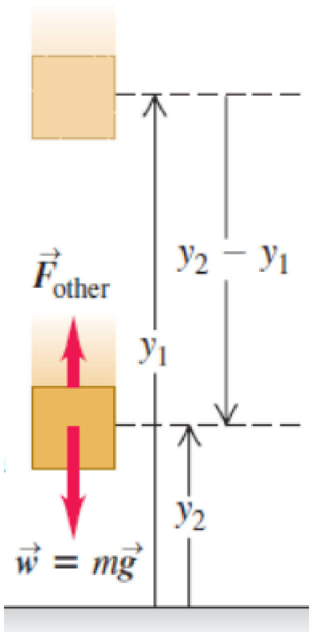
Chapter 7: Potential Energy & Energy Conservation

Gravitational potential energy, U_{grav}

The weight of an object is constant near the surface of Earth.

$$\left|\vec{F}_{\text{grav}}\right| = mg$$

The work done by gravity to move an object from a height y_1 to y_2 is



$$\begin{aligned} W_{\text{grav}} &= \int \vec{F}_{\text{grav}} \cdot d\vec{s} \\ &= -mg \int_{y_1}^{y_2} dy \\ &= -mg\left(y_2 - y_1\right) \\ &= mgy_1 - mgy_2 \\ &= U_{\text{grav},1} - U_{\text{grav},2} \\ &= -\Delta U_{\text{grav}} \end{aligned}$$

which is equal to the negative change in potential energy.

Thus, the change in the gravitational potential energy is,

$$\Delta U_{\text{grav}} = mg\left(y_2 - y_1\right)$$

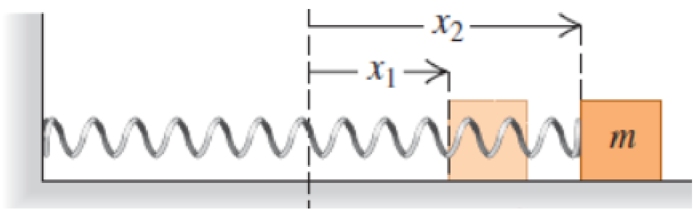
The potential energy of a spring, U_{spring}

The force of a spring with $x_{\text{eq}} = 0$ is given by

$$F_s = -kx$$

The work done by a spring to change length from x_1 to x_2 is

$$\begin{aligned} W_{\text{spring}} &= \int \vec{F}_s \cdot d\vec{s} \\ &= -k \int_{x_1}^{x_2} x \, dx \\ &= -k \left(\frac{1}{2} x_2^2 - \frac{1}{2} x_1^2 \right) \\ &= \frac{1}{2} k x_1^2 - \frac{1}{2} k x_2^2 \\ &= U_{\text{spring},1} - U_{\text{spring},2} \\ &= -\Delta U_{\text{spring}} \end{aligned}$$



Thus, the change in the spring's potential energy is

$$\Delta U_{\text{spring}} = \frac{1}{2} k \left[(x_2)^2 - (x_1)^2 \right]$$

Internal energy

The change in the internal energy of a system is given as

$$\Delta U_{\rm int} = U_{\rm int,\, final} - U_{\rm int,\, initial}$$

Note that the internal energy can be changed by several phenomena.

Examples are from dissipative forces such as friction

$$\Delta U_{\rm int,\, friction} = -W_{\rm friction}$$

and explosions

$$\begin{aligned} \Delta U_{\rm int,\, explosion} &= -W_{\rm explosion} \\ &= -Q \end{aligned}$$

where Q is the energy released from an explosion

Conservation of energy

The change in the energy of a system is given by

$$\Delta E = \Delta K + \Delta U_{\text{int}} + \Delta U_g + \Delta U_s + \cdots = W_{\text{ext}}$$

In words, this is stated as “The total change in energy of a system is equal to the sum of the changes in the internal energy, the kinetic energy, and the potential energy, where this sum is equal to the total external work done on the system.

You put fresh batteries into a flashlight. Then you turn it on and leave it on until the bulb gradually dims and finally goes out. Which statement best describes the involvement of energy in this process?

1. The energy has been used up by the bulb and no longer exists anywhere.
2. All the energy that the batteries originally had when new still exists somewhere.
3. The energy of the batteries was converted to heat by the bulb, so it no longer exists.
4. The amount of energy in the flashlight, batteries, and bulb remain the same, because energy is conserved.
5. The energy has moved from one end of the batteries to the other.

