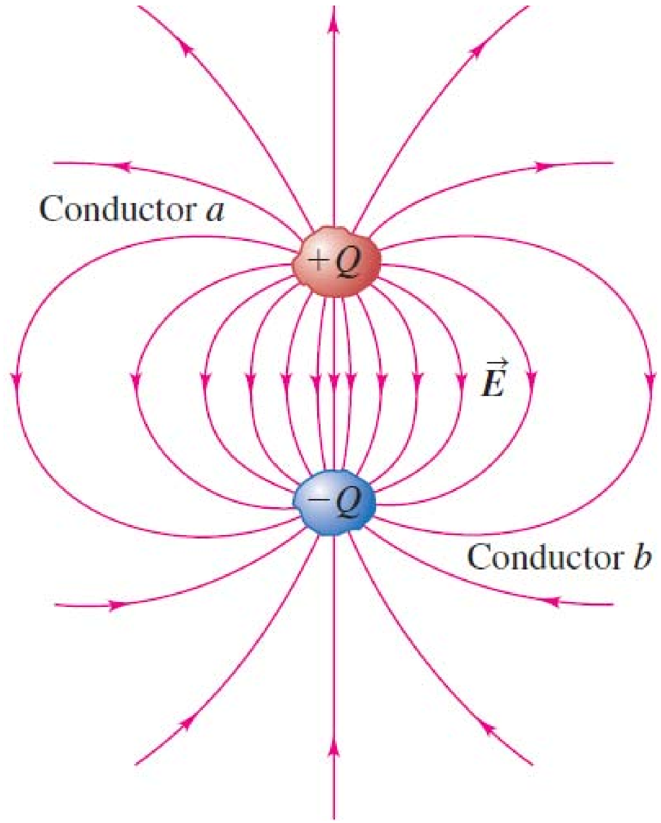
Chapter 24: Capacitance and Dielectrics

- Capacitors and capacitance
- Capacitors in series and parallel
- Energy in a capacitor
- Energy density of the electric field
- Dielectrics
- Molecular model of induced charges
- Gauss's law in dielectrics

Capacitors



Any two conductors separated by an insulator (or a vacuum) form a capacitor. The two conductors have charges with equal magnitude and opposite sign, and the net charge on the capacitor as a whole remains zero.

In circuit diagrams a capacitor is represented by either of these symbols,

In either symbol the vertical lines (straight or curved) represent the conductors and the horizontal lines represent wires connected to either conductor.

Capacitance

The electric field at any point in the region between the conductors is proportional to the magnitude of charge Q on each conductor.

The potential difference V between the conductors is also proportional to Q.

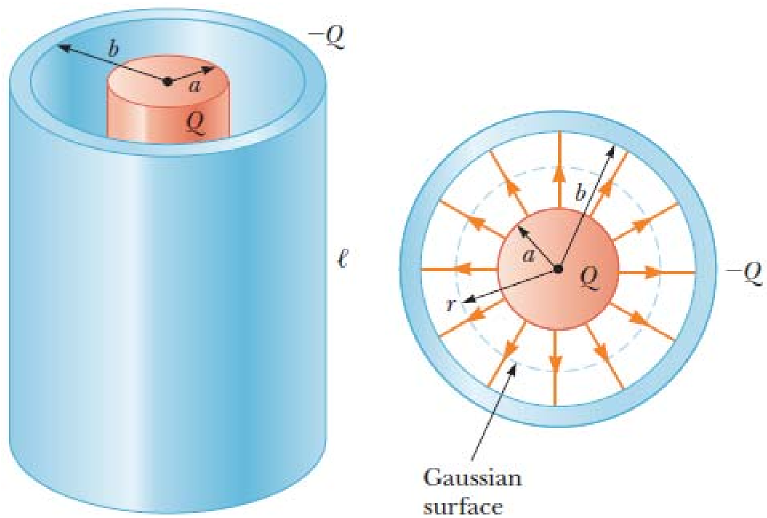
The capacitance is defined as the ratio of charge to potential difference,

$$\boxed{C = \frac{Q}{V}}$$

The capacitance is a measure of the ability of a capacitor to store energy.

