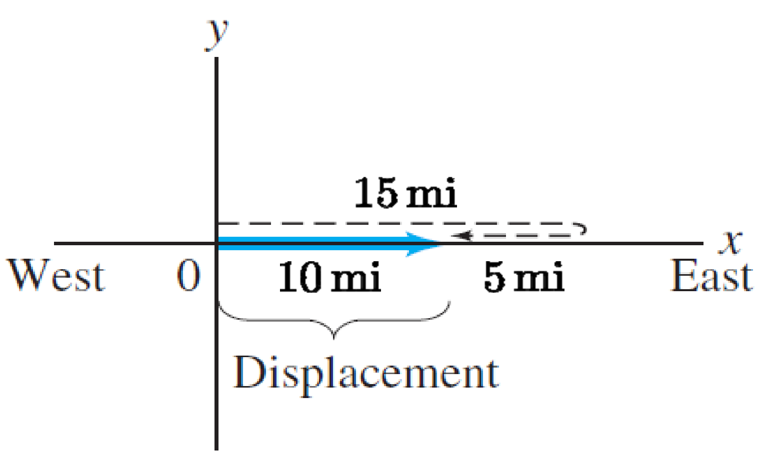
Chapter 2: Motion along a straight line

<u>Distance vs. displacement</u>

- The **<u>distance</u>** traveled by an object is path dependent.
- The **<u>displacement</u>** is the change in position of an object.

<u>Example</u>: A car travels 15 miles east and 5 miles west.



distance traveled = 20 miles
displacement = 10 miles

Velocity

Average velocity

The **average velocity** is the displacement $\Delta\vec{s}$ of an object divided by the change in time, Δt

$$\vec{v}_{\mathbf{avg}} = \frac{\Delta\vec{s}}{\Delta t}$$

Instantaneous velocity

For infinitesimally short times, the instantaneous velocity is given by

$$\begin{aligned}\vec{v} &= \lim_{\Delta t \to 0} \frac{\Delta\vec{s}}{\Delta t} \\ &= \frac{d\vec{s}}{dt}\end{aligned}$$

Acceleration

Average acceleration

The **average acceleration** is the change in velocity $\Delta\vec{v}$ of an object divided by the change in time, Δt

$$\vec{a}_{\mathbf{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$

Instantaneous acceleration

For infinitesimally short times, the instantaneous acceleration is given by

$$\begin{aligned}\vec{a} &= \frac{d\vec{v}}{dt}\\ &= \frac{d}{dt}\left(\frac{d\vec{s}}{dt}\right)\\ &= \frac{d^2\vec{s}}{dt^2}\end{aligned}$$

Note that $\left(\frac{d\vec{s}}{dt}\right)^2 \neq \frac{d^2\vec{s}}{dt^2}$

Motion under constant acceleration

Assume we are traveling in the x-direction. The acceleration is $a_x = \dfrac{dv_x}{dt}$

We can rewrite the instantaneous acceleration equation as $dv_x = a_x\, dt$

If $a_x =$ constant, then

$$\begin{aligned}\int_{v_{0x}}^{v_x} dv'_x &= \int_0^t a_x\, dt' \\ &= a_x \int_0^t dt' \\ &= a_x t\end{aligned}$$

where we have set the initial time to be zero when the velocity is some constant, v_{0x}.

Therefore, the velocity as a function of time for constant acceleration is given by

$$\boxed{v_x = v_{0x} + a_x t}$$

Motion under constant acceleration (cont.)

The equation for the instantaneous velocity can be rewritten as $dx = v_x\, dt$

Substituting our previously boxed equation for the velocity and again integrating with respect to the time gives

$$\begin{aligned}\int_{x_0}^{x} dx' &= \int_0^t (v_{0x} + a_x t')\, dt' \\ &= \int_0^t v_{0x} dt' + \int_0^t a_x t'\, dt' \\ &= v_{0x} \int_0^t dt' + a_x \int_0^t t'\, dt' \\ &= v_{0x} t + \frac{1}{2} a_x t^2\end{aligned}$$

where we have set the initial time to be zero when the position is some constant, x_0.

