Chapter 23: Electric potential

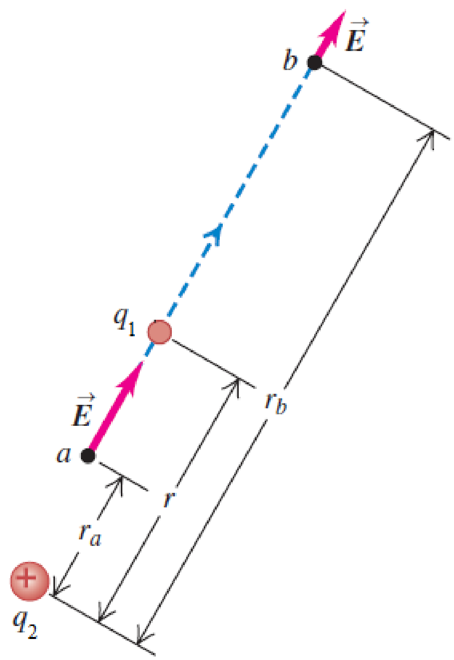
- Electric potential energy
- Electric potential
- Calculating electric potential
- Equipotential surfaces
- Potential gradient

Work done on point charges

We know the relationship between the work due to a force and the distance,

$$W = \int \vec{F} \cdot d\vec{s}$$

If we are concerned with small spherical charges, then the force is simply Coulomb's law between charged particles. Thus,



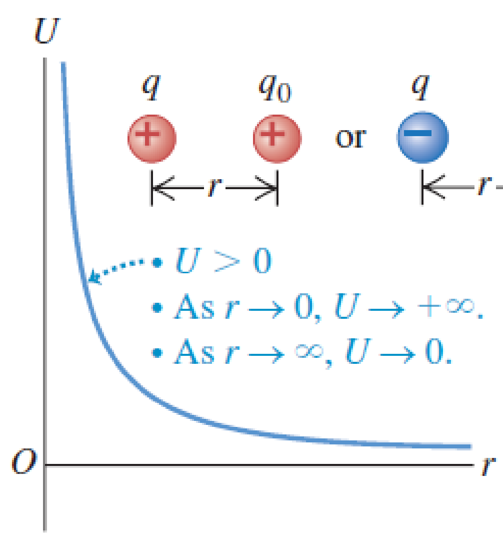
$$\begin{aligned} W &= \frac{q_1 q_2}{4\pi\epsilon_0} \int_{r_A}^{r_B} \frac{1}{r^2} \hat{r} \cdot d\vec{\ell} \\ &= \frac{q_1 q_2}{4\pi\epsilon_0} \int_{r_A}^{r_B} \frac{dr}{r^2} \\ &= \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{1}{r_A} - \frac{1}{r_B} \right) \end{aligned}$$

Electric potential energy

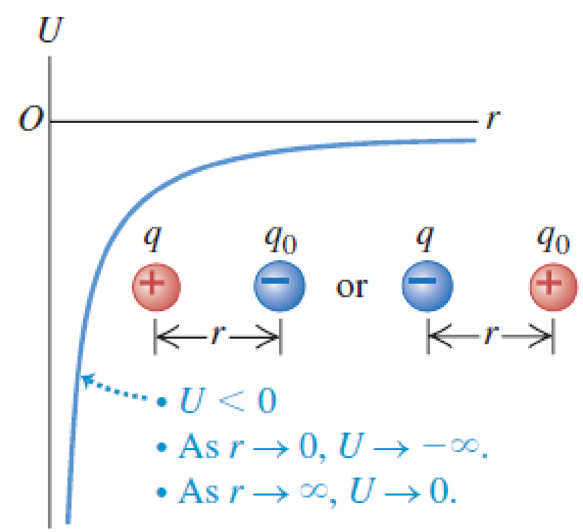
We know that the work due to the conservative electric force is related to the potential energy, $W = -\Delta U$. Setting $r_A \to \infty$ and $r_B \to r$, we find

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

q and q_0 have the same sign.



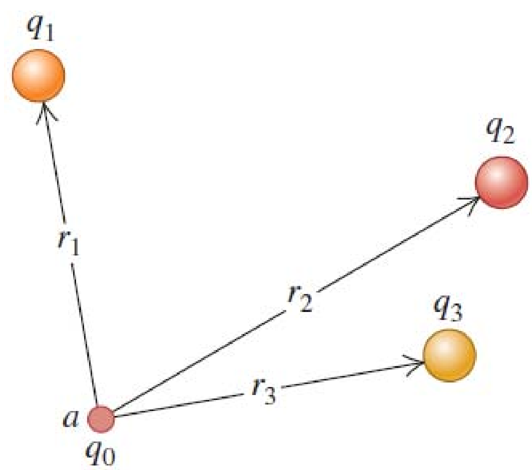
q and q_0 have opposite signs.