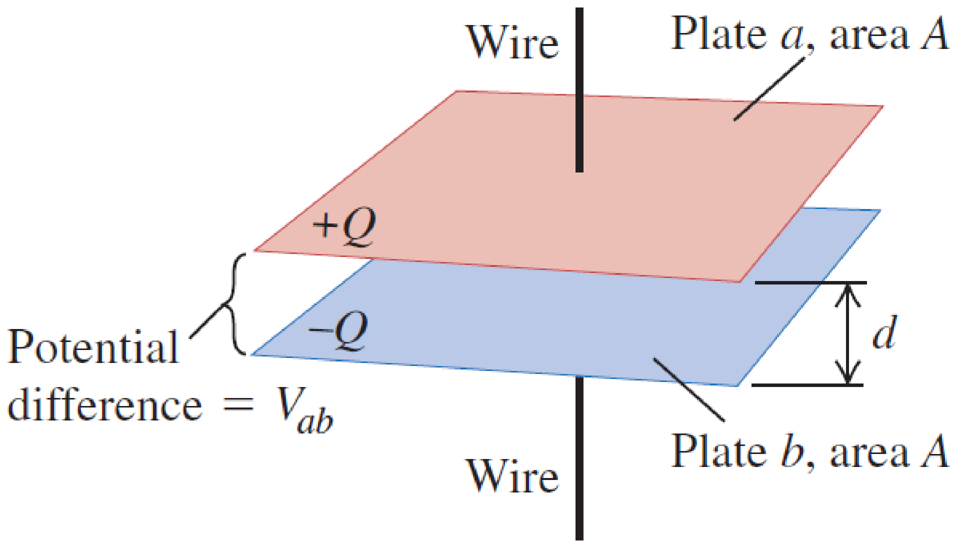
The SI unit of capacitance is called one **farad.** (F = C/V).

A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness, radius $b > a$, and charge $-Q$. Find the capacitance of this cylindrical capacitor if its length is ℓ.
Hint: ignore fringe effects when finding the field in the capacitor.



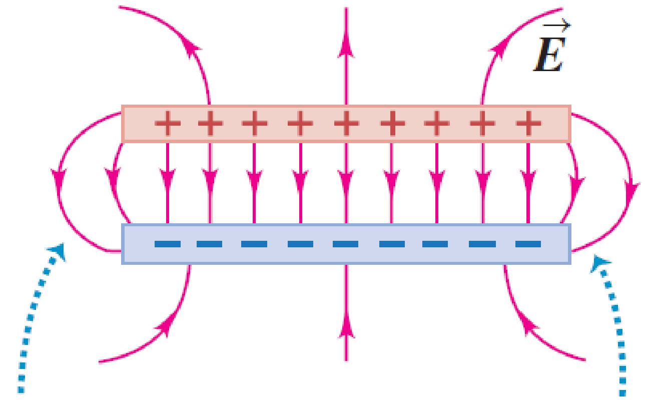
Parallel plate capacitor

The simplest form of capacitor is the parallel plate capacitor. It consists of two parallel conducting plates, each with area *A* separated by a distance *d* that is small in comparison with their dimensions.



Capacitance between two parallel plates

The field between these plates is essentially uniform (ignoring small edge effect).



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

Because the field is uniform, the potential difference between the two plates is

$$V = Ed = \frac{Qd}{A\epsilon_0}$$

Thus, the capacitance follows as

$$C = \epsilon_0 \frac{A}{d}$$

The plates of a parallel-plate capacitor are maintained with a constant voltage by a battery as they are pushed together, without touching. How is the amount of charge on the plates affected during this process?

A. The amount of charge on the plates decreases during this process.

B. The amount of charge on the plates becomes zero.

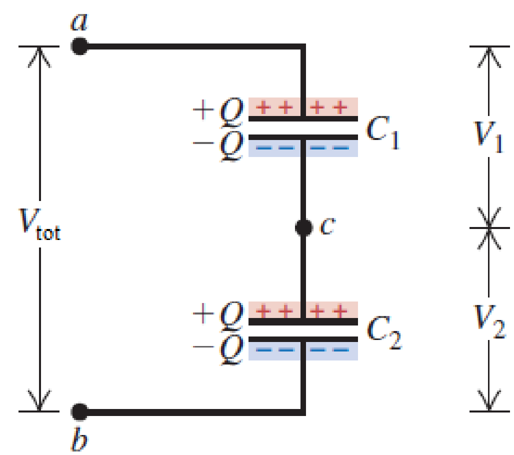
C. The amount of charge on the plates increases during this process.