Therefore, the position as a function of time for constant acceleration is given by

$$\boxed{x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2}$$

Motion under constant acceleration (cont.)

For a time-independent equation we have the average velocity between the initial time ($t = 0$) and final time to be

$$a_x = \frac{v_x - v_{0x}}{t}$$

Solving for t and substituting this into the equation $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives

$$x = x_0 + v_{0x}\frac{v_x - v_{0x}}{a_x} + \frac{1}{2}a_x\left(\frac{v_x - v_{0x}}{a_x}\right)^2$$

We can then simplify the equation algebraically to get

$$\boxed{2a_x\left(x - x_0\right) = v_x^2 - v_{0x}^2}$$

Summary of equations of one-dimensional motion

Kinematic equations

$$v_x = \frac{dx}{dt}$$

$$a_x = \frac{dv_x}{dt}$$

Constant acceleration ONLY

$$\left.\begin{aligned} v_x &= v_{0x} + a_x t \\ x &= x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \end{aligned}\right\} \text{time-dependent}$$

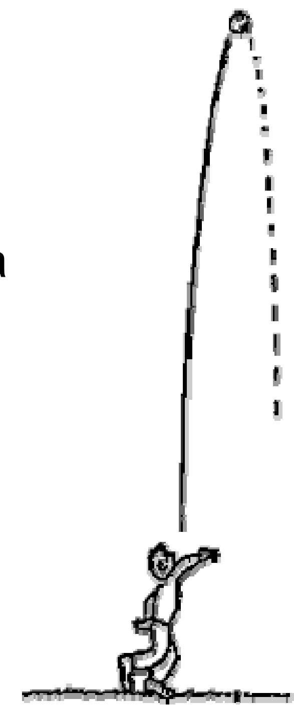
$$\left.\begin{aligned} v_x^2 &= v_{0x}^2 + 2a_x (x - x_0) \end{aligned}\right\} \text{time-independent}$$

Free falling objects

When near the surface of the Earth, the acceleration due to gravity is a constant with magnitude of $g = 9.8\ m/s^2$.

