Chapter 10: dynamics of rotational motion

Torque

A force acting on an object can affect its translational and rotational motion. The torque, $\vec{\tau}$, caused by a force, $\vec{F}$, applied to an object at a distance, $\vec{r}$, from the axis of rotation is given by

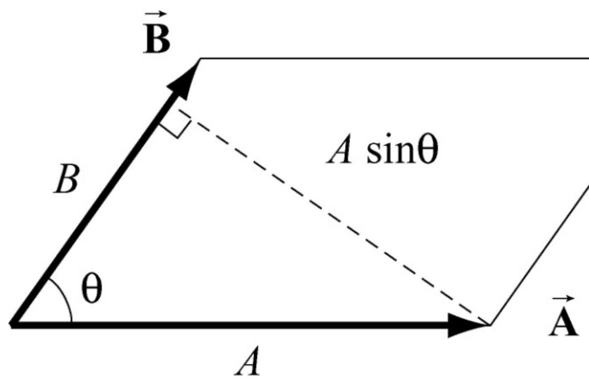
$$\vec{\tau} = \vec{r} \times \vec{F}$$

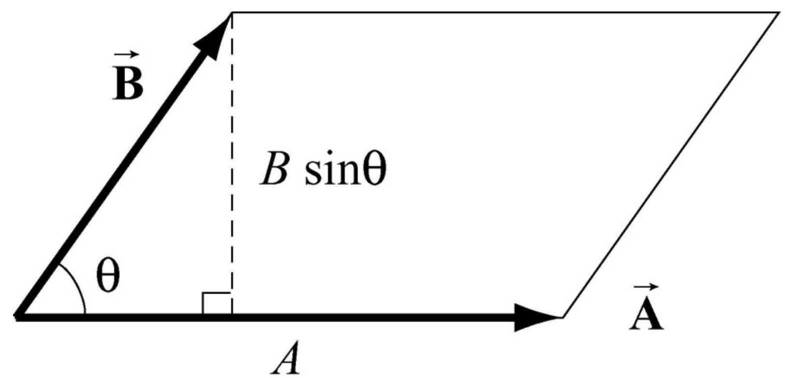
The magnitude of the torque may be calculated by multiplying the magnitude of the force applied with the magnitude of the distance and the sine of the angle between them,

$$\tau = rF\sin\theta$$

Cross product and the right-hand-rule

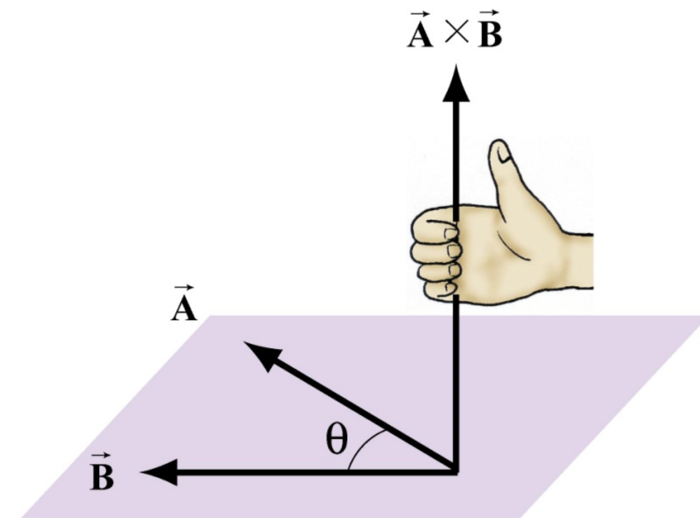
- Magnitude: $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = AB\sin\theta = A\left(B\sin\theta\right) = \left(A\sin\theta\right)B \quad (0 \le \theta \le \pi)$