D. The amount of charge on the plates remains constant.

Capacitors in series

The equivalent capacitance of the series combination is defined as the capacitance of a single capacitor for which the charge is the same as for the combination, when the electric potential difference is the same.

For the two capacitor series circuit, we can write the electric potential differences between points a and c (V_1), c and b (V_1), and a and b (V_{tot}) as



$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_{\text{tot}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Therefore we can write an equivalent capacitance of the two capacitors in series as

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

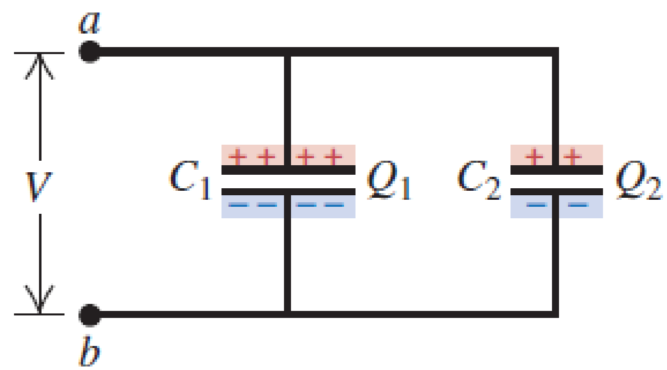
In general, for N number of capacitors <u>all</u> in series, we may write

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$$

Capacitors in parallel

For the case of two capacitors in parallel, the upper and lower plates of the two capacitors each form an equipotential surface.

The potential difference across each individual capacitor is the same, and it is equal to V; however, the charges on each capacitor may differ.



For the two capacitor parallel circuit, we can write the on each capacitor as

$$Q_1 = VC_1 \quad \text{and} \quad Q_2 = VC_2$$

The total charge on the capacitors is simply $Q_{\text{tot}} = Q_1 + Q_2 = V(C_1 + C_2)$. Therefore the equivalent capacitance is

$$C_{\text{eq}} = C_1 + C_2$$

In general, for N number of capacitors all in parallel, we may write

$$C_{\text{eq}} = C_1 + C_2 + \cdots + C_N$$

When two or more capacitors are connected in series across a potential difference

A. each capacitor carries the same amount of charge.

B. the equivalent capacitance of the combination is less than the capacitance of any of the capacitors.

C. the potential difference across the combination is the algebraic sum of the potential differences across the individual capacitors.

D. All of the above choices are correct.

E. None of the above choices are correct.

When two or more capacitors are connected in parallel across a potential difference

A. the potential difference across each capacitor is the same.

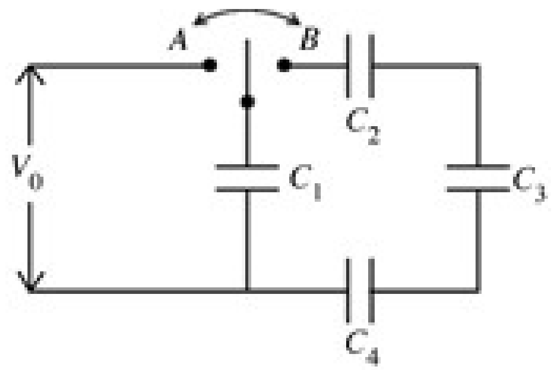
B. the equivalent capacitance of the combination is less than the capacitance of any of the capacitors.

C. each capacitor carries the same amount of charge.

D. All of the above choices are correct.

E. None of the above choices are correct.

The four identical capacitors in the circuit shown in the figure are initially uncharged. Let the charges on the capacitors be Q_1, Q_2, Q_3, and Q_4 and the potential differences across them be V_1, V_2, V_3, and V_4. The switch is thrown first to position *A* and kept there for a long time. It is then thrown to position *B*. Which of the following conditions is true with the switch in position *B*?



1. $V_1 + V_2 + V_3 + V_4 = V_0$
2. $Q_1 = 3\ Q_2$
3. $V_1 = V_0$
4. $Q_1 = Q_2$
5. $V_1 = V_2 = V_3 = V_4$

Energy stored in a capacitor

The electric potential energy of a charge in a static electric field is simply the charge multiplied by the electric potential energy. In a capacitor, however, the electric potential between the plates changes as we accumulate more charge. Therefore we must integrate,

$$\begin{aligned} U &= \int_0^Q V\,dq \\ &= \int_0^Q \left(\frac{q}{C}\right) dq \\ &= \frac{Q^2}{2C} \end{aligned}$$

Substituting our equation $C = Q/V$, we may write $U = \frac{1}{2}QV = \frac{1}{2}CV^2$.