Note that we define the potential energy at the position of the particle relative to all other charges with respect to an initial position infinitely far from the charges.

Electric potential energy with multiple charges

Suppose the electric field $\vec{E}$ in which charge q_0 moves is caused by several point charges $q_1, q_2, q_3, \ldots$ at distances $r_1, r_2, r_3, \ldots$ from q_0. The potential energy associated with the charge q_0 is



$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right)$$

The above equation can be written in a more compact form using a summation symbol. In general, the potential energy associated with any charge q_i can be written as

$$U = \frac{q_i}{4\pi\epsilon_0}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{q_j}{r_{ji}} \qquad \text{where} \qquad r_{ji} = |\vec{r}_j - \vec{r}_i|$$

Electric potential

Electric potential is the electric potential energy per unit charge,

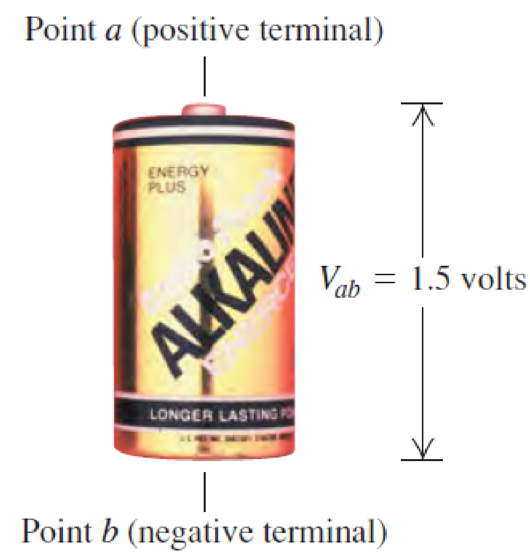
$$V = \frac{U}{q_0}$$

The potential energy and charge are both scalars, and therefore, the electric potential is a scalar.

The SI unit of electric potential is the Volt (V).

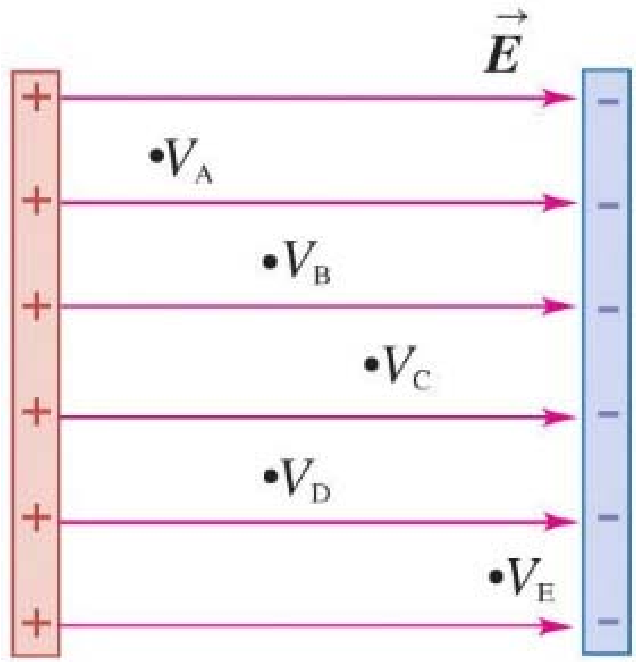
1 V = 1 Volt = 1 J/C = 1 Joule/Coulomb

In electric circuits, the potential difference between two points is often referred to as the voltage.



The figure shows the electric potential V at five locations in a uniform electric field. At which point is the electric potential the largest?