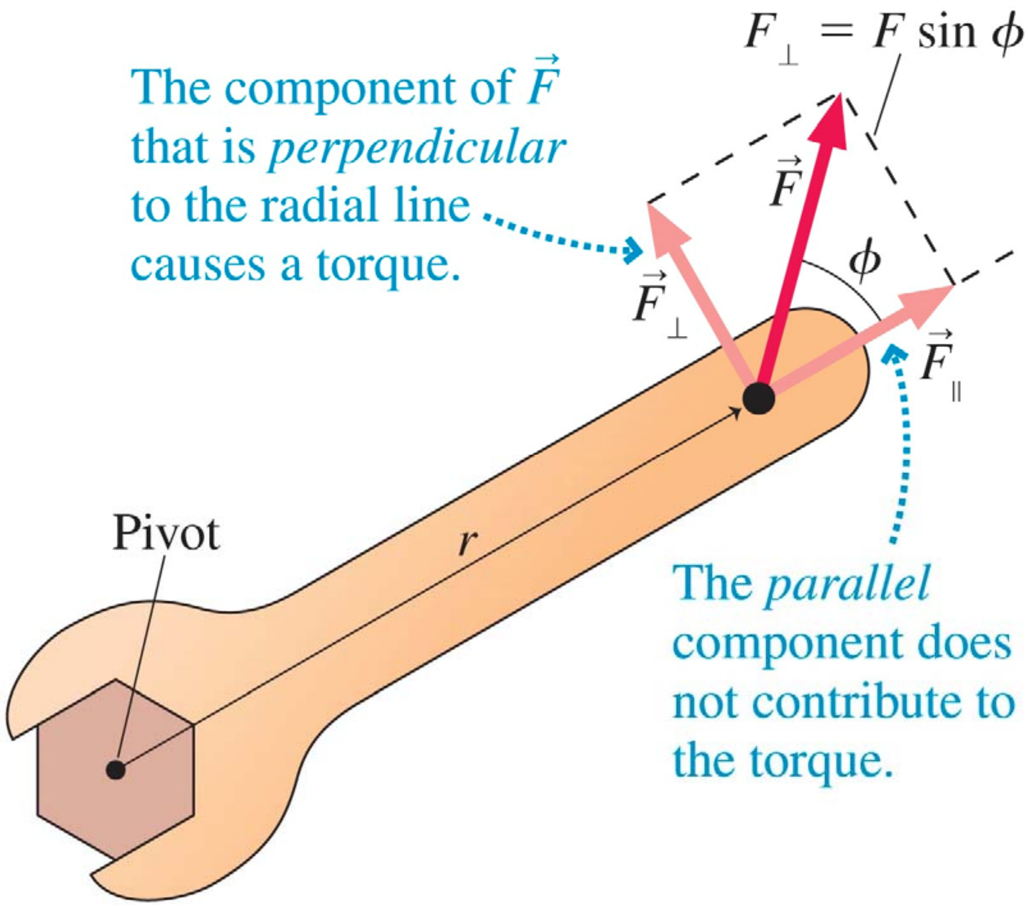
  - – area of the parallelogram

- Direction: determined by the Right-Hand-Rule



Interpretation of Torque

Torque is due to the component of the force *perpendicular* to the radial line.



$$\tau = rF_\perp = rF\sin\phi$$

Torque (cont.)

Just as force is proportional to the acceleration of a rigid object, the torque has a proportional relationship with the angular acceleration. We can split the applied force into tangential $F_\perp$ and radial F_r components. Note that only $F_\perp$ causes a torque, where $rF_r \sin 0^\circ = 0$. Knowing $F_\perp = ma$, we may rewrite our torque equation,

$$\tau = F_\perp r = mr^2\alpha$$

For rigid bodies, there are many mass elements that undergo an angular acceleration when applying the torque. Let us assume the torque causes an angular acceleration in the direction of the z-axis (rotating in the xy-plane). There is a torque on the ith element such that

$$\tau_{iz} = m_i r_i^2 \alpha_{iz} \quad \text{where } r^2 = x^2 + y^2$$

The sum of the torques on all of the mass elements in a rigid body is

$$\sum_i^N \tau_{iz} = \sum_i^N m_i r_i^2 \alpha_{iz}$$

Torque (cont.)