Conservation of mechanical energy

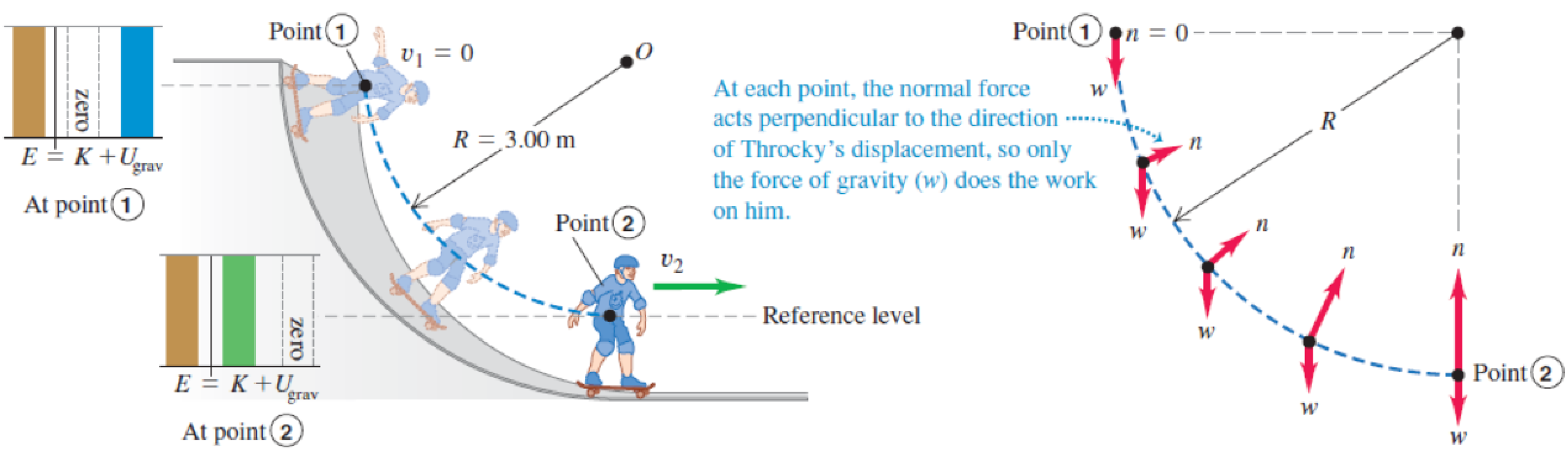
When there is no net work done on a system and no change in the internal energy, then

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s + \cdots = 0$$

In words, this is stated as "when mechanical energy is conserved, the total change in energy is zero. Therefore, the sum of the changes in the kinetic energy and the potential energies is zero."

Conservative transfer of mechanical energy

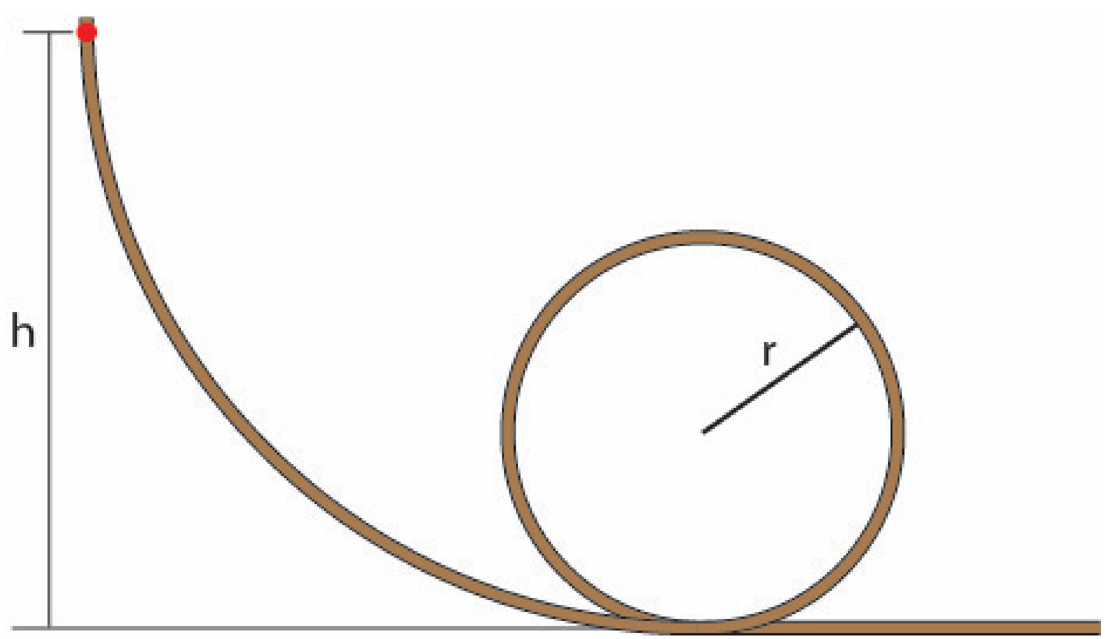
(quarter pipe)



