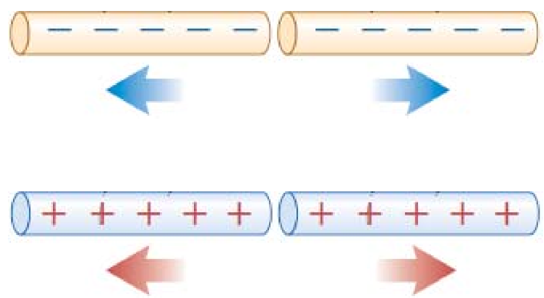
Chapter 21: Electric charge and electric field

- Electric charge
- Conductors, insulators, and induced charges
- Coulomb’s law
- Electric field and electric forces
- Electric field calculations
- Electric field lines
- Electric dipoles

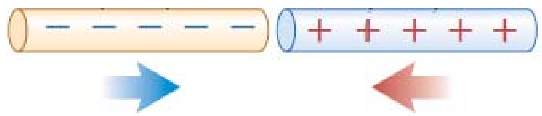
Electric charge

The fundamental quantity in electrostatics is the electric charge. There are two kinds of charge, *positive* and *negative*.

Objects with the same net charge repel each other.



Objects with opposite net charges attract each other.



Electric charge and the structure of matter

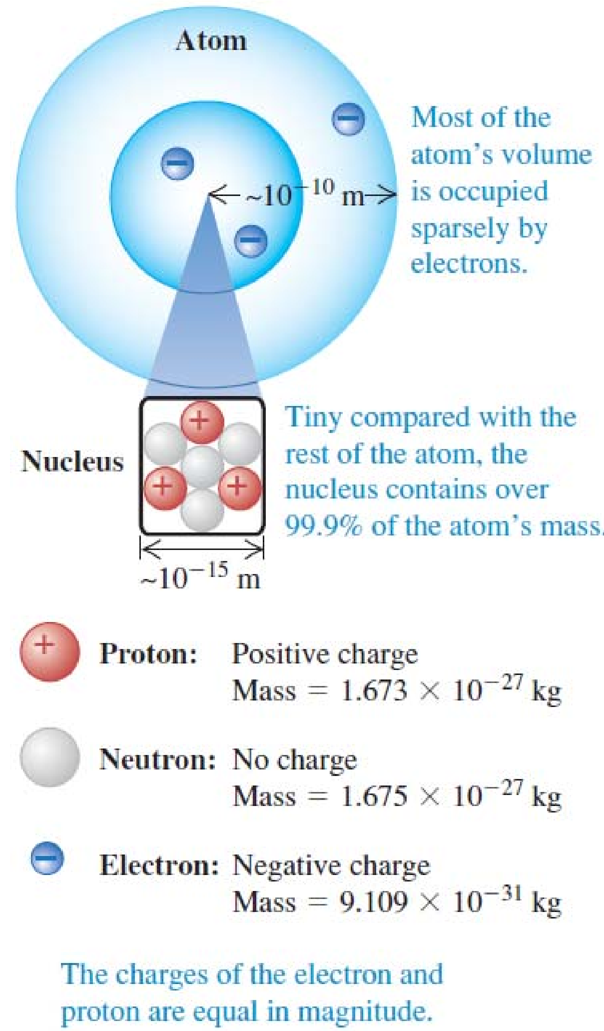
The structure of atoms can be described in terms of three particles: the negatively charged electron, the positively charged proton, and the uncharged neutron.

Over 99.9% of the mass of any atom is concentrated in its nucleus.

The negative charge of the electron has exactly the same magnitude as the positive charge of the proton.

$$e = 1.602176487(40) \times 10^{-19}\ \text{C}$$

If one or more electrons are removed from an atom, what remains is called a positive ion. A negative ion is an atom that has gained one or more electrons.