The electric potential energy stored in a charged capacitor is equal to the amount of work required to charge it

$$W = U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

Electric field energy density

The energy density of a capacitor is simply the energy stored by the capacitor divided by the volume of the capacitor. For a parallel plate capacitor, the volume is the area of a plate *A* multiplied by the distance between the plates *d*. Therefore, a parallel plate capacitor has the uniform energy density

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}QV^2}{Ad}$$

Substituting our equation for the capacitance of a parallel plate capacitor, $C = \epsilon_0 A/d$, and the expression for the electric potential in terms of the constant electric field between the plates, $V = Ed$, we arrive at the expression

$$u = \frac{1}{2}\epsilon_0 E^2$$

This last expression is fundamental for vacuum, not just parallel plate capacitors!

The voltage applied across a given parallel-plate capacitor is doubled. How is the energy stored in the capacitor affected?

A. The energy stored in the capacitor is decreased to one-fourth of its original value.

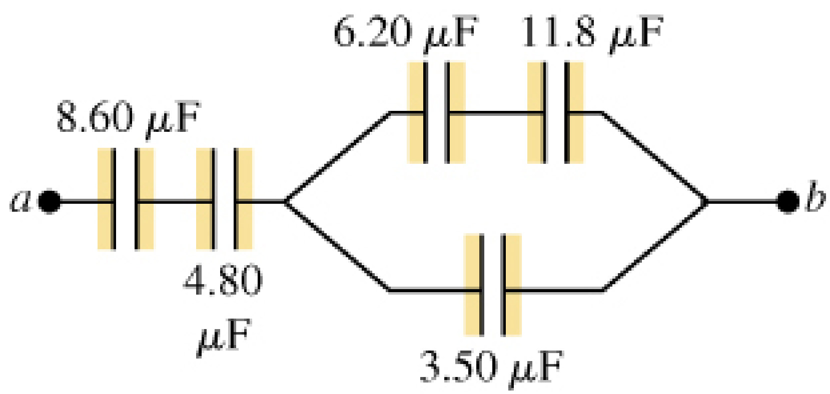
B. The energy stored in the capacitor is decreased to one-half of its original value.

C. The energy stored in the capacitor remains constant.

D. The energy stored in the capacitor quadruples its original value.

E. The energy stored in the capacitor doubles its original value.

For the capacitor network shown in the figure, the potential difference across *ab* is 12.0 V.



(a) Find the total energy stored in this network.

(b) Find the energy stored in the 4.80 μF capacitor.

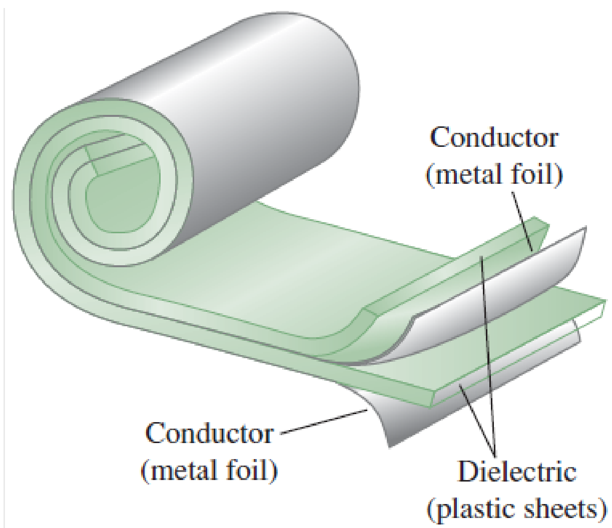
Dielectrics

A dielectric is an insulating material, and when subject to an external electric field, it shifts the charges to microscopically polarize throughout its volume. This polarization reduces the electric field inside the capacitor.

Placing a solid dielectric between the plates of a capacitor serves 3 functions:

(1) it solves the mechanical problem of maintaining two large metal sheets at a very small separation without actual contacting.
(2) using a dielectric increases the maximum possible potential difference between the capacitor plates.
(3) the capacitance of a capacitor of given dimensions is greater when there is a dielectric material between the plates than when there is vacuum.

A common type of capacitor uses dielectric sheets to separate the conductors.



When a dielectric is subjected to a sufficiently large electric field, it experiences a partial ionization and permits conduction through it. This effect is known as <u>dielectric breakdown</u>.