The sum of the torques on all of the mass elements in a rigid body is

$$\sum_i^N \tau_{iz} = \sum_i^N m_i r_i^2 \alpha_{iz}$$
$$= \left(\sum_i^N m_i r_i^2\right) \alpha_z$$

where $\alpha_{iz} = \alpha_z$ because <u>all</u> mass elements have the same angular acceleration about the axis of rotation in a rigid object.

We previously defined the moment of inertia of an object about the axis of rotation as $I = \sum_i^N m_i r_i^2$. If the rotation is about the principle axis in the *z*-direction, then

$$\tau_z = I_{zz} \alpha_z$$

$$(\tau = I\alpha)$$

Which factor does the torque on an object *not* depend on?

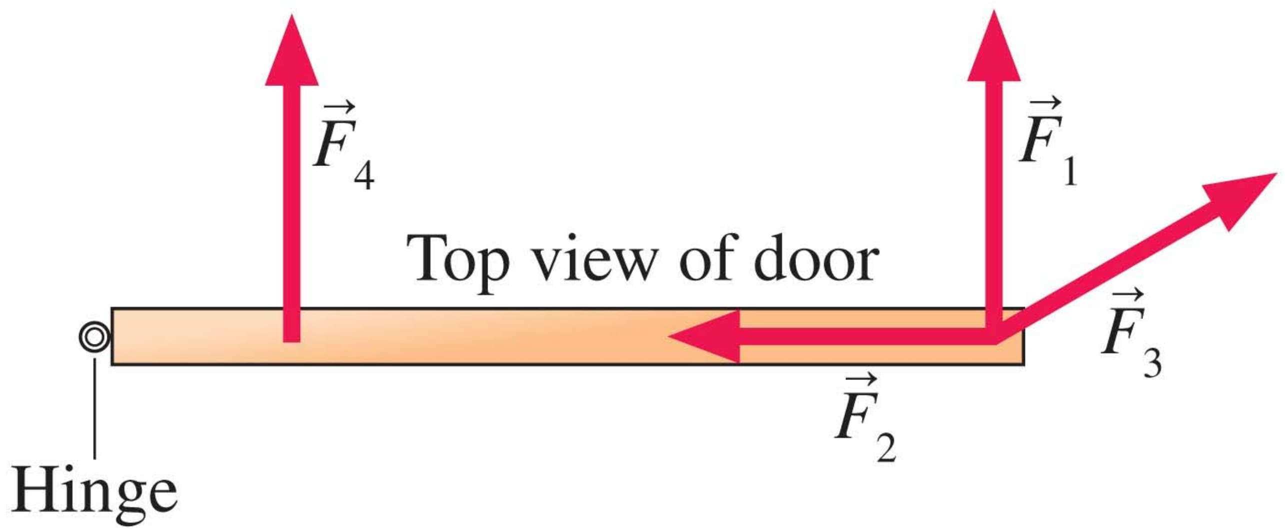
A. The magnitude of the applied force.
B. The object's angular velocity.
C. The angle at which the force is applied.
D. The distance from the axis to the point at which the force is applied.

A net torque applied to an object causes

A. a linear acceleration of the object.
B. the object to rotate at a constant rate.
C. the angular velocity of the object to change.
D. the moment of inertia of the object to change.

The four forces shown have the same strength. Which force would be most effective in opening the door?

