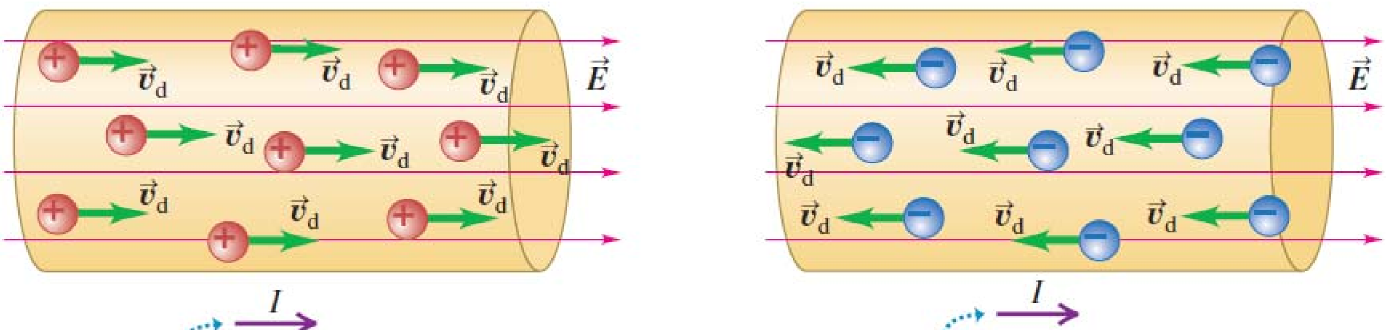
Chapter 25: Current, Resistance, and electromotive force

- Current and current density
- Resistivity
- Resistors
- Circuits and electromotive force (emf)
- Energy and power in circuits
- Conduction in metals

Current

A current is any motion of charge from one location to another.

There is a very slow net motion or drift of the moving charged particles as a group in the direction of the electric force. This motion is from the drift velocity $\vec{v}_d$ of particles. When the flowing particles are electrons, the sign of the charge is negative, and the current points in the opposite direction of the drift velocity.



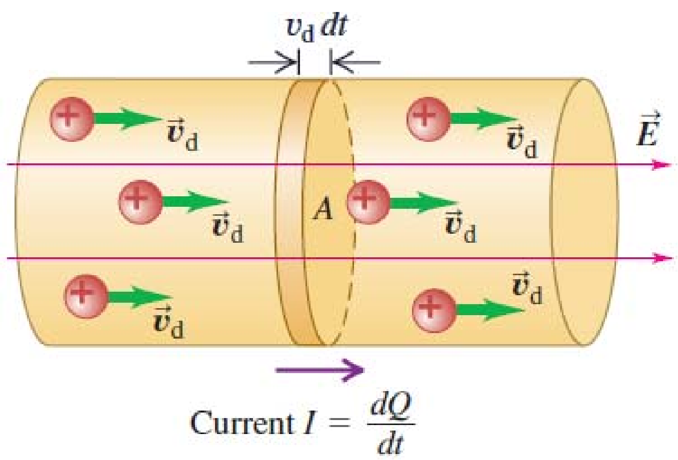
A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

We define the magnitude of the current through the cross-sectional area to be the net charge flowing through the area per unit time, $I = \dfrac{dQ}{dt}$.

Current density

The concentration of moving charges is given by n, which is the number of moving charges per unit volume.



For current running through a cylindrical wire, the charges move through a volume of $A\,|\vec{v}_d|\,dt$.

The concentration multiplied by charge and volume is the total charge that passed through a cross sectional area of the wire, $dQ = qnA\,|\vec{v}_d|\,dt$.

The current from our definition of the magnitude, $I = dQ/dt$, follows as

$$\vec{I} = qnA\vec{v}_d$$

For a uniform current flowing in the wire, the current density is

$$\vec{J} = \frac{\vec{I}}{A} = qn\vec{v}_d$$

The 12-gauge copper wire in a typical residential building has a cross-sectional area of 3.31×10^{-6} m^2. It carries a constant current of 10.0 A. What is the drift speed of the electrons in the wire? Assume each copper atom contributes one free electron to the current. The density of copper is 8.92 g/cm^3 and has a molar mass of 63.5 g/mol.

Molar mass density is $\rho = \dfrac{M}{V}$

Electron density is $n = \dfrac{N}{V} = \dfrac{N\rho}{M}$

$$v_d = \frac{I}{nqA} = \frac{mI}{qAN\rho}$$

$$v_d = \frac{(10.0\,\mathrm{A})\,(0.0635\,\mathrm{kg/mol})}{(1.60\times10^{-19}\,\mathrm{C})\,(3.31\times10^{-6}\,\mathrm{m^2})\left(6.02\times10^{23}\,\mathrm{mol^{-1}}\right)(8920\,\mathrm{kg/m^3})} = 2.23\times10^{-4}\,\mathrm{m/s}$$

Resistivity

The current density in a conductor depends on the electric field (for example, caused by the potential difference at the terminals of a battery connected to a circuit) and on the properties of the material.