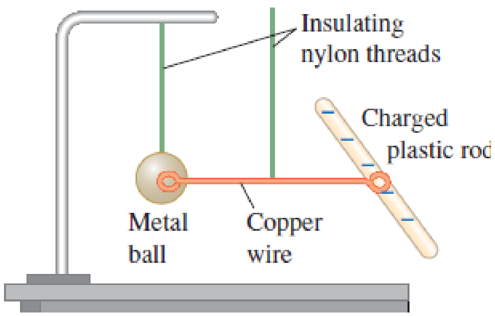


Conductors, insulators, and charging objects

Conductors permit the easy movement of charge through them.

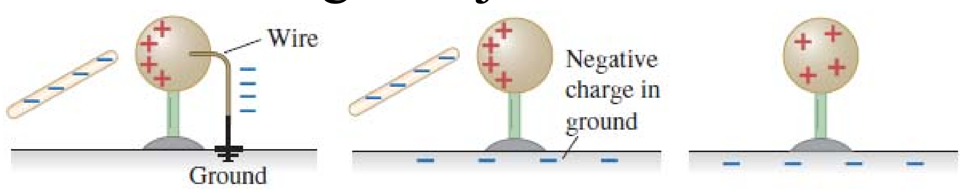
Insulators resist the movement of charge through them.

Charging by conduction occurs when you bring an object with a net surface charge in contact with a neutral object or allow charges to be displaced to the other surface with a conducting wire. Surface charges are repelled from the charged object onto the surface of the neutral object.

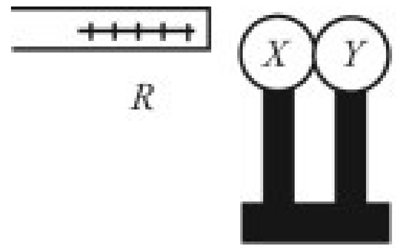


The wire conducts charge from the negatively charged plastic rod to the metal ball.

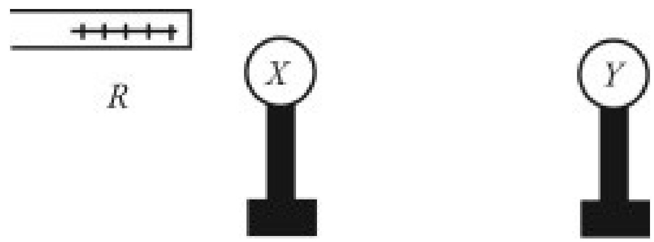
Charging by induction involves placing a charge object near a conductor connected to a ground wire. The conductor takes the opposite charge through charge displacement in the ground wire due to the presence of the charged object. Then, severing the wire causes the net charge to be trapped on the conductor after the charged object is removed.



X and *Y* are two uncharged metal spheres on insulating stands, and are in contact with each other. A positively charged rod *R* is brought close to *X* as shown.



Sphere *Y* is now moved away from *X*



What are the final charge states of *X* and *Y*?

A. Both X and Y are neutral.

B. X is positive and Y is neutral.

C. X is negative and Y is positive.

D. X is neutral and Y is positive.

E. Both X and Y are negative.

Coulomb's law

Coulomb's law: the magnitude of the electric force between two *point charges* is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$F = k\frac{|q_1 q_2|}{r^2}$$

where $k = 8.987551787\times10^{9}\ \mathrm{Nm^2/C^2}$.

The value of k is known to such a large number of significant figures because this value is closely related to the speed of light in vacuum.

Coulomb's law may also be written as

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

where $k = \dfrac{1}{4\pi\epsilon_0}$.