A. Force F_1
B. Force F_2
C. Force F_3
D. Force F_4
E. Either F_1 or F_3



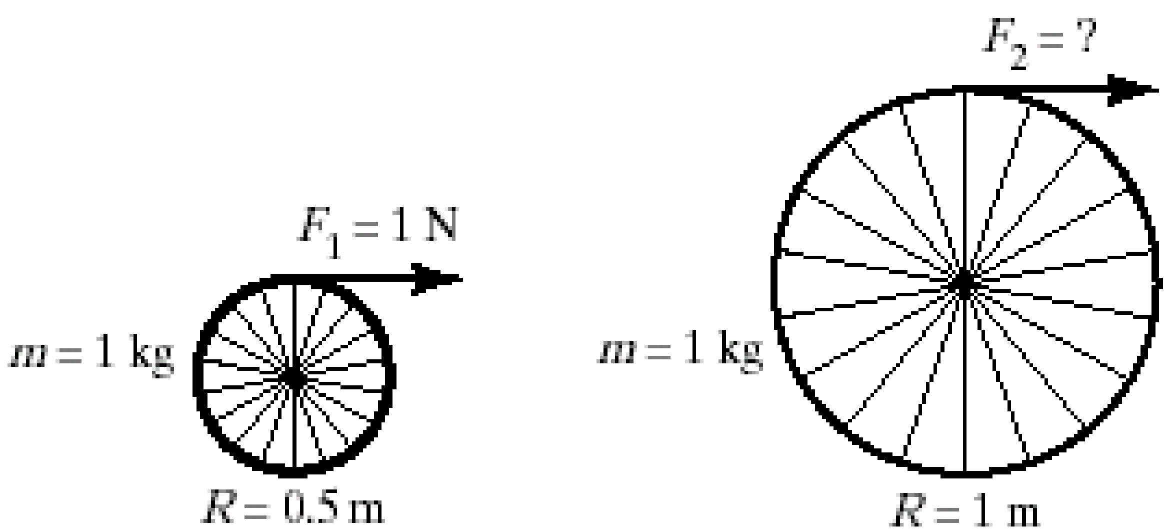
Two wheels with fixed hubs, each having a mass of 1 kg, start from rest, and forces are applied as shown. The larger wheel has twice the radius of the smaller wheel. Assume the hubs and spokes are massless, so that the rotational inertia is $I = mR^2$. In order to impart identical angular accelerations, how large is F_2?

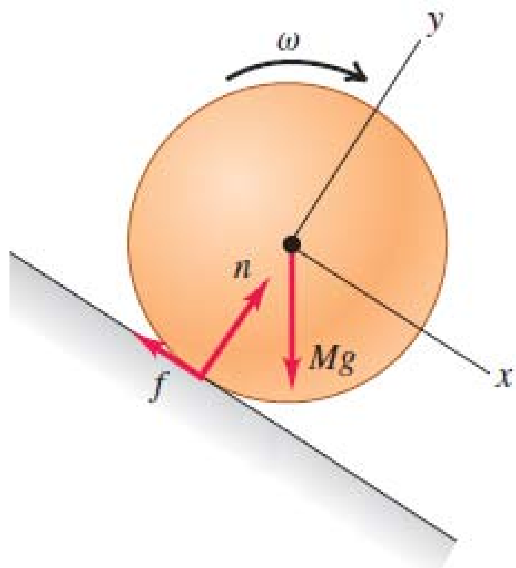
A. 0.25 N

B. 0.5 N

C. 1 N

D. 2 N





A solid sphere, $I_{\mathbf{sph}} = \frac{2}{5}MR^2$, with mass 2.10 kg rolls without slipping down

a slope angled at 32.0º.

(a) Find the acceleration.
(b) Find the friction force.
(c) Find the minimum coefficient of friction needed to prevent slipping.