Example problem (classical: loop-the-loop)

A object of mass 2.50 kg on a frictionless track is released from rest at a height of 12.0 m. There is a loop of radius 1.50 m after the track becomes level as shown in the below diagram.

(a) What is the magnitude of the velocity of the object at the top of the loop-the-loop?
(b) What is the normal force (effective weight) of the object at the top of the loop-the-loop?



Change in potential energy of a conservative force

The conservation of mechanical energy for a system with a conservative force is given by

$$\Delta U + \Delta K = 0$$

Using the work energy theorem, $W = \Delta K$, we substitute the work for the change in the kinetic energy,

$$\Delta U = -W \longrightarrow dU = -\vec{F} \cdot d\vec{s}$$

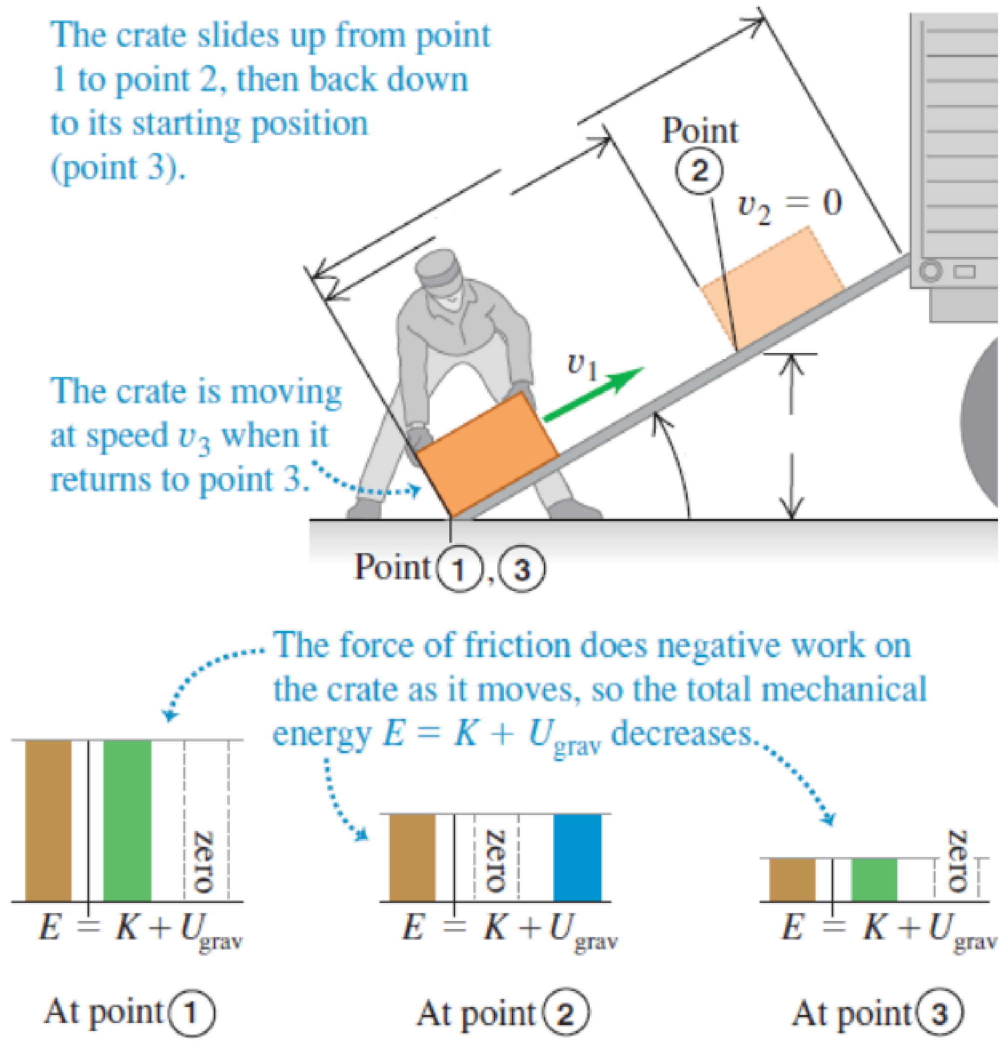
Therefore, we may write the force in terms of the potential