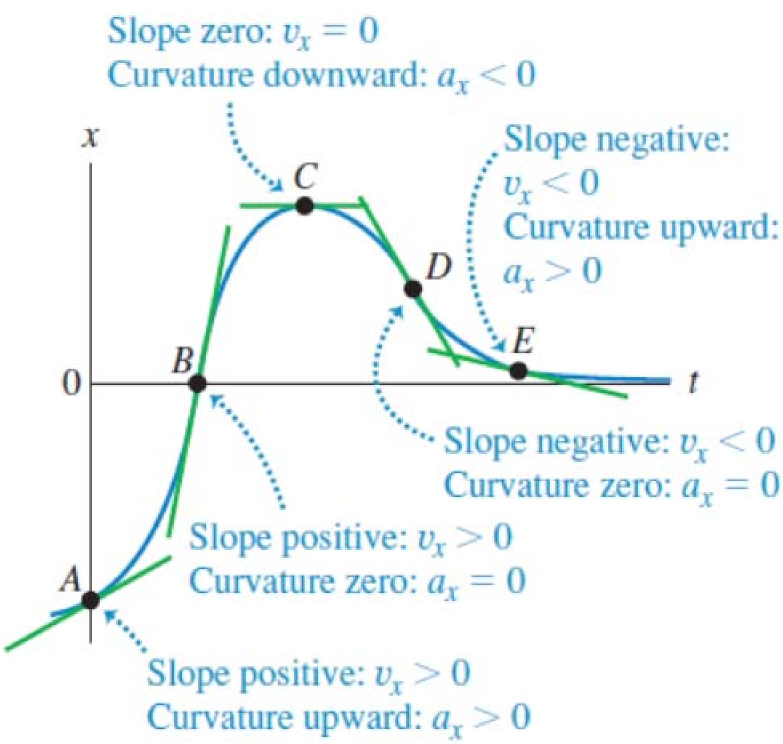
EXAMPLE

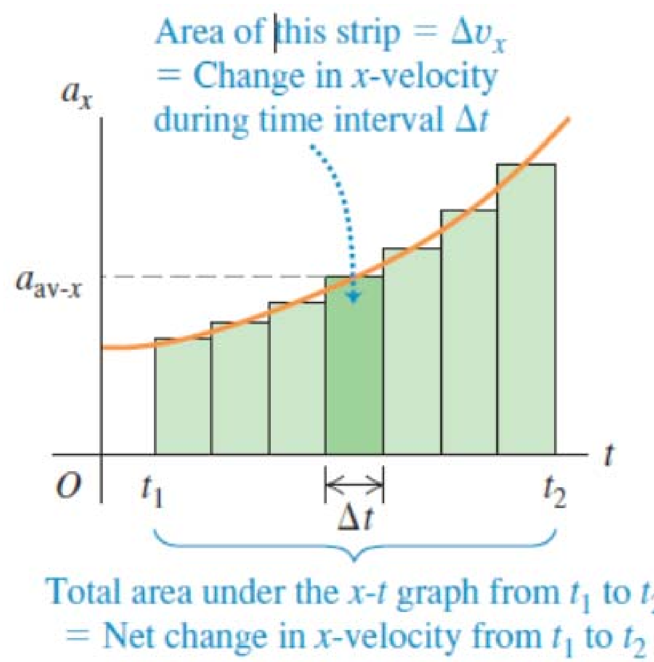
A ball thrown directly upward from ground level and reaches a maximum height of h. If we measure h, then find v_0.



Graph relationships: Calculus in a nutshell

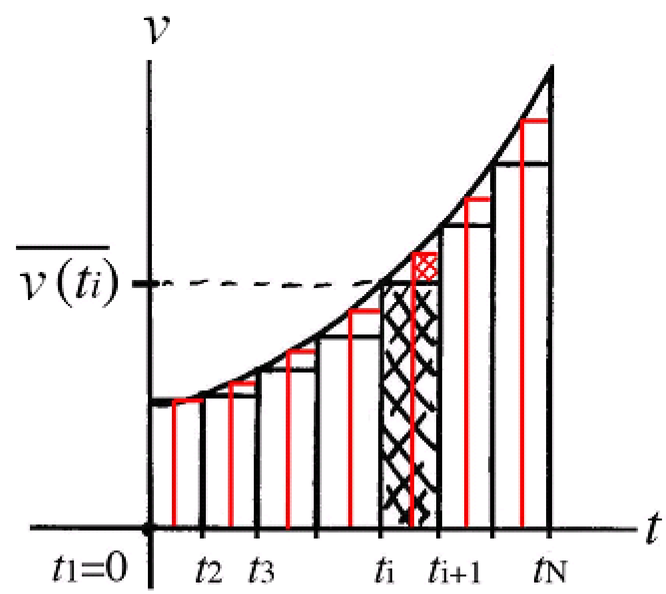
- For functions of a single variable, the **derivative** is the slope of a line.
- For functions of a single variable, the **integral** is the area under the curve between two specified values of the independent variable.





Position as the integral of the velocity

The area under the graph of the *x*-component of the velocity vs. time is the change in position



$$\int_{t'=0}^{t'=t} v(t')dt' \equiv \lim_{\Delta t_i \to 0} \sum_{i=1}^{i=N} v(t_i)\Delta t_i = Area(v,t)$$

$$\int_{t'=0}^{t'=t} v(t')dt' = \int_{t'=0}^{t'=t} \frac{dx}{dt}dt' = \int_{x'=x(t=0)}^{x'=x(t)} dx' = x(t) - x_0$$