1. V_A
2. V_B
3. V_C
4. V_D
5. V_E



Suppose you have two negative point charges. As you move them farther and farther apart, the potential energy of this system relative to infinity

A. stays the same.

B. decreases.

C. increases.

Calculating the electric potential

We use the following equation to find the electric potential due to a single point charge,

$$V = \frac{q}{4\pi\epsilon_0}\frac{1}{r}$$

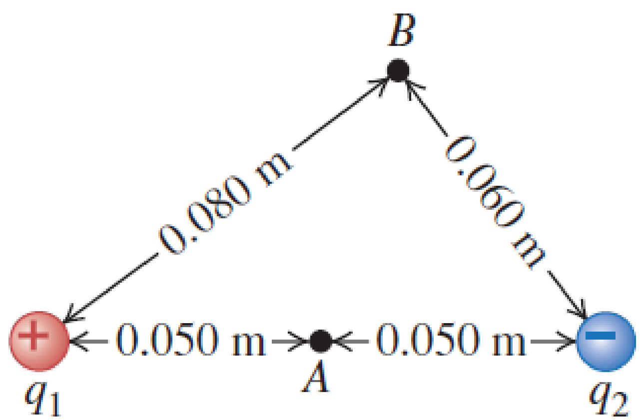
For a electric potential due to a collection of charges, we sum all the single point charge electric potentials,

$$V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i}$$

When we have a continuous distribution of charge, the summation becomes an integral,

$$V = \frac{1}{4\pi\epsilon_0}\int_0^q \frac{dq'}{r}$$

Two point charges $q_1 = 2.20$ nC and $q_2 = -6.70$ nC are 0.100 m apart. Point A is midway between them, and point B is 0.080 m from q_1 and 0.060 m from q_2. Take the electric potential to be zero at infinity.

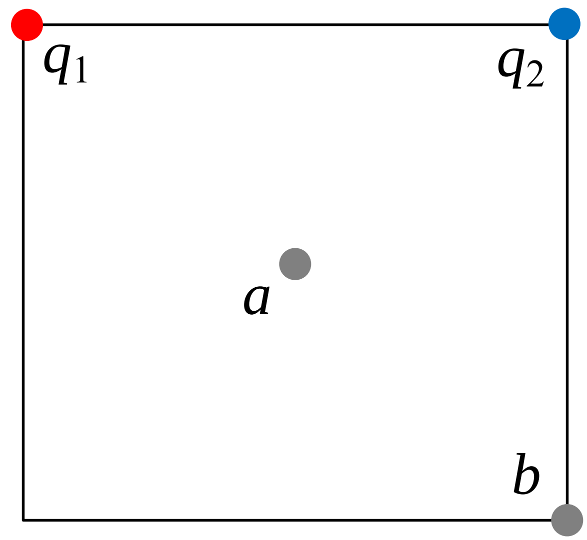


(a) Find the potential at point A.

(b) Find the potential at point B.

(c) Determine the work done by the electric field on a charge of 2.40 nC that travels from point B to point A.

Point charges $q_1 = +2.00\ \mu C$ and $q_2 = -2.00\ \mu C$ are placed at adjacent corners of a square for which the length of each side is 3.50 cm . Point *a* is at the center of the square, and point *b* is at the empty corner closest to q_2. Take the electric potential to be zero at a distance far from both charges.

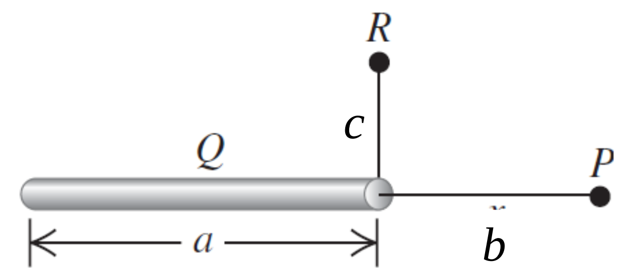


(a) What is the electric potential at point *a* due to q_1 and q_2?

(b) What is the electric potential at point *b*?

(c) A point charge $q_3 = -2.00\ \mu C$ moves from point *a* to point *b*. How much work is done on q_3 by the electric forces exerted by q_1 and q_2?