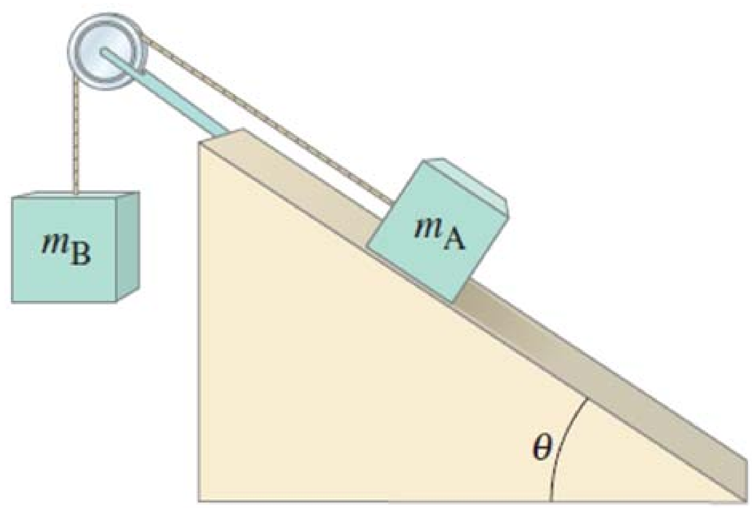
Two blocks of mass m_A and m_B are connected by a light string that passes over a cylindrical pulley of mass m_p. Mass A is on a frictionless incline at an angle θ from the horizontal as shown in the diagram. Assume there is no slipping between the string and the pulley.

(a) Draw free-body diagrams of both objects.

(b) Determine the magnitude of the acceleration of the blocks.

(c) Determine the tension in the string between m_A and the pulley.

(d) Determine the tension in the string between m_B and the pulley.



Work done by a torque

The work done by a torque on an object is given by

$$W_{\text{rot}} = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

We also know that $\tau = I\alpha$, and we can recast this equation as

$$W_{\text{rot}} = \int_{\theta_1}^{\theta_2} (I\alpha) \, d\theta$$

Using the definitions, $\alpha = \dfrac{d\omega}{dt}$ and $\omega = \dfrac{d\theta}{dt}$, the work may be rewritten as

$$W_{\text{rot}} = \int_{\omega_1}^{\omega_2} I\omega \, d\omega$$

For a rigid object <u>not changing shape</u>, $W_{\text{rot}} = \dfrac{1}{2} I \left(\omega_2^2 - \omega_1^2\right)$

Two cylinders of the same size and mass roll down an incline. Cylinder *A* has most of its weight concentrated at the rim, while cylinder *B* has most of its weight concentrated at the center. Which reaches the bottom of the incline first?

1. *A*
2. *B*
3. Both reach the bottom at the same time.

Angular momentum

The angular momentum is defined as $\vec{L} = \vec{r} \times \vec{p}$.

Just as linear momentum is related to the force momentum, angular momentum is related to the torque via

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

The angular momentum of a rigid object rotating about an axis can also be found by summing the contributions of all elements, where

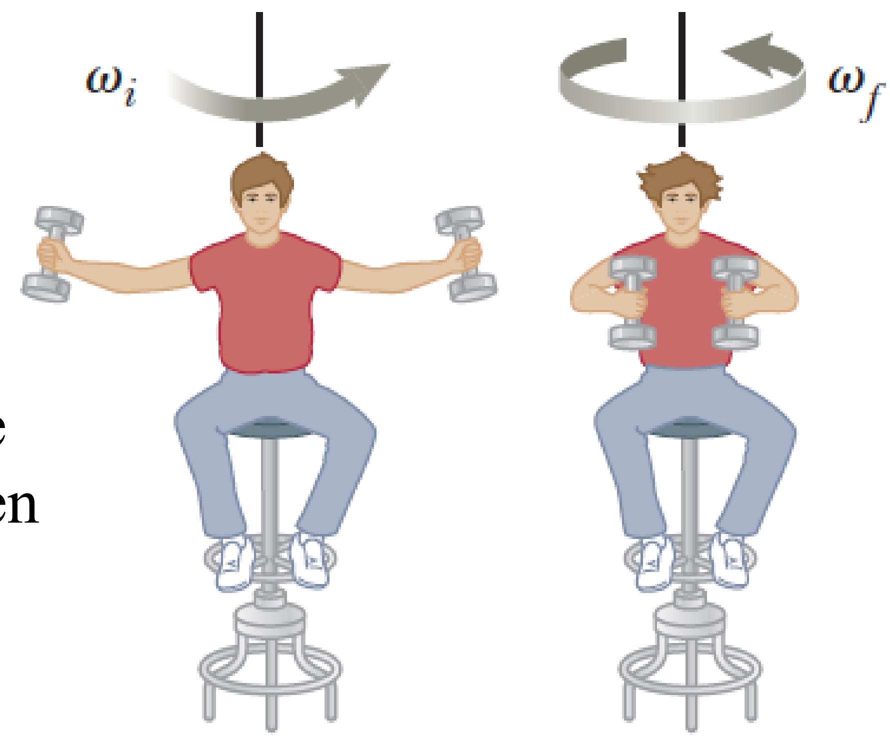
$$\begin{aligned} \left|\vec{L}\right| &= \left(\sum_i^N m_i r_i^2\right)\omega \\ &= I\omega \end{aligned}$$