$$\vec{F} = -\nabla U$$

where $\nabla = \hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}$ in Cartesian coordinates.

Non-conservative transfer of energy

(sliding up and down a ramp with friction)



Complete the following statement: A force that acts on an object is said to be *conservative* if

1. it obeys Newton's laws of motion.

2 it results in a change in the object's kinetic energy.

3 it always acts in the direction of motion of the object.

4 the work it does on the object is independent of the path of the motion.

5 the work it does on the object is equal to the increase in the object's kinetic energy.

If you raise an object to a greater height at constant velocity, you are increasing the

A. kinetic energy.

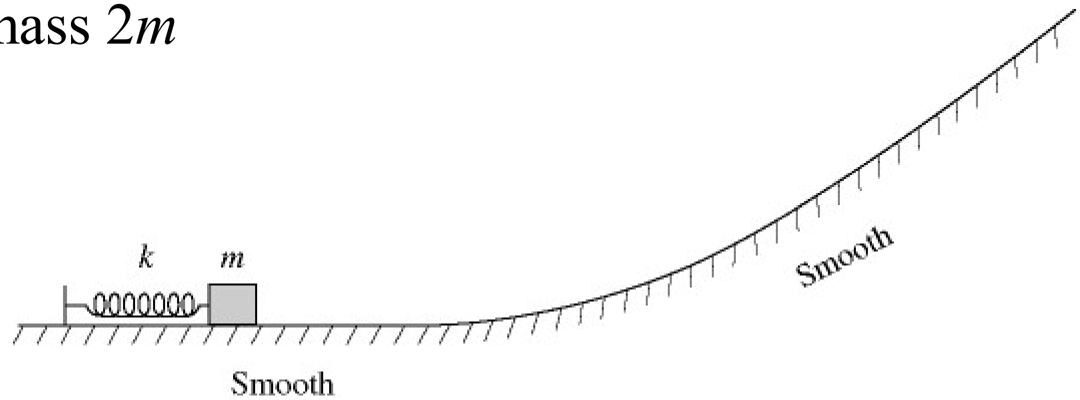
B. heat.

C. potential energy.

D. chemical energy.

E. thermal energy.

A box of mass m is pressed against (but is not attached to) an ideal spring of force constant k and negligible mass, compressing the spring a distance x. After it is released, the box slides up a frictionless incline as shown in the figure and eventually stops. If we repeat this experiment with a box of mass $2m$



A. both boxes will reach the same maximum height on the incline.

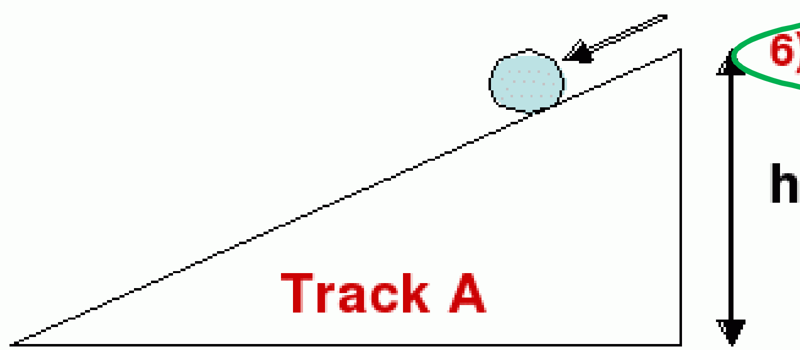
B. both boxes will have the same speed just as they move free of the spring.