For a given material at a given temperature, if the current density $\vec{J}$ is directly proportional to $\vec{E}$, then the material follows Ohm's law (material is said to be Ohmic).

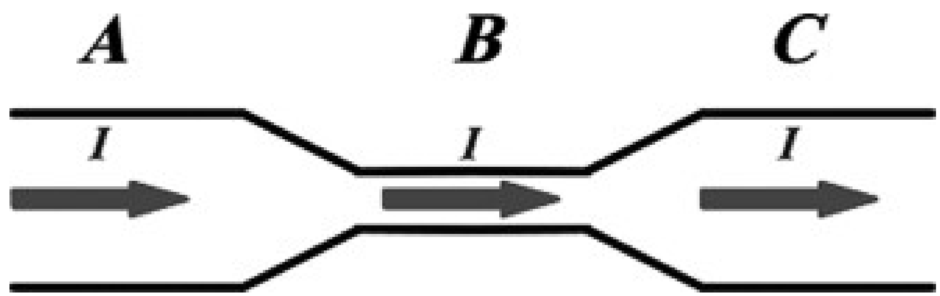
We define the resistivity of a material as the ratio of the magnitudes of electric field and current density,

$$\rho = \frac{E}{J}$$

The conductivity is the reciprocal of the resistivity,

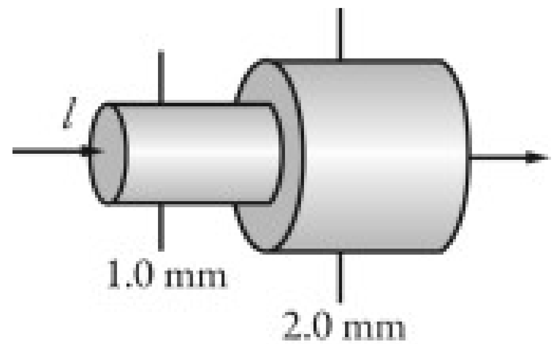
$$\sigma = \frac{1}{\rho}$$

The figure shows a steady electric current passing through a wire with a narrow region. What happens to the drift velocity of the moving charges as they go from region ***A*** to region ***B*** and then to region ***C***?



1. The drift velocity decreases from A to B and increases from B to C.
2. The drift velocity decreases all the time.
3. The drift velocity increases from A to B and decreases from B to C.
4. The drift velocity remains constant.

The figure shows two connected wires that are made of the same material. The current entering the wire on the left is 2.0 A and in that wire the electron drift speed is v_d. What is the electron drift speed in the wire on the right side?



A. $4v_d$

B. $2v_d$

C. $v_d/2$

D. $v_d/4$

Resistivities of some materials at 20 ºC

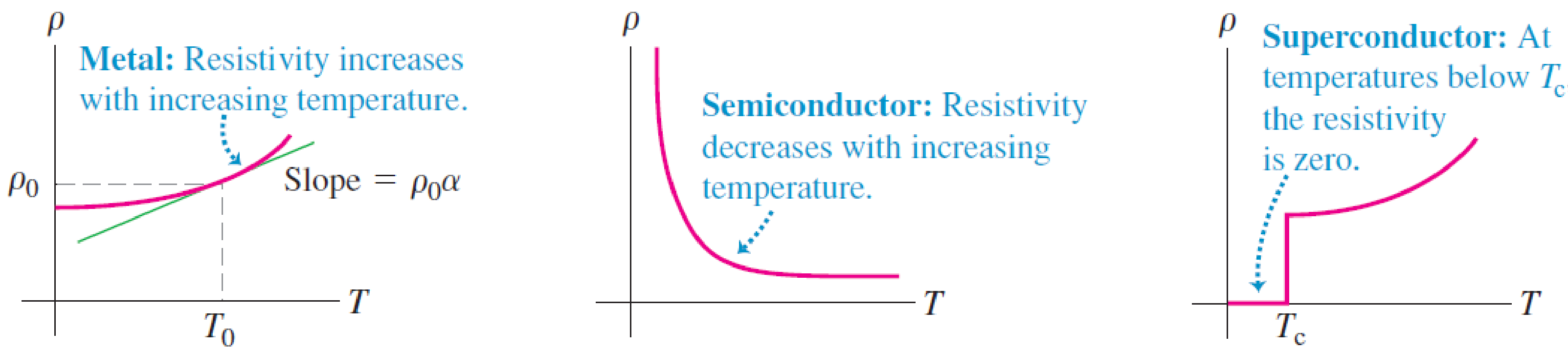
	Substance	ρ ($\Omega \cdot m$)
Conductors		
Metals	Silver	1.47×10^{-8}
	Copper	1.72×10^{-8}
	Gold	2.44×10^{-8}
	Aluminum	2.75×10^{-8}
	Tungsten	5.25×10^{-8}
	Steel	20×10^{-8}
	Lead	22×10^{-8}
	Mercury	95×10^{-8}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}
	Nichrome	100×10^{-8}
Semiconductors		
	Pure carbon (graphite)	3.5×10^{-5}
	Pure germanium	0.60
	Pure silicon	2300
Insulators		
	Amber	5×10^{14}
	Glass	$10^{10}-10^{14}$
	Lucite	$>10^{13}$
	Mica	$10^{11}-10^{15}$
	Quartz (fused)	75×10^{16}
	Sulfur	10^{15}
	Teflon	$>10^{13}$
	Wood	$10^{8}-10^{11}$