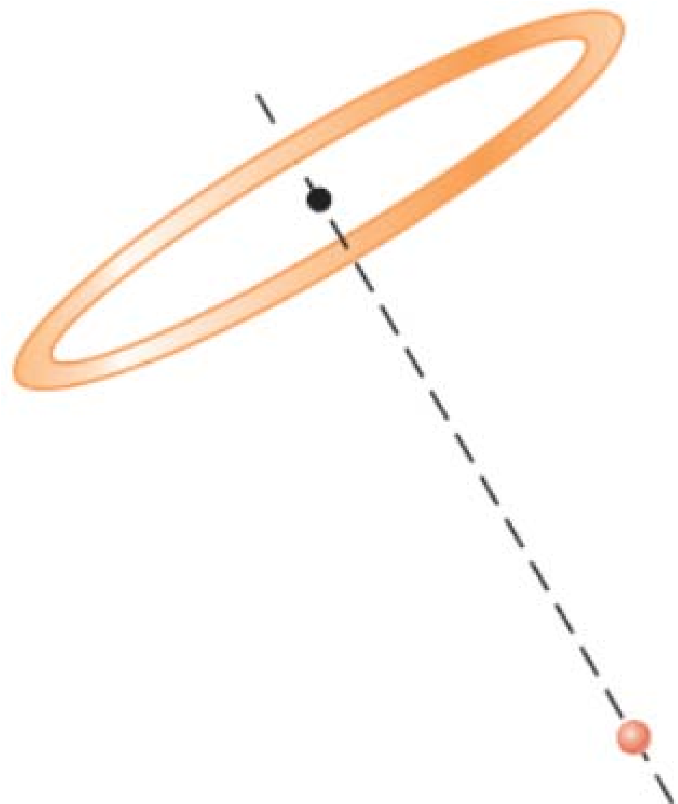
Electric charge is distributed uniformly along a thin rod of length *a*, with total charge *Q*. Take the potential to be zero at infinity.



(a) Find the potential at point P a distance *b* to the right of the rod.

(b) Find the potential at point R a distance *c* above the right-hand end of the rod.

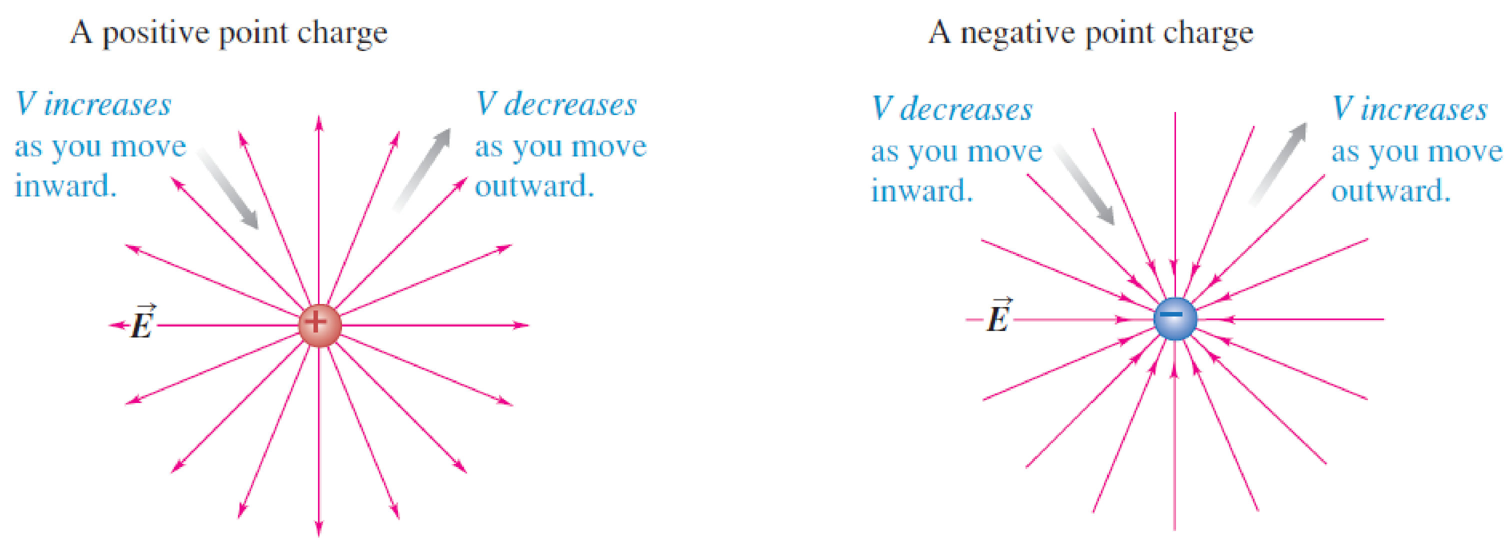
A uniformly charged thin ring has radius 13.0 cm and total charge 24.0 nC . An electron is placed on the ring's axis a distance 30.0 cm from the center of the ring and is constrained to stay on the axis of the ring. The electron is then released from rest. Determine the speed of the electron when it reaches the center of the ring.