Dielectric constant

The presence of a dielectric between two capacitor plates reduces the electric potential difference between the two plates V relative to the electric potential difference without the presence of the dielectric V_0. The charge accumulated on the plates is the same for either situation.

The dielectric constant follows as $K = \frac{V_0}{V} = \frac{C}{C_0}$.

Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas®	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Dielectric strengths of some materials

The maximum electric-field magnitude that a material can withstand without the occurrence of breakdown is called its dielectric strength.

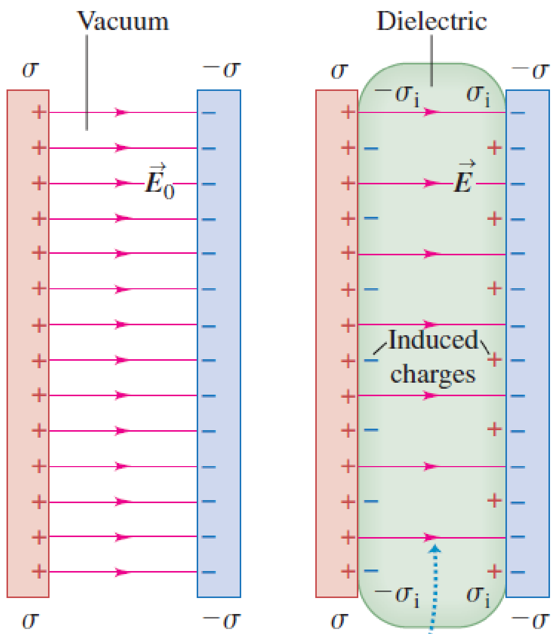
Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Dielectric Constant, K	Dielectric Strength, E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

Capacitance of a dielectric filled capacitor

The polarization of the dielectric in a capacitor causes an induced surface charge from the opposing negative and positive charges shifting in the material. The induced surface charge density σ_i in terms of the given charge density σ,

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

The permittivity ϵ of the dielectric is defined as

$$\epsilon = K\epsilon_0$$

The electric field of the capacitor follows as $E = \dfrac{\sigma}{\epsilon}$.

The capacitance when the dielectric is present is $C = KC_0 = K\epsilon_0\dfrac{A}{d} = \epsilon\dfrac{A}{d}$.

The energy density of the electric field inside the dielectric is

$$u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2$$

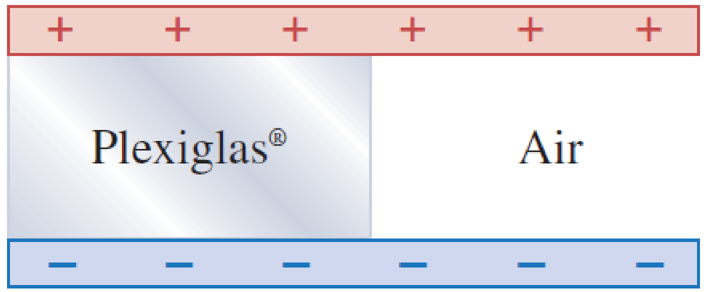
Two capacitors, C_1 and C_2, are connected in series across a source of potential difference. With the potential source still connected, a dielectric is now inserted between the plates of capacitor C_1. What happens to the charge on capacitor C_2?

A. The charge on C_2 remains the same.

B. The charge on C_2 increases.

C. The charge on C_2 decreases.

A parallel-plate capacitor is made from two plates 12.0 cm on each side and 4.50 mm apart. Half of the space between these plates contains only air, but the other half is filled with Plexiglas® of dielectric constant 3.40. An 18.0 V battery is connected across the plates.



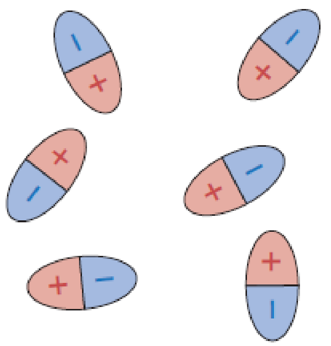
(a) What is the capacitance of this combination?

(b) How much energy is stored in the capacitor?