Temperature dependence of resistivity

The resistivity of a metallic conductor nearly always increases with increasing temperature. Over a small temperature range (around 100 °C), the resistivity of a metal can be represented approximately by the equation,

$$\rho(T) = \rho_0 [1 + \alpha (T - T_0)]$$

The resistivity of semiconductors decrease with increasing temperature, because at higher temperatures, more electrons are excited from the atoms and become mobile.

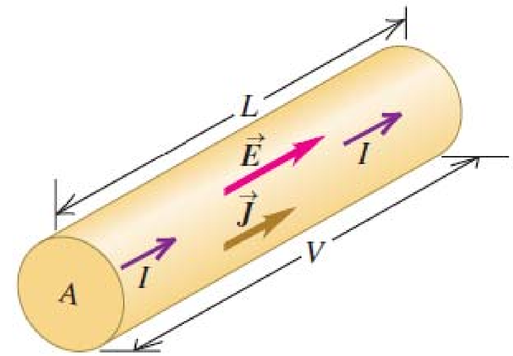


Some materials can have a resistivity of zero when the temperature of the material becomes low enough. This is called superconductivity. Metals can become superconductors, but some of the high temperature superconductors are insulating ceramics at room temperature!

Resistance and Ohm's law

For a material with a resistivity that follows Ohm's law, $\rho = \dfrac{E}{J}$, we can divide both sides of the equation by the cross-sectional area to give

$$\frac{\rho}{A} = \frac{E}{JA} = \frac{E}{I}$$



For a constant field inside a conductive cylinder, the potential difference over the length of the cylinder L is given by $V = EL$. Thus we may rewrite Ohm's law as

$$V = I\frac{\rho L}{A}$$

Defining the resistance of the cylinder in terms of its dimensions and the resistivity, $R = \dfrac{\rho L}{A}$, we can write Ohm's law in the form

$$V = IR$$

Just like the resistivity, the resistance of a conductor also changes with temperature. For small temperature changes,

$$R(T) = R_0 \left[1 + \alpha (T - T_0)\right]$$

A cylindrical wire has a resistance R and resistivity ρ. If its length and diameter are BOTH cut in half, what will be its resistance?

A. $R/2$

B. R

C. $2R$

D. $4R$

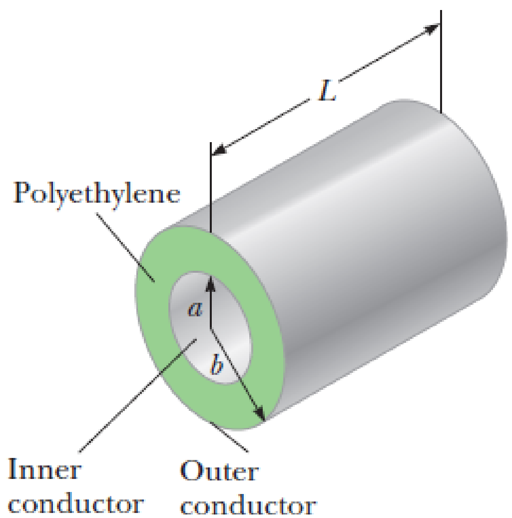
A cylindrical wire has a resistance R and resistivity ρ. If its length and diameter are BOTH cut in half, what will be its resistivity?