Finding the electric potential from the field

When we are given a collection of point charges, it is usually the easiest to calculate the potential V. In some problems in which the electric field is known or can be found easily, it is easier to determine V from $\vec{E}$,

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}$$



A conducting sphere contains positive charge distributed uniformly over its surface. All potentials are measured relative to infinity. Which statement about the potential due to this sphere are true?

A. The potential at the center is the same as the potential at infinity.

B. The potential at the center of the sphere is zero.

C. The potential is lowest, but not zero, at the center of the sphere.

D. The potential at the center of the sphere is the same as the potential at the surface.

E. The potential at the surface is higher than the potential at the center.

An insulating sphere contains positive charge distributed uniformly throughout its volume. All potentials are measured relative to infinity. Which statements about the potential due to this sphere are true?

A. The potential at the surface is higher then the potential at the center.

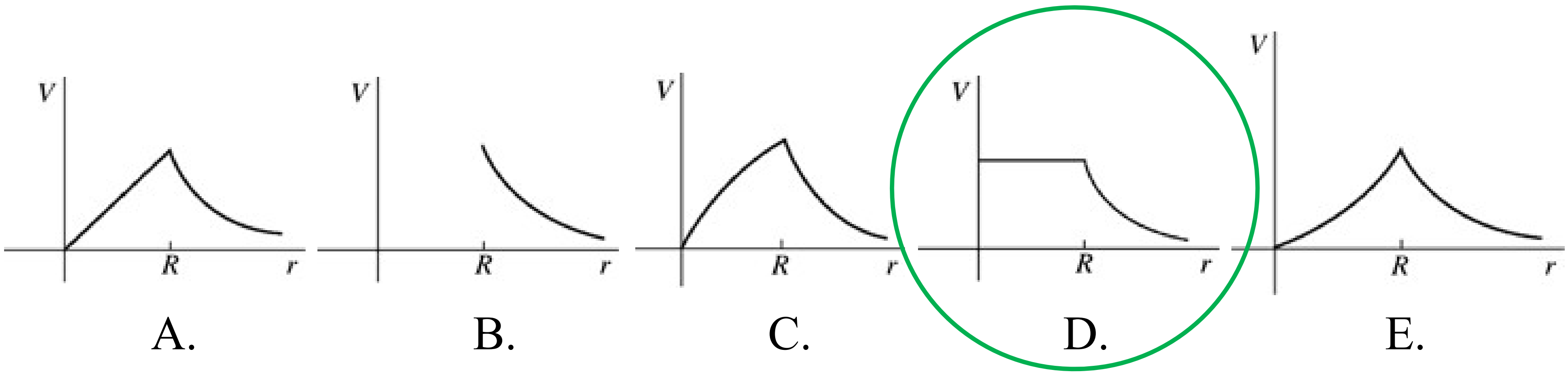
B. The potential at the center of the sphere is the same as the potential at the surface.

C. The potential is highest at the center of the sphere.

D. The potential at the center is the same as the potential at infinity.

E. The potential at the center of the sphere is zero.

A conducting sphere of radius R carries an excess positive charge and is very far from any other charges. Which one of the following graphs best illustrates the potential (relative to infinity) produced by this sphere as a function of the distance r from the center of the sphere?