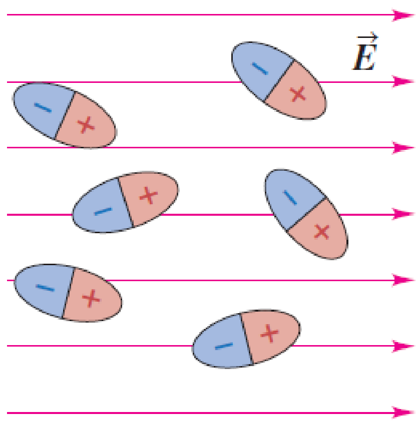
Molecular reorientation

Some molecules, such as H_2O are neutral charged but they have an uneven distribution of charges with excess positive charge concentrated on one side of the molecule and negative charge on the other.

In a temperature bath they are macroscopically isotropic (no orientation), but they become aligned when an electric field is applied.



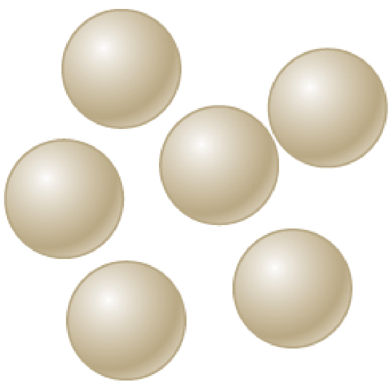
In the absence of an electric field, polar molecules orient randomly.



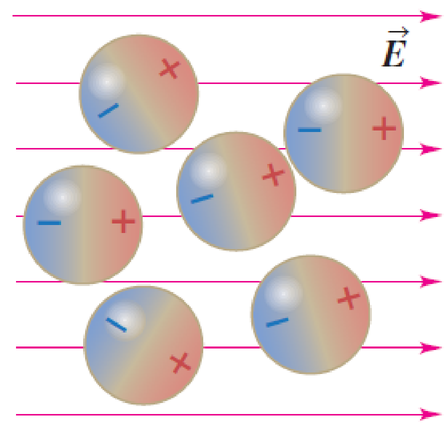
When an electric field is applied, the molecules tend to align with it.

Molecular polarization

A molecule that is not ordinarily polar can also become a dipole when it is placed in an electric field because the field pushes the positive charges in the molecules in the direction of the field and pushes the negative charges in the opposite direction.



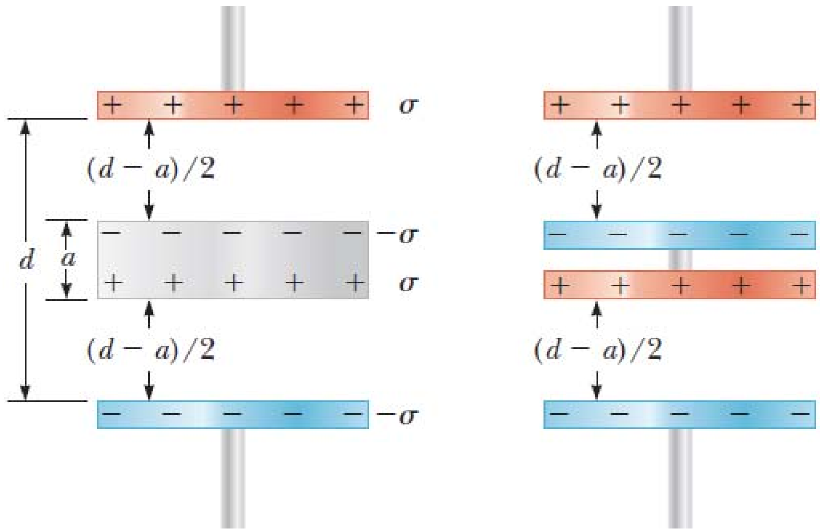
In the absence of an electric field, nonpolar molecules are not electric dipoles.



An electric field causes the molecules' positive and negative charges to separate slightly, making the molecule effectively polar.

If the electrons where allowed to move to other molecules as with the electrons in the conduction band of a conductor, then they would be called free electrons. In a dielectric material, no electrons are passed between molecules, and all electrons are called bound electrons.

A parallel-plate capacitor has a plate separation *d* and plate area *A.* An uncharged metallic slab of thickness *a* is inserted midway between the plates. Find the capacitance of the device.



Gauss's law in dielectrics

If an electric field causes a material to microscopically polarize, then the dielectric constant is not that of vacuum. Thus, the field inside a polarizable material (dielectric) will be different than the results calculated in previous chapters that assumed negligible dielectric constants.

Gauss's law for a dielectric material with a net enclosed charge of Q is given by

$$\oiint K\vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$