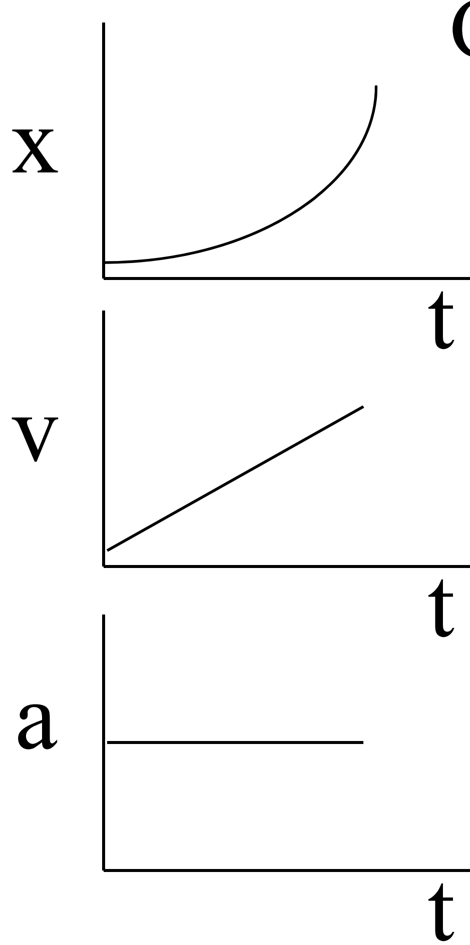
Relationships between graphs

Slope of the Line Calculates:

Velocity

Acceleration

Jerk



Area Under The Curve Calculates:

Nothing useful to us

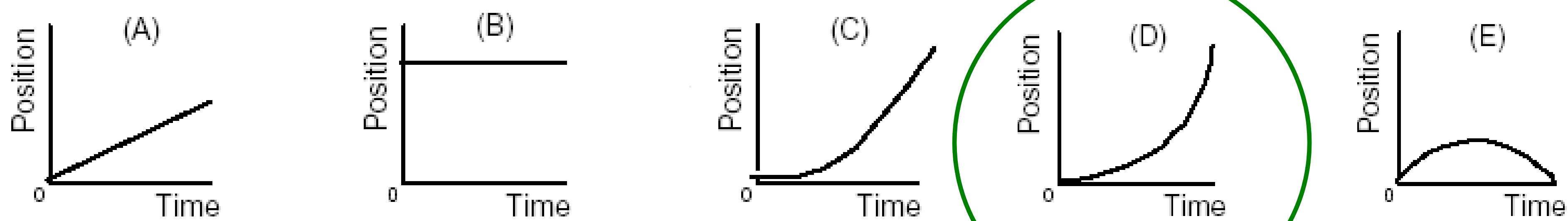
Displacement

Change in Velocity

The area under a velocity-versus-time graph of an object is

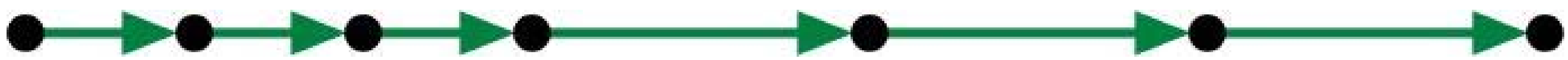
A. the object’s speed at that point.
B. the object’s acceleration at that point.
C. the distance traveled by the object.
D. the displacement of the object.
E. This topic was not covered in this chapter.

Position versus time graphs for five objects are shown below. All axes have the same scale. Which object had the highest instantaneous velocity during the interval?

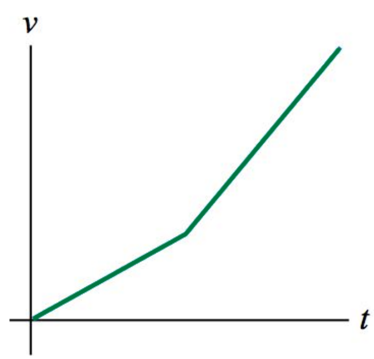


1 2 3 4 5

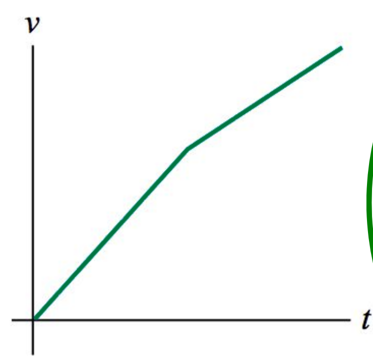
Here is a motion diagram of a car moving along a straight stretch of road:



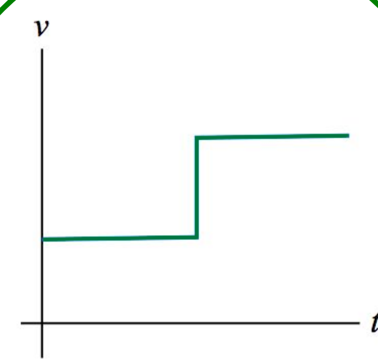
Which of the following *velocity*-*versus*-*time* graphs matches this motion diagram?



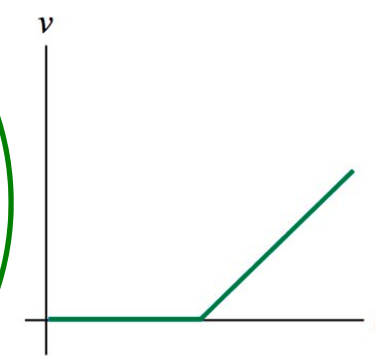
A.



B.

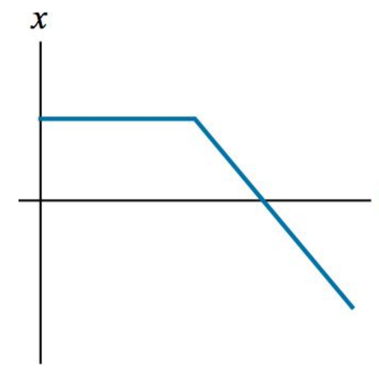


C.

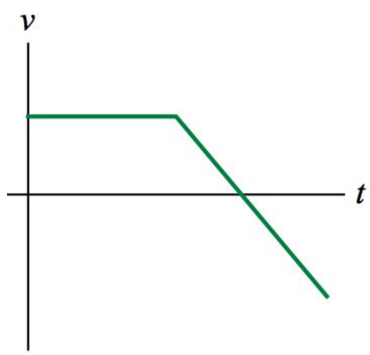


D.

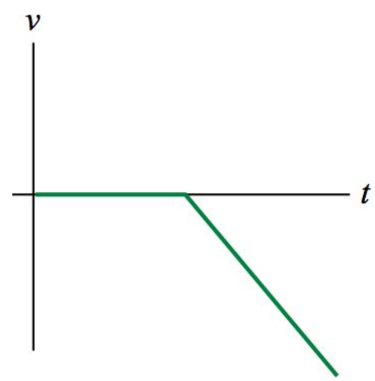
A graph of position versus time for a basketball player moving down the court appears like so:



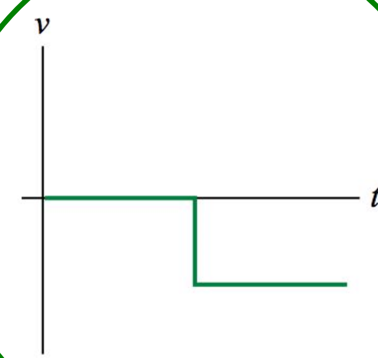
Which of the following velocity graphs matches the above position graph?



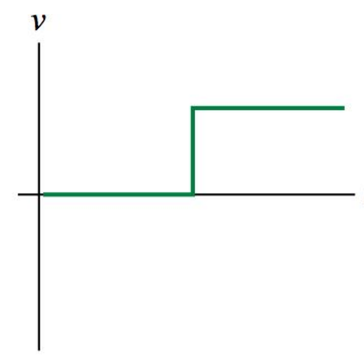
A.



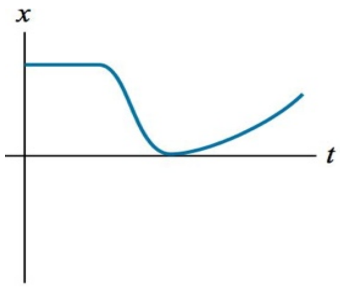
B.