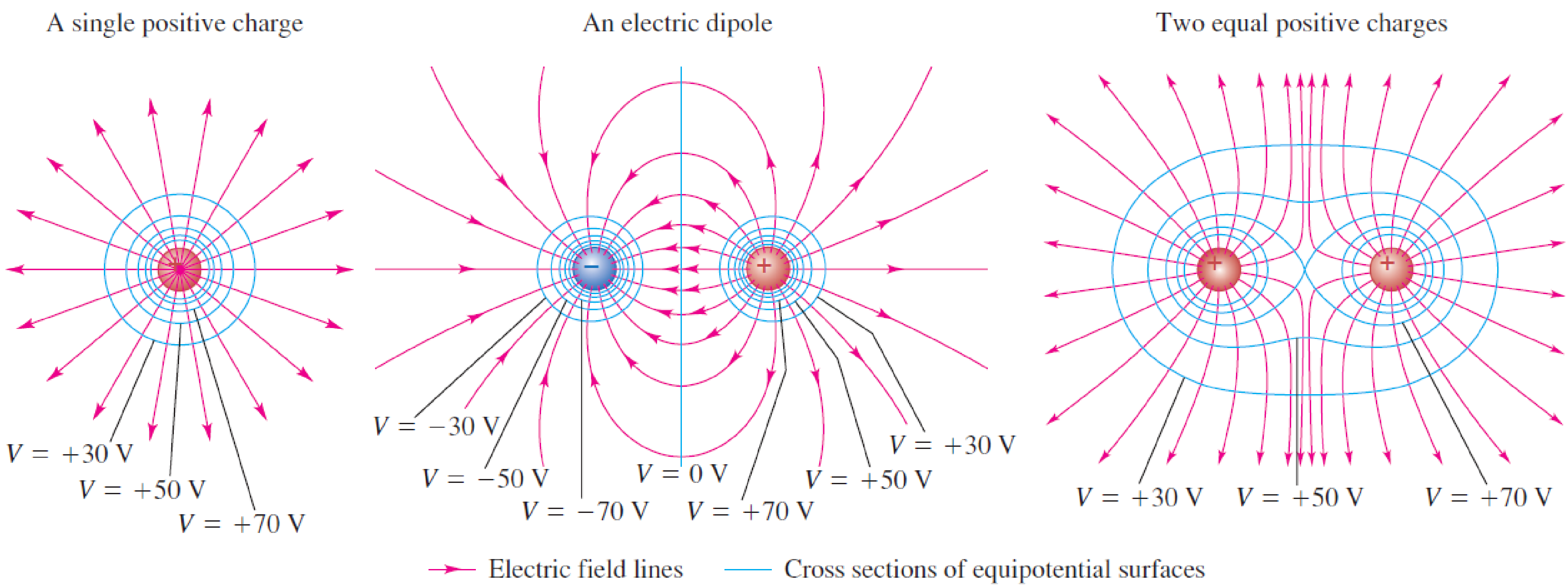


A. B. C. D. E.

Equipotential surfaces

The potential at various points in an electric field can be represented graphically by equipotential surfaces.

Because potential energy does not change as a test charge moves over an equipotential surface, the electric field can do no work on such a charge. Therefore, $\vec{E}$ must be perpendicular to the surface at every point so that the electric force is always perpendicular to the displacement of a charge moving on the surface.



A positive charge is moved from point A to point B along an equipotential surface. How much work is performed or required in moving the charge?

A. Work is required in moving the positive charge from point A to point B.

B. No work is performed or required in moving the positive charge from point A to point B.

C. Work is both performed and required in moving the charge from point A to point B.

D. Work is performed in moving the positive charge from point A to point B.

Electric field and the gradient of the potential

We may rewrite the expression $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}$, by using the fundamental theorem of line integrals (also called the gradient theorem).

Thus, we may calculate the electric field from the electric potential via

$$\vec{E} = -\nabla V$$

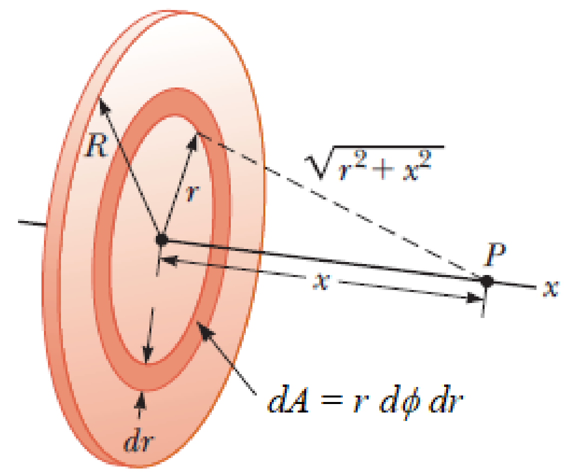
In Cartesian coordinates, the "del" symbol is given by

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

For a radially dependent potential in spherical coordinates, only the radial derivative is nonzero. The radial component of the electric field is

$$E_r = -\frac{\partial V}{\partial r}$$

A uniformly charged disk of radius R lies in a plane perpendicular to the x axis. A uniformly charged disk has radius R and surface charge density σ.



(a) Find the electric potential at a point P along the perpendicular central axis of the disk.

(b) Find the electric field at point P.

In a region of space, the electric potential is $V(x,y,z) = Axy - Bx^2 + Cy$, where A, B, and C are positive constants.

(a) Calculate the x-component of the electric field.

(b) Calculate the y-component of the electric field.

(c) Calculate the z-component of the electric field.

At which point is the electric field equal to zero?

A. $x = 0$, $y = 0$, $z = 0$

B. $x = -C/A$, $y = 0$, $z = -2BC/A^2$

C. $x = -C/A$, $y = -2BC/A^2$, $z = C/A$