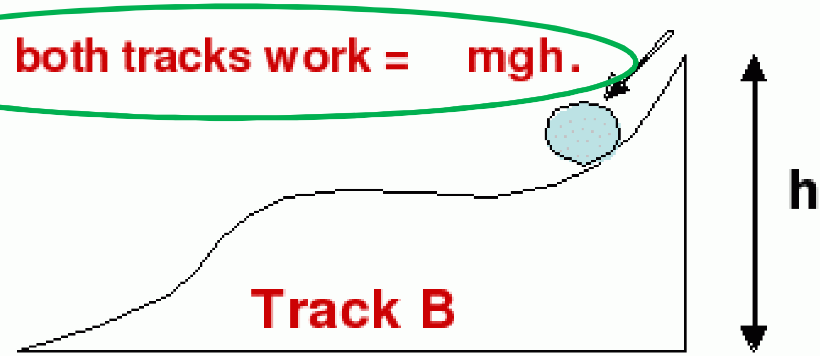
C. the lighter box will go twice as high up the incline as the heavier box.

D. just as it moves free of the spring, the lighter box will be moving twice as fast as the heavier box.

Two Identical marbles of mass m start at height h and roll down two different tracks, as shown. What is the magnitude of the work done by gravity on the marble as it rolls from height h to the ground on each track?

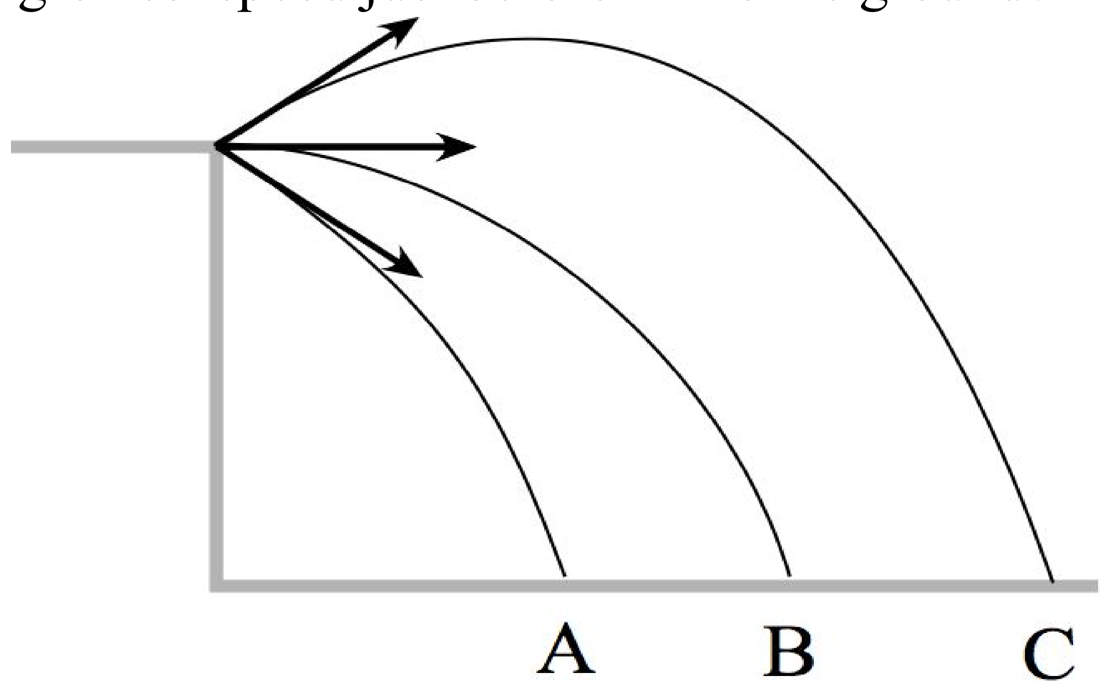
1) On track A, work = mgh. On track B, work < mgh.
2) On track A, work < mgh. On track B, work = mgh.
3) On track A, work > mgh ; On track B, work = mgh.
4) On both tracks work < mgh.
5) On both tracks work > mgh.
6) On both tracks work = mgh.