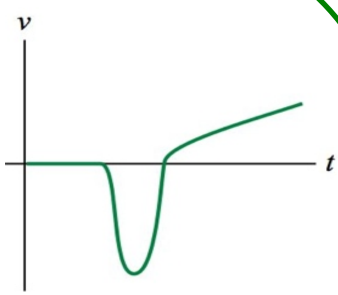
C.



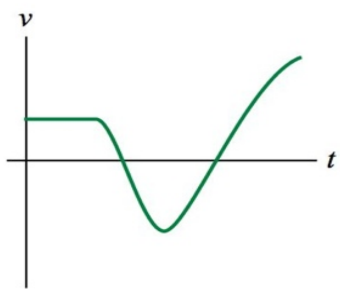
D.



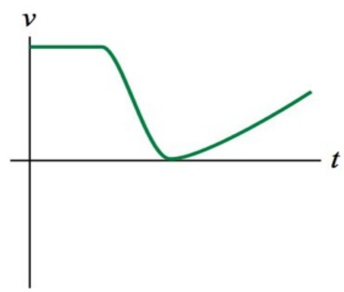
A particle moves with the position-versus-time graph shown at left. Which graph best illustrates the velocity of the particle as a function of time?



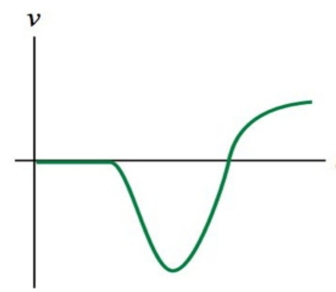
A.



B.

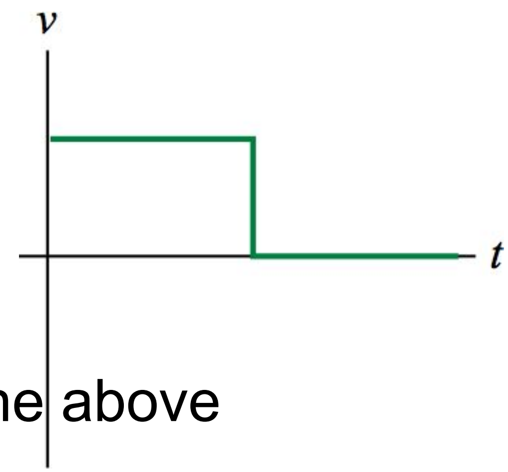


C.

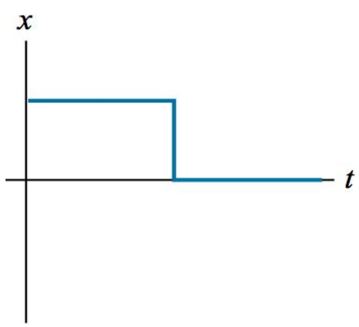


D.

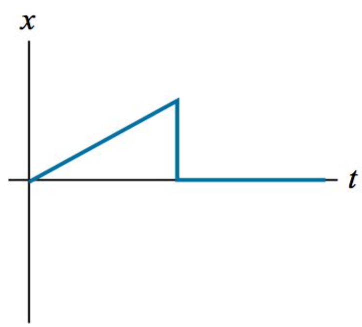
A graph of velocity versus time for a hockey puck shot into a goal appears like so:



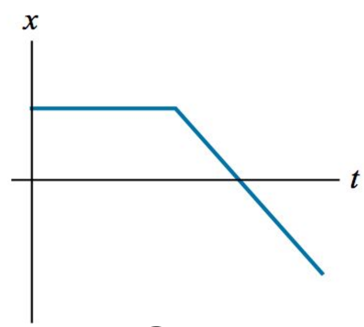
Which of the following position graphs matches the above velocity graph?



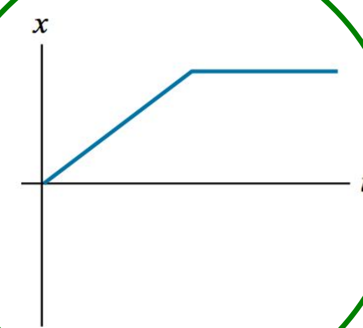
A.