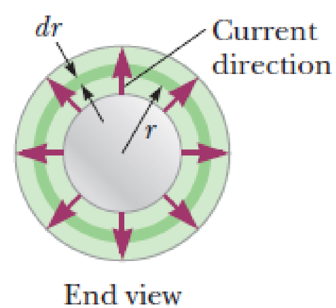
A. $\rho/2$
B. ρ
C. 2ρ
D. 4ρ

A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with polyethylene plastic. Current leakage through the plastic, in the *radial* direction, is unwanted.

The cable is designed to conduct current along its length, but that is *not* the current being considered in this problem.

The radius of the inner conductor is $a = 0.500$ cm, the radius of the outer conductor is $b = 1.75$ cm, and the length is $L = 15.0$ cm. The resistivity of the plastic is 1.0×10^{13} Vm. Calculate the resistance of the plastic between the two conductors.





You are given a copper bar of dimensions 3 cm × 5 cm × 8 cm and asked to attach leads to it in order to make a resistor. If you want to achieve the SMALLEST possible resistance, you should attach the leads to the opposite faces that measure

A. 5 cm × 8 cm.

B. 3 cm × 8 cm.

C. 3 cm × 5 cm.

D. Any pair of faces will make the same resistance.

A wire of resistivity ρ must be replaced in a circuit by a wire of the same material but 4 times as long. If, however, the resistance of the new wire is to be the same as the resistance of the original wire, the diameter of the new wire must be

A. 1/2 the diameter of the original wire.

B. the same as the diameter of the original wire.

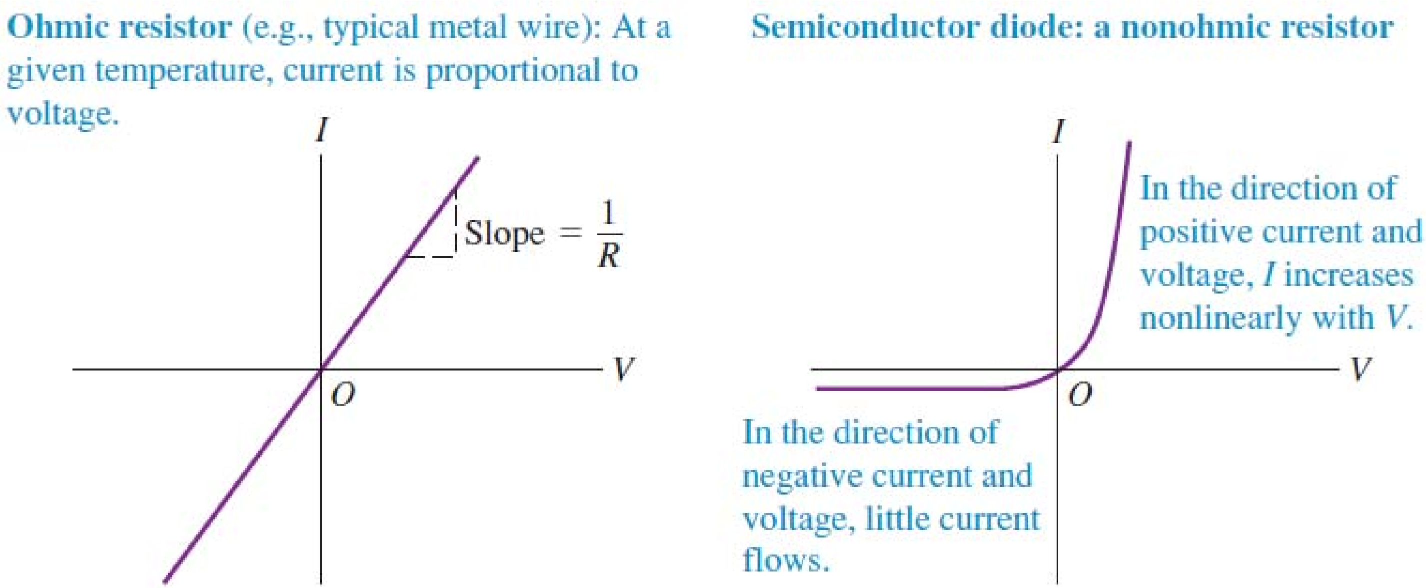
C. 2 times the diameter of the original wire.

D. 4 times the diameter of the original wire.

Resistors

A circuit device made to have a specific value of resistance between its ends is called a resistor.

For a resistor that obeys Ohm's law, a graph of current as a function of electric potential difference (voltage) is a straight line.



In devices that do not obey Ohm's law, such as a semiconducting diode, the relationship of voltage to current may not be a direct proportion, and it may be different for the two directions of current.

