Chapter 6: Work & Kinetic Energy

The work done on an object from a constant force is given by

$$W_i = F_i d \cos \alpha_i$$

In words, the above equation is stated, "the work from a constant applied force is equal to the magnitude of the force vector on the object multiplied by the magnitude of the displacement vector of the object and the cosine of the angle between them."

The total work done on an object subject to constant forces is the sum of all the work

$$\begin{aligned} W_{\text{tot}} &= (ma)\, d \cos \alpha \\ &= \sum_{i=1}^{N} F_i d \cos \alpha_i \\ &= \sum_{i=1}^{N} W_i \end{aligned}$$

Work

The work done on an object from a force is given by

$$W_i = \int_{s_1}^{s_2} \vec{F}_i \cdot d\vec{s}$$

The total work done on an object is the sum of the work done on an object by each force.

$$\begin{aligned} W_{\text{tot}} &= \int_{s_1}^{s_2} (m\vec{a}) \cdot d\vec{s} \\ &= \int_{s_1}^{s_2} \left(\sum_{i=1}^{N} \vec{F}_i \right) \cdot d\vec{s} \\ &= \sum_{i=1}^{N} W_i \end{aligned}$$

The work done by a force depends on the direction traveled relative to the direction of the force,

$$W_i = \int_{s_1}^{s_2} \vec{F}_i \cdot d\vec{s}$$
$$= \int_{s_1}^{s_2} \cos(\alpha_i)\ F_i\, ds$$

where α_i is the angle between the force vector and the infinitesimal displacement vector.

Example: work done by kinetic friction

The force of kinetic friction, $\vec{F}_k$, always opposes the direction of motion. Therefore,

$$W_k = \int_{s_1}^{s_2} \vec{F}_k \cdot d\vec{s}$$
$$= \int_{s_1}^{s_2} \cos(180^\circ)\ F_k\, ds$$
$$= -\int_{s_1}^{s_2} \mu_k N\, ds$$

Work by a constant force

When the force doing work is a constant, then

$$
\begin{aligned}
W_i &= \int_{s_1}^{s_2} \vec{F}_i \cdot d\vec{s} \\
&= \int_{s_1}^{s_2} \cos(\alpha_i)\ F_i\, ds \\
&= F_i \int_{s_1}^{s_2} \cos(\alpha_i)\ ds
\end{aligned}
$$

When the motion is along a straight line, the angle is constant, and the equation reduces to

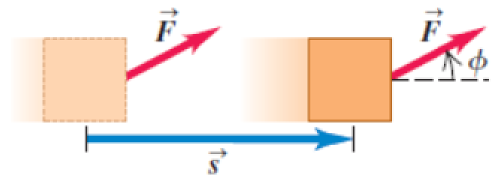
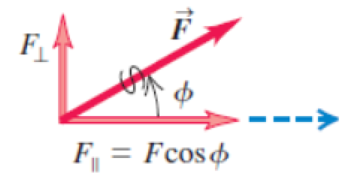
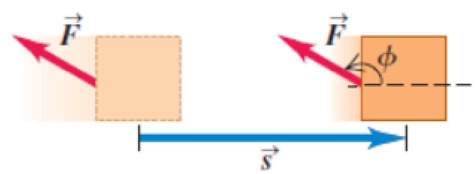
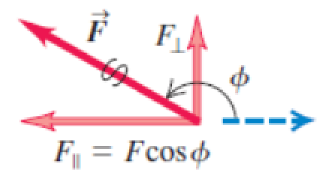
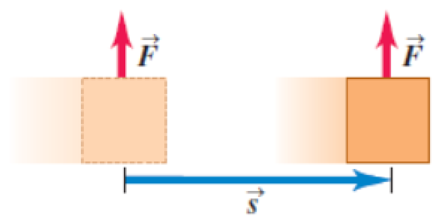
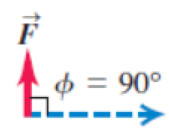
$$W_i = F_i \cos(\alpha_i)\,(s_2 - s_1)$$

For constant, conservative forces, only the initial and final positions are relevant,