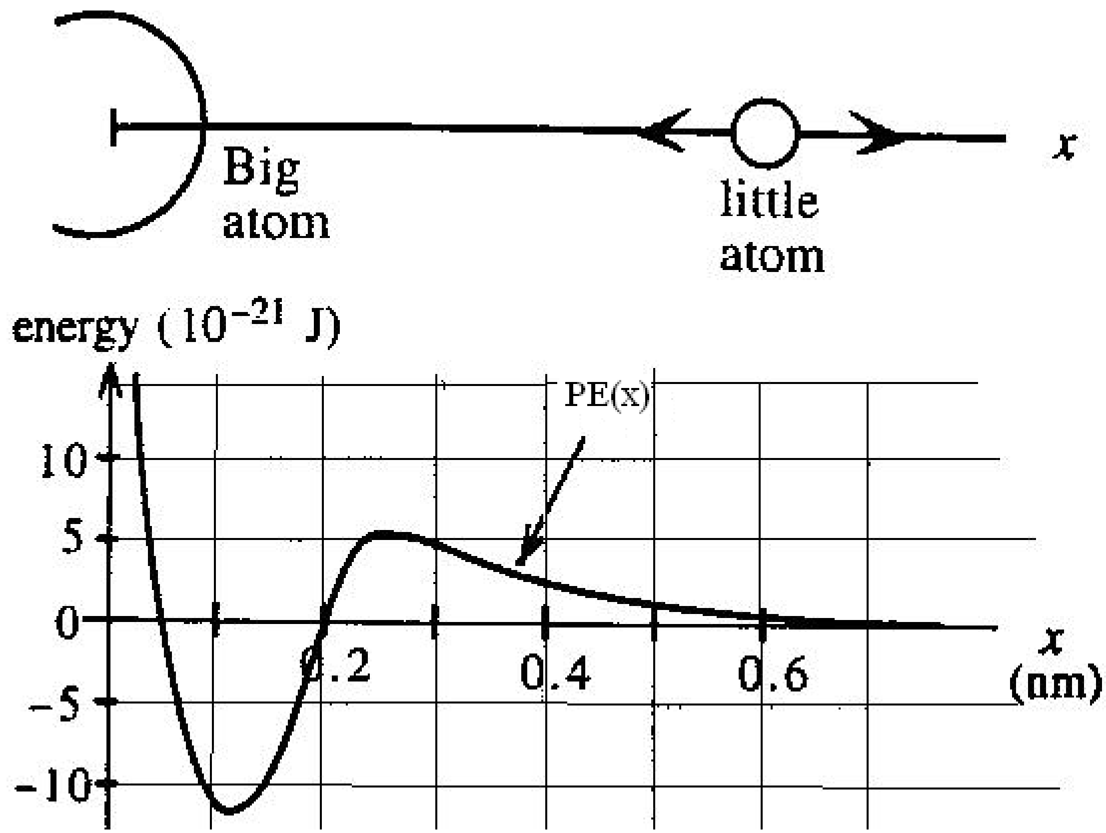


Three balls are thrown off a cliff with the same speed, but in different directions. Which ball has the greatest speed just before it hits the ground?

A. Ball A
B. Ball B
C. Ball C
D. All balls have the same speed



Two atoms interacting with each other have the potential energy graph shown. Assume the big atom's mass is much greater than the little atom's and that all the motion is along x.



Imagine that the little atom approaches the big atom from infinity with an initial $K = 5 \times 10^{-21}$ J. How close to the big atom does it get? (nm=nanometer=10^{-9} m)

(i) $x = 0$ (ii) $x = 0.04$ nm (iii) $x = 0.2$ nm

(iv) $x = 0.3$ nm (v) other (specify)