When two point charges are a distance d part, the electric force that each one feels from the other has magnitude F. In order to make this force twice as strong, the distance would have to be changed to

A. $d/\sqrt{2}$

B. $d/2$

C. $2d$

D. $d/4$

E. $\sqrt{2}d$

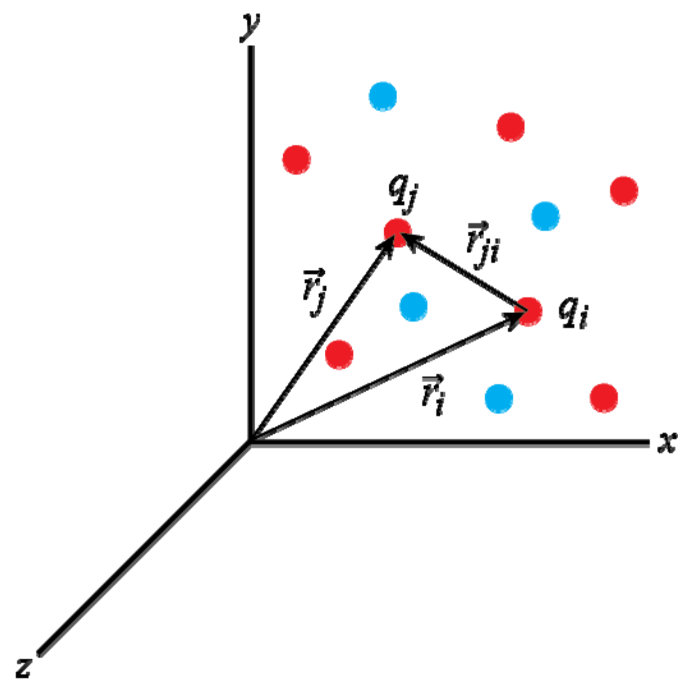
Superposition of forces

The superposition of forces for electric charges states that the total force acting on a charge is the *vector sum* of all the forces acting on that charge from all other charges.

For N number of charges, the force on the ith charge due to all other $N-1$ charges is

$$\sum_{\substack{j=1 \\ j\neq i}}^{N-1} \vec{F}_{j\,\text{on}\,i} = k \sum_{\substack{j=1 \\ j\neq i}}^{N-1} \frac{q_i q_j}{r_{ji}^2}\hat{r}_{ji}$$

where $\vec{r}_{ji} = \vec{r}_j - \vec{r}_i$



Electric field

The electric force on a charged body is exerted by the electric field created by other charged bodies.

The electric field is defined as the unit of force per unit charge, N/C. Thus the electric field at a specific location can found by measuring the force on a test charge and dividing by that test charge,

$$\vec{E} = \frac{\vec{F}_0}{q_0}\,.$$

This is the same concept as the gravitational field, also called the gravitational acceleration,