Also, just as the linear momentum in a closed system is conserved, the angular momentum in a closed system is also conserved,

$$\vec{L}_{\text{initial}} = \vec{L}_{\text{final}}$$

A pizza pie dough is approximately a cylinder with a uniform mass distribution. If the pizza dough with a radius of 8.0 cm is thrown into the air at a speed of 15.0 rad/s, and then lands back onto the chef's hand at a speed of 10.0 rad/s, then what is the radius of the stretched pizza dough?

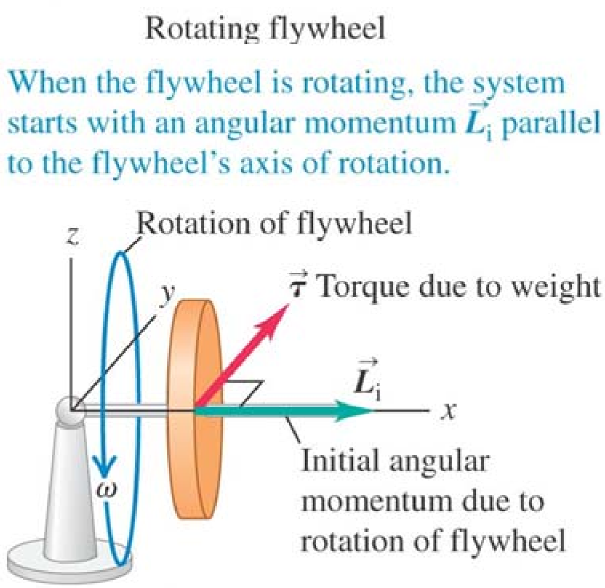
A student sits on a freely rotating stool holding two weights, each of mass 3.00kg (approximate as point masses). When his arms are extended horizontally, the weights are 0.700 m from the axis of rotation and the student rotates with an angular speed of 1.00 rad/s. The moment of inertia of the student plus stool is a constant, 3.00 kg·m^2. While spinning, the student pulls the weights inward horizontally to a position 0.200 m from the axis of rotation.

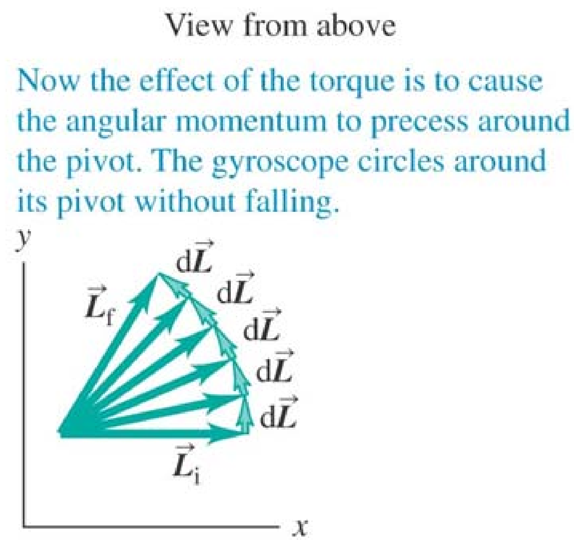


Calculate the angular speed of the student after the weights have been pulled inward.