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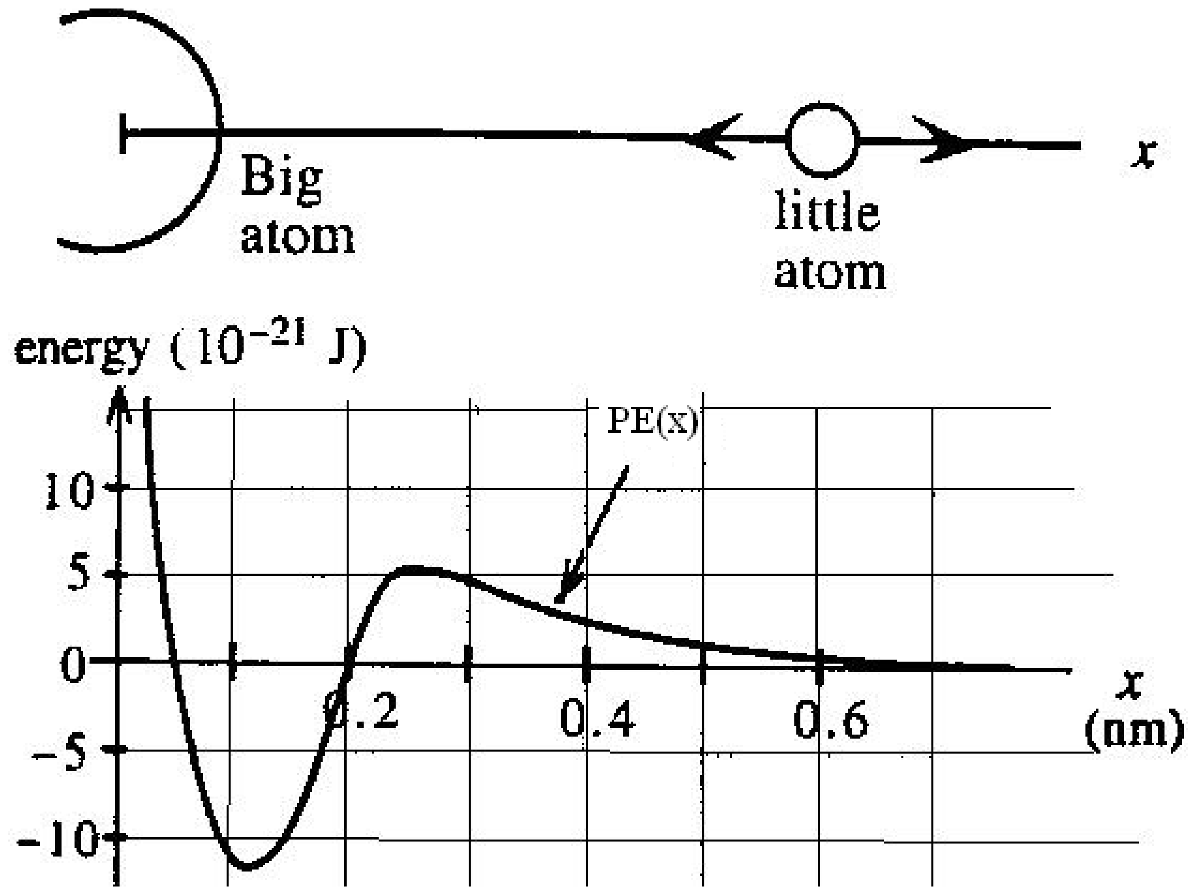


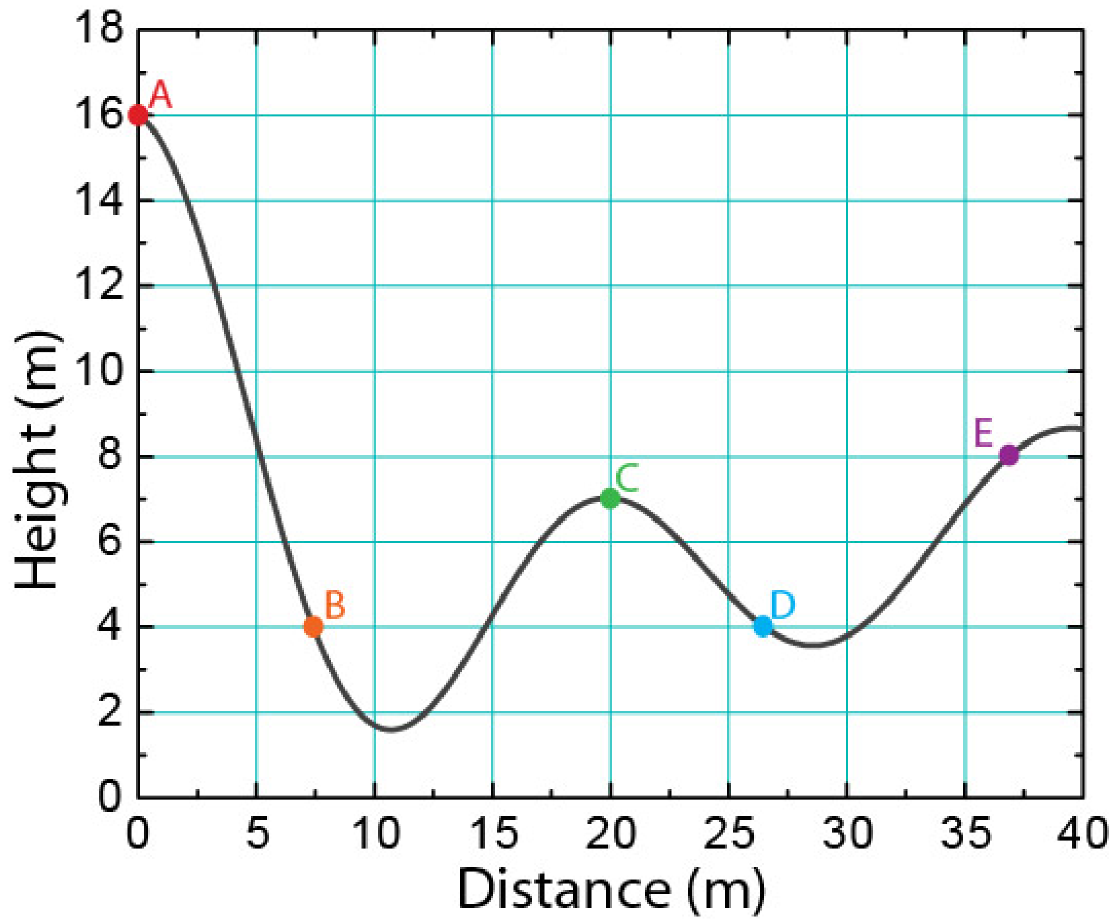
Imagine that at t=0, the little atom is at position x = 0.11 nm and has a kinetic energy of 5 x 10^{-21} J. About how much energy would we have to add to break the bond?