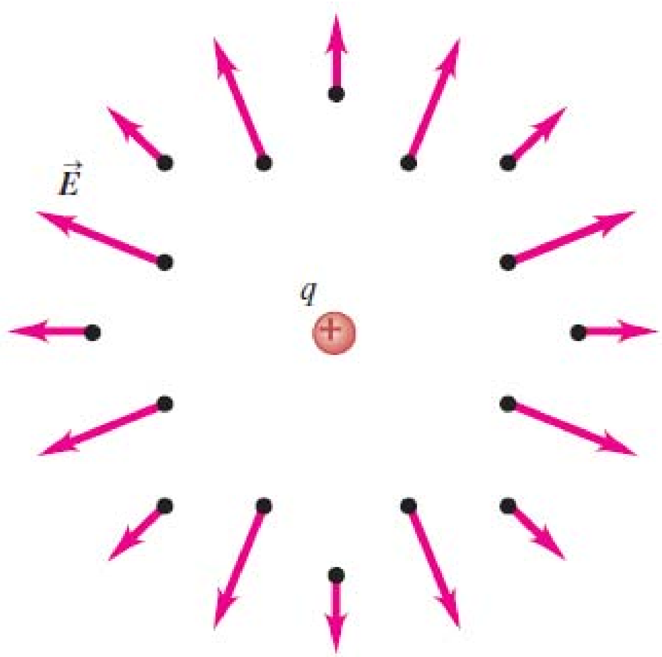
$$\vec{g} = \frac{\vec{F}_0}{m_0}\,.$$

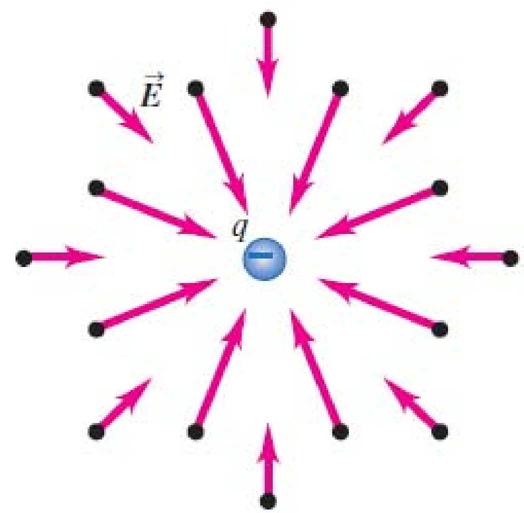
Electric field of a point charge

The electric field of a point charge is given by the relationship

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$
$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

The field direction points outward for positive charges and points inward for negative charges.





Superposition of electric fields

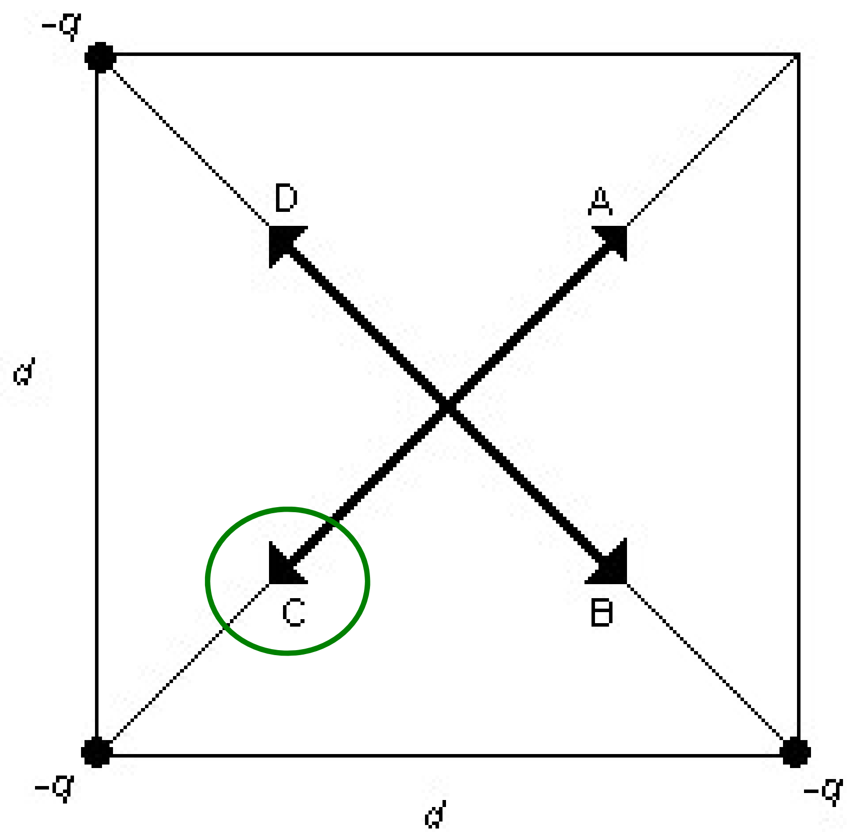
We already know that forces may be summed together as vectors,

$$\vec{F}_0 = \vec{F}_{1\,\mathrm{on}\,0} + \vec{F}_{2\,\mathrm{on}\,0} + \vec{F}_{3\,\mathrm{on}\,0} + \cdots$$