$$W_i = F_i d \cos(\theta)$$

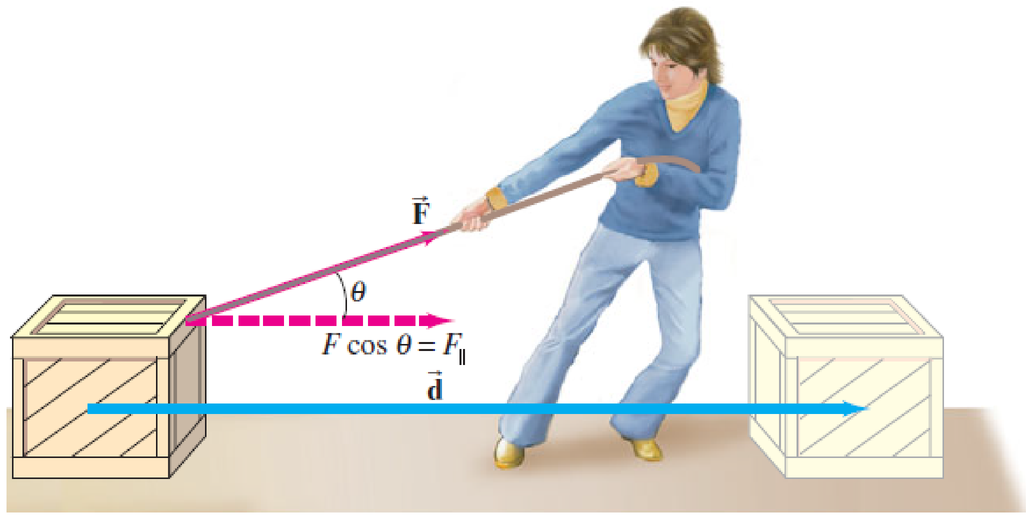
where d is the displacement between the initial and final positions and θ is the angle between the force vector and the displacement vector.

Work depends on the direction of a constant force and the displacement

Direction of Force (or Force Component)	Situation	Force Diagram
(a) **Force $\vec{F}$ has a component in direction of displacement:** $W = F_{\parallel}s = (F\cos\phi)s$ Work is *positive*.		
(b) **Force $\vec{F}$ has a component opposite to direction of displacement:** $W = F_{\parallel}s = (F\cos\phi)s$ Work is *negative* (because $F\cos\phi$ is negative for $90° < \phi < 180°$).		
(c) **Force $\vec{F}$ (or force component $F_{\perp}$) is perpendicular to direction of displacement:** The force (or force component) does *no* work on the object.		

Example: Work from a constant force

A 380-kg piano slides 2.9 m down a 25° incline and is kept from accelerating by a man who is pushing back on it *parallel to the incline* (Fig. 6–36). Determine: (*a*) the force exerted by the man, (*b*) the work done on the piano by the man, (*c*) the work done on the piano by the force of gravity, and (*d*) the net work done on the piano. Ignore friction.

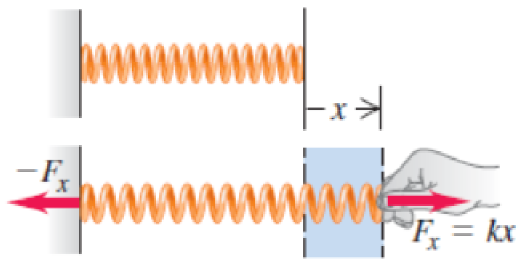


A person moves a 5.00kg box 10.0m by pulling on a rope making an angle of 30.0^{o} above the horizontal. The floor and box have a coefficient of kinetic friction of 0.500 and she pulls with a force of 30.0N.

(a) How much work does she do?
(b) How much work is done by gravity?
(c) How much work is done by friction?

Work done to stretch a spring
(non-constant force)

The force needed to stretch an ideal spring is proportional to the spring's elongation: $F_x = kx$.