(1) 1 x 10^{-21} J (2) 6 x 10^{-21} J (3) 12 x 10^{-21}

(4) none, the bond is already broken.

A bead of mass 0.150 kg slides without friction on a track as shown in the diagram.

The bead initially starts from rest at point A.



(a) Determine the change in the gravitational potential energy from point A to point B.

(b) Determine the work done by gravity from point A to point B.

(c) Determine the change in kinetic energy from point C to Point D.

(d) Determine the kinetic energy at point E.

(e) Determine the velocity at point E.

Leonard Jones potential

The modified Leonard-Jones potential is a common semi-empirical model in solid state physics, chemistry and materials science. The potential has the form

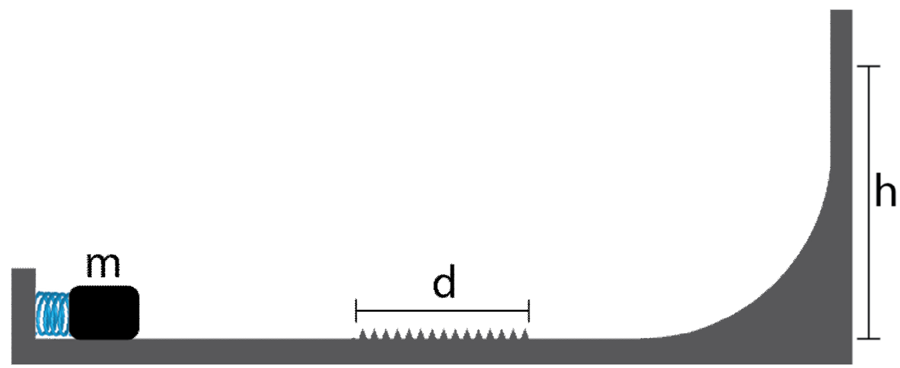
$$U(r) = A\left(\frac{B^{12}}{r^{12}} - 2\frac{B^{6}}{r^{6}}\right) + C$$

This potential estimates the attractive potential between polarizable atoms/molecules.

<u>Problem</u>

If we take A = 2.65 kJ/mol, B = 1.03 nm (where [nm] = nanometer), and C = 0.7 kJ/mol for a system of two noble atoms, then what is the distance between the nobel atom and the adjacent nobel atom when in equilibrium?

A block of mass m = 0.50 kg is connected to a spring on a horizontal surface that is compressed by a distance of $\Delta x = 10$ cm. When the spring is released, the block moves with a velocity until it reaches a patch of length d = 0.25 m with a coefficient of kinetic friction, $\mu_k = 0.15$. The block then continues to moves up a frictionless ramp to a height of 0.35 m.



(a) Determine the kinetic energy of the block between the rough patch of friction and the ramp.

(b) Determine the work done on the block by the patch of friction.

(c) Determine the kinetic energy of the block between the spring and the patch of friction.

(d) Determine the spring constant.