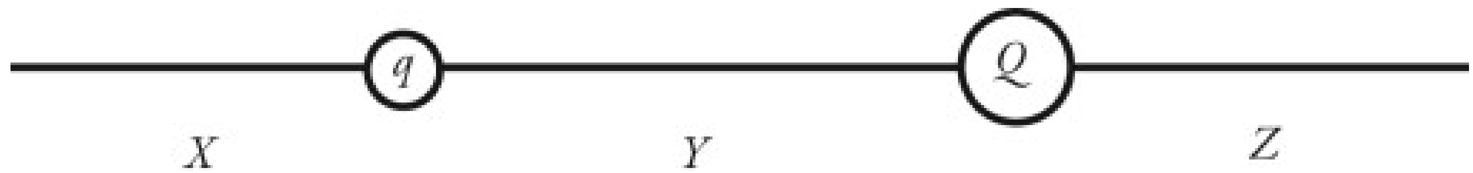
By dividing both sides of the equation by the test charge q_0,

$$\begin{aligned}\vec{E}_0 &= \frac{1}{q_0}\left(\vec{F}_{1\,\mathrm{on}\,0} + \vec{F}_{2\,\mathrm{on}\,0} + \vec{F}_{3\,\mathrm{on}\,0} + \cdots\right) \\ &= \frac{1}{q_0}\vec{F}_{1\,\mathrm{on}\,0} + \frac{1}{q_0}\vec{F}_{2\,\mathrm{on}\,0} + \frac{1}{q_0}\vec{F}_{3\,\mathrm{on}\,0} + \cdots \\ &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots\end{aligned}$$

Three equal negative point charges are placed at three of the corners of a square of side *d* as shown in the figure. Which of the arrows represents the direction of the net electric field at the center of the square?



The figure shows two unequal point charges, q and Q, of opposite sign. Charge Q has greater magnitude than charge q. In which of the regions X, Y, Z will there be a point at which the net electric field due to these two charges is zero?



A. only region Y

B. only region X

C. only region Z

D. only regions X and Z

E. all three regions

Charge distributions

Just like mass distributions may be discussed using mass densities, charge distributions may be discussed using charge densities.

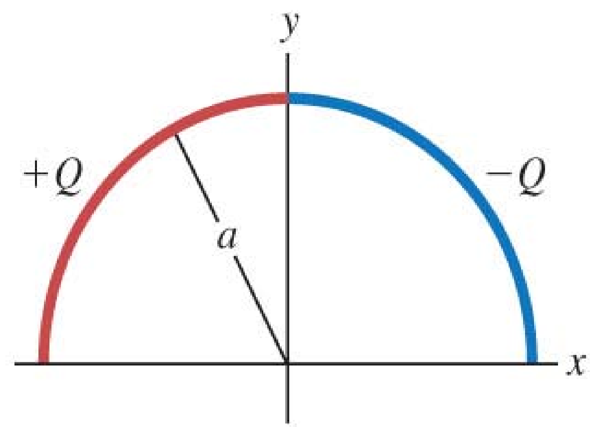
Three types of charge densities are based on dimensionality,

$$\lambda = \frac{dq}{d\ell} \quad \text{(linear charge density)}$$

$$\sigma = \frac{dq}{dA} \quad \text{(surface charge density)}$$

$$\rho = \frac{dq}{dV} \quad \text{(volume charge density)}$$

A semicircle of radius *a* is in the first and second quadrants, with the center of curvature at the origin. Positive charge $+Q$ is distributed uniformly around the left half of the semicircle, and negative charge $-Q$ is distributed uniformly around the right half of the semicircle.



(a) What is the magnitude of the net electric field at the origin produced by this distribution of charge?