A rotating flywheel

The magnitude of the angular momentum stays the same, but its direction changes continuously.



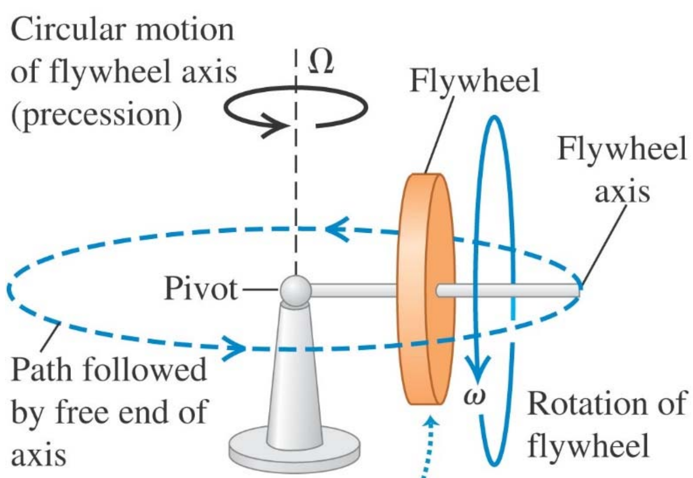


Flywheel not spinning: If we hold it with the flywheel axis horizontal and let go, the free end of the axis simply drops owing to gravity

Flywheel spinning: a steady circular motion of the axis in a horizontal plane, combined with the spin motion of the flywheel about the axis will occur.

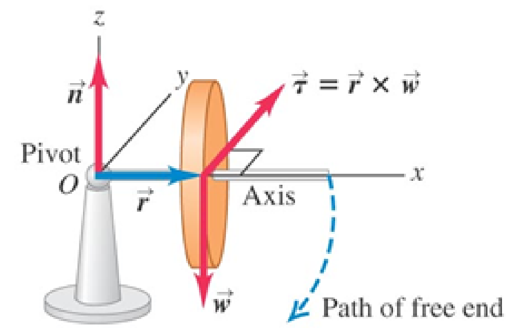
Gyroscopes and precession

For a gyroscope, the axis of rotation changes direction. The motion of this axis is called *precession*.



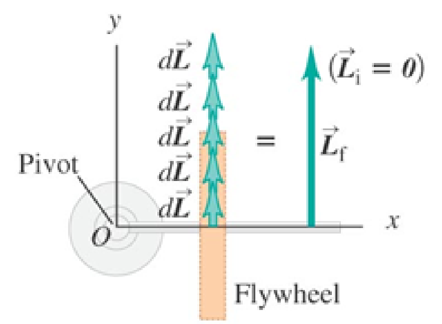
When the flywheel and its axis are stationary, they will fall to the table surface. When the flywheel spins, it and its axis "float" in the air while moving in a circle about the pivot.

Nonrotating flywheel falls



When the flywheel is not rotating, its weight creates a torque around the pivot, causing it to fall along a circular path until its axis rests on the table surface.

View from above as flywheel falls



In falling, the flywheel rotates about the pivot and thus acquires an angular momentum $\vec{L}$. The *direction* of $\vec{L}$ stays constant.

The rate at which the axis moves, $d\phi/dt$, is called the <u>precession angular speed</u>,

$$\Omega = \frac{d\phi}{dt} = \frac{\left|d\vec{L}\right| / \left|\vec{L}\right|}{dt} = \frac{\tau_z}{L_z} = \frac{mgr}{I\omega}$$