Resistor color codes

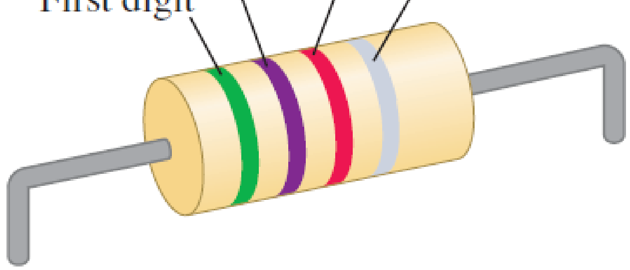
Resistors for compact, high-frequency applications are fabricated with a rectangular box geometry to specific surface mount technology (SMT) specifications.

Resistors used in breadboards and common electronics are often cylindrical, a few millimeters in diameter and length, and have wires coming out of the ends.

The cylindrical resistors are usually color coded to specify their value of resistance.

Color Codes for Resistors

Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9



Color	Tolerance
Gold	5%
Silver	10%
No band	20%

Electromotive force

When a circuit forms a closed loop, we call it a complete circuit. When the loop is left open, it is an incomplete circuit.

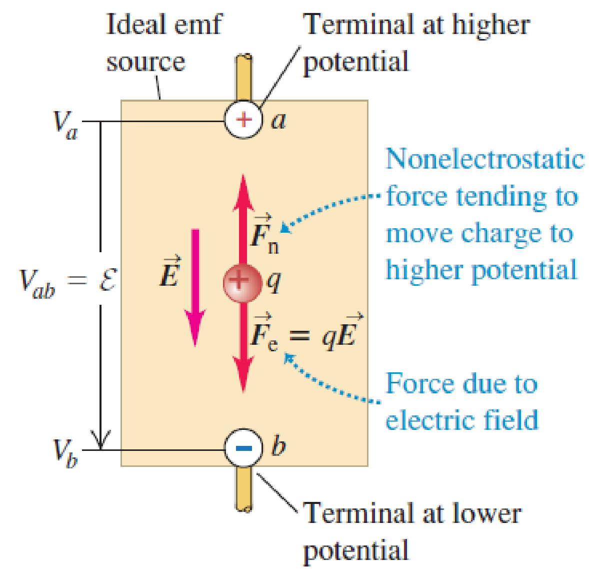
The influence that makes current flow from lower to higher potential is called the electromotive force (emf), $\mathcal{E}$. The emf is not a force; it is an energy per unit charge quantity. It has the same units as the electric potential!!!

$$\mathcal{E} = V \quad \text{(ideal emf source)}$$

Every complete circuit with a steady current must include some device that provides emf.

For a simple circuit with a resistor and an ideal emf source,

$$\mathcal{E} = V = IR$$

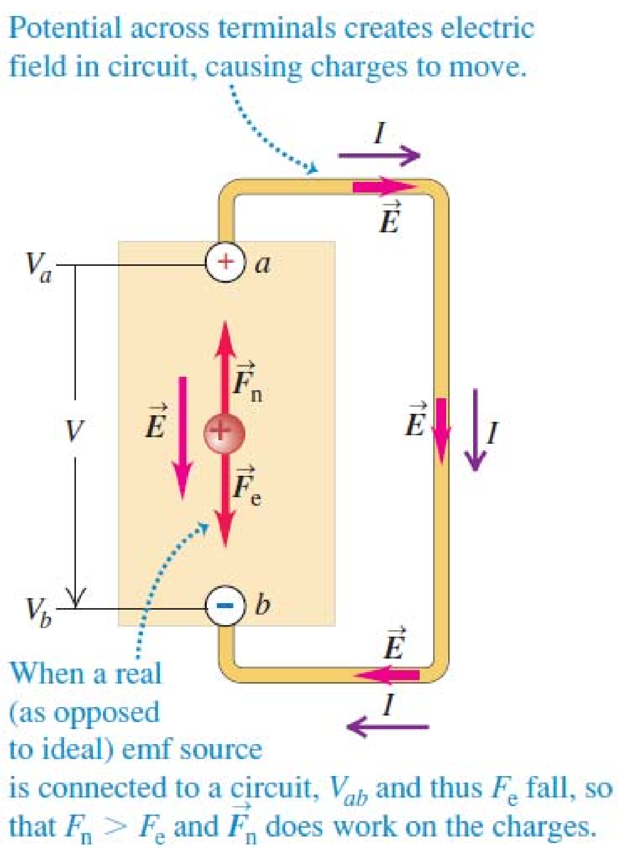


When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

Unideal emf source with internal resistance

Real sources of emf in a circuit are not ideal. Any real