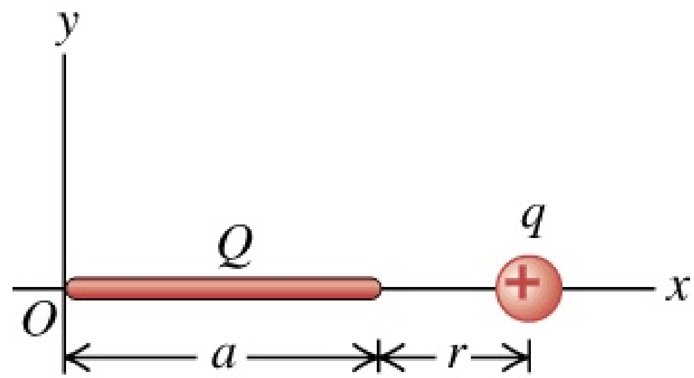
$$E_y = \frac{k\lambda}{a}\int_{\pi/2}^{\pi} \sin\theta\, d\theta - \frac{k\lambda}{a}\int_{0}^{\pi/2} \sin\theta\, d\theta = 0$$

$$E_x = \frac{k\lambda}{a}\int_{\pi/2}^{\pi} \cos\theta\, d\theta - \frac{k\lambda}{a}\int_{0}^{\pi/2} \cos\theta\, d\theta = 2\frac{k\lambda}{a} \quad \text{and} \quad \lambda = \frac{Q}{\frac{\pi}{2}a} \Longrightarrow E = \frac{4}{\pi}k\frac{Q}{a^2}$$

(b) What is the direction of the net electric field at the origin produced by this distribution of charge?

$+x$ direction

A positive charge Q is distributed uniformly along the x-axis from $x = 0$ to $x = a$. A positive point charge q is located on the positive x-axis at $x = a + r$, a distance r to the right of the end of Q.



(a) Calculate the x-component of the electric field produced by the charge distribution Q at points on the positive x-axis where $x > a$.

(b) Calculate the y-component of the electric field produced by the charge distribution Q at points on the positive x-axis where $x > a$.

(c) Calculate the magnitude of the force that the charge distribution Q exerts on q.

(d) What is the direction of the force that the charge distribution Q exerts on q?

Two large, flat, horizontally oriented plates are parallel to each other, a distance d apart. Half way between the two plates the electric field has magnitude E. If the separation of the plates is reduced to $d/2$ what is the magnitude of the electric field half way between the plates? Assume the surface charge density of the plates is constant.