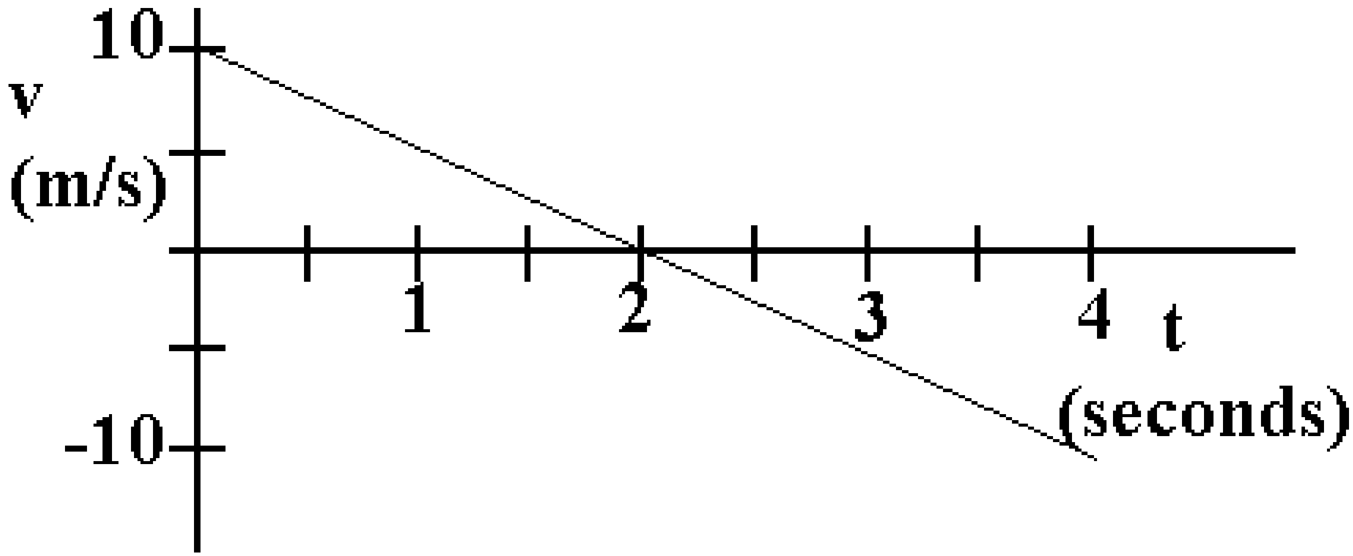
B.



C.



D.



Draw the position vs. time graph.

Mike jumps out of a tree and lands on a trampoline. The trampoline sags 2 feet before launching Mike back into the air. At the very bottom, where the sag is the greatest, Mike's acceleration is:

A. Upward

A. Downward

A. Zero

Motion under a time-dependent acceleration

Assume we know the time dependence of the acceleration, $a_x(t)$

The velocity can be calculated from the acceleration knowing $a_x = \frac{dv_x}{dt}$, which can be rewritten to give

$$v_x(t) = v_{0x} + \int_0^t a_x(t')\, dt'$$

The position can be calculated from the velocity knowing $v_x = \frac{dx}{dt}$, which can be rewritten to give

$$x(t) = x_0 + \int_0^t v_x(t')\, dt'$$

Finding the velocity and acceleration from a time-dependent position

Assume instead that we know the time dependence of the position, $x(t)$

The time-dependent velocity can be calculated by finding the slope of $x(t)$

$$v_x = \frac{dx}{dt}$$

The time-dependent acceleration can be calculated by finding the slope of $v_x(t)$

$$a_x = \frac{dv_x}{dt}$$

EXAMPLE

The acceleration is given by the equation $a_x(t) = \alpha - \beta t + \gamma t^3$

(a) Answer the following

(i) What are the units of a_x?

(ii) What are the units of α?

(iii) What are the units of β?

(iv) What are the units of γ?

(b) Find the equation for the velocity as a function of time.

(c) Find the equation for the position as a function of time.