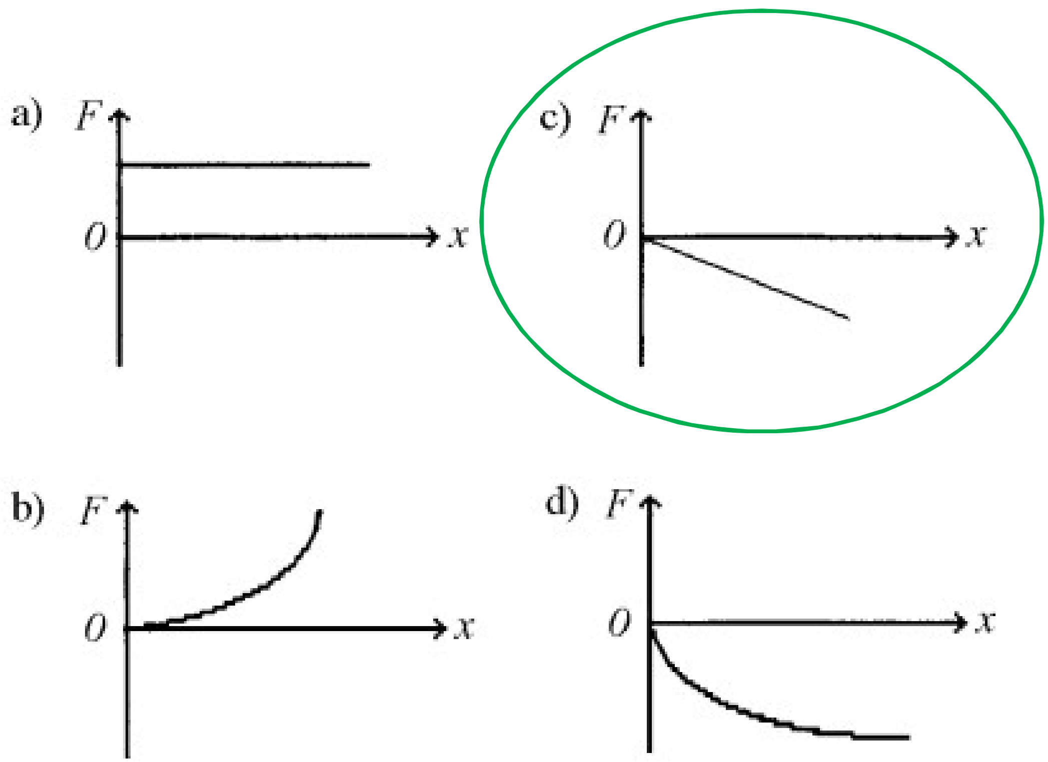


For simplicity we set $x_{eq} = 0$.

The stretching force is opposite of the spring's force, $F_s = -kx$

$$
\begin{aligned}
W_{\text{stretch}} &= \int_{x_1}^{x_2} \left[-F_s(x)\right] dx \\
&= \int_{x_1}^{x_2} kx\, dx \\
&= \frac{1}{2}kx^2\Big|_{x_1}^{x_2} \\
&= \frac{1}{2}k\left(x_2^2 - x_1^2\right)
\end{aligned}
$$

Which of the graphs in the figure illustrates Hooke's Law?



Example: work done by a spatially dependent force in one dimension

Let us assume that an object is subject to the force

$$F(x) = 5.0\,\mathrm{N} - \left(2.0\,\frac{\mathrm{N}}{\mathrm{m}}\right)x + \left(1.0\,\frac{\mathrm{N}}{\mathrm{m}^2}\right)x^2$$

(a) Calculate the work done by the force from $x = 0.0$ m to $x = 2.0$ m.

(b) Calculate the work done by the force from $x = 2.0$ m to $x = 4.0$ m.

Translational kinetic energy

The translational kinetic energy of a moving object relative to a stationary point is one-half the mass multiplied by the velocity squared. Symbolically, the kinetic energy may be written as

$$K = \frac{1}{2}mv^2$$

Another important quantity that we will often use is the *change* in translational kinetic energy of an object. We may write the change in translational kinetic energy equation as

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

Work and kinetic energy (for constant *a*)

We previously discovered that the total work done on an object from a constant force is given by

$$W_{\text{tot}} = (ma)\, d \cos\alpha$$

For simplicity, let us assume that the sum of the forces are along the direction of motion for an object traveling in one-dimension, or

$$W_{\text{tot}} = ma\, d$$

Our time-independent one-dimensional kinematic equation for constant acceleration was given as

$$2ad = v^2 - v_0^2$$

Solving for the acceleration and substituting into the expression for W_{tot} gives

$$W_{\text{tot}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

Therefore,

$$W_{\text{tot}} = \Delta K$$

Work-kinetic energy theorem (general derivation)

Starting with the definition of acceleration in one-dimension, we have

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = v_x\frac{dv_x}{dx}$$

We also know that the work is defined as

$$W_{\text{tot}} = \int F_x dx = \int (ma_x)\, dx$$

Substituting in the first definition into the second gives

$$W_{\text{tot}} = \int F_x dx = \int m v_x \frac{dv_x}{dx} dx = \int_{v_{x0}}^{v_x} m v_x' dv_x' = \frac{1}{2} m \left(v_x^2 - v_{x0}^2\right)$$

If the net work done on an object is zero, what can you determine about the object's kinetic energy?

A. The object's kinetic energy is zero.

B. The object's kinetic energy is increasing.

C. The object's kinetic energy is decreasing.

D. The object's kinetic energy remains the same.

If the net work done on an object is positive, what can you conclude about the object's motion?

A. The object is at rest; its position is constant.

B. The object is slowing down.

C. The object is speeding up.

D. The object is moving with a constant velocity.

Consider two carts, of masses *m* and 2*m*, at rest on an air track. If you push first one cart for 3 s and then the other for the same length of time, exerting ***equal*** force on each, the kinetic energy of the light cart is

1. larger than
2. equal to
3. smaller than

the kinetic energy of the heavy car.

Power

Power is the time rate of doing work. The average power is the amount of work done in time divided by that time.

$$P_{\text{avg}} = \frac{\Delta W}{\Delta t}$$

The instantaneous power is the limit of the average power as Δt approaches zero.

$$\begin{aligned} P &= \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} \\ &= \frac{dW}{dt} \end{aligned}$$

When a force acts on a particle moving with a velocity, the instantaneous power is the scalar product of $\vec{F}$ and $\vec{v}$.

$$P = \vec{F} \cdot \vec{v}$$

When its 75 kW (100-hp) engine is generating full power, a small single-engine airplane with mass 700 kg gains altitude at a rate of 2.5 m/s. What fraction of the engine power is being used to make the airplane climb?

(The remainder is used to overcome the effects of air resistance and of inefficiencies in the propeller and engine.)