A. 0

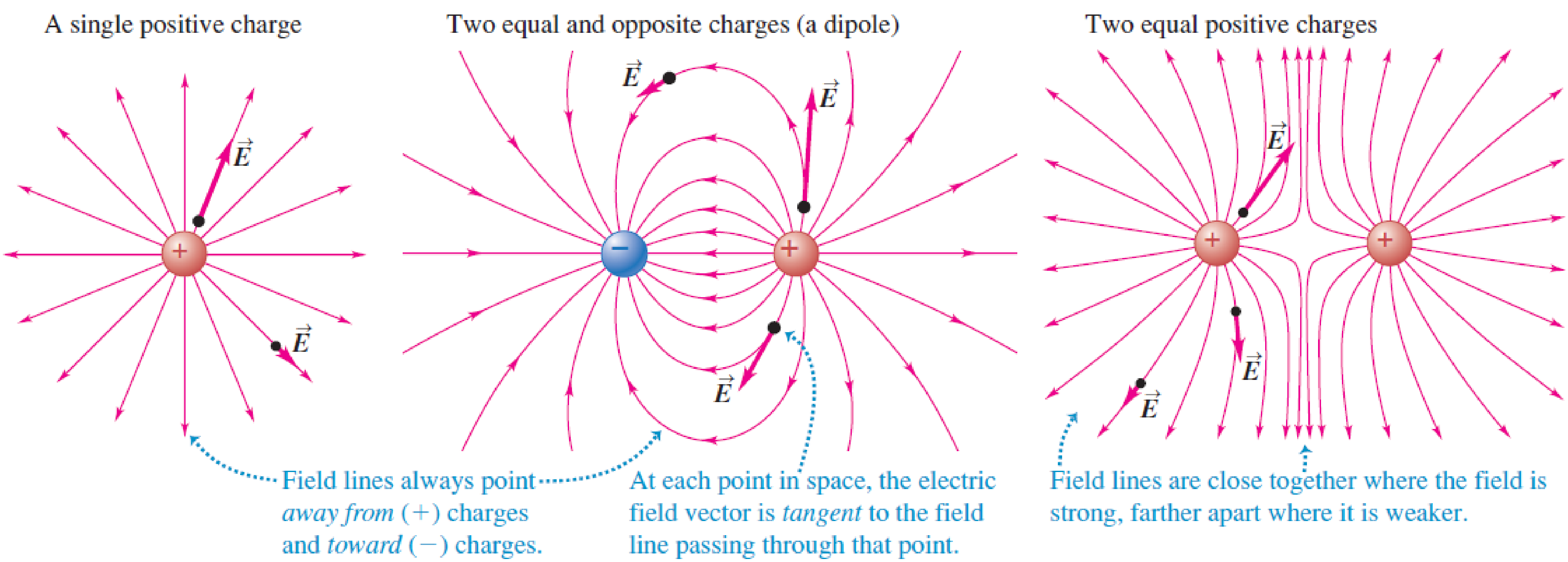
B. E

C. $2E$

D. $E/2$

Electric field lines

An <u>electric field line</u> is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric field vector at that point.



Field lines <u>never</u> intersect!!!

The figure shows some of the electric field lines due to three point charges arranged along the vertical axis. All three charges have the same magnitude.

What are the signs of the three charges?

A. top charge is negative, middle is negative, and bottom is positive

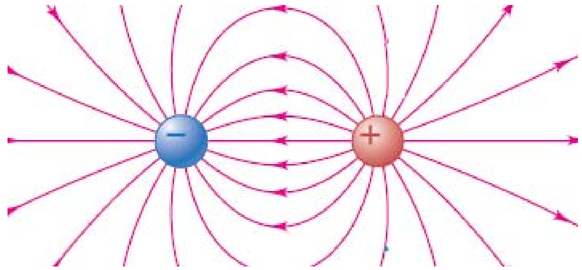
B. top charge is positive, middle is negative, and bottom is negative

C. top charge is positive, middle is positive, and bottom is positive

D. top charge is positive, middle is negative, and bottom is positive

Electric dipoles

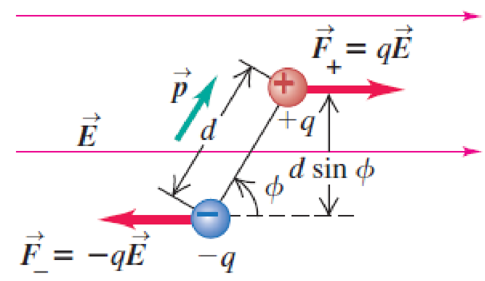
An electric dipole p is a pair of point charges with equal magnitude and opposite sign (a positive charge q and a negative charge $-q$) separated by a distance d.



$$\vec{p} = q\vec{d}$$

The magnitude of the torque on an electric dipole due to a uniform external electric field is given by

$$\tau = \sum F_i r_i \sin\phi = qE\frac{d}{2}\sin\phi - (-q)\,E\frac{d}{2}\sin\phi = qEd\sin\phi$$



Defining the electric dipole moment $\vec{p} = q\vec{d}$, it becomes $\tau = pE\sin\phi$.

The potential energy for a dipole in a uniform external electric field is

$$U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$$

Two point charges, $q_1 = -4.5$ nC and , $q_2 = 4.5$ nC, are separated by 3.1 mm to form an electric dipole.

(a) Find the electric dipole moment (magnitude and direction).

(b) The charges are in a uniform electric field whose direction makes an angle of 36.9° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude 7.2×10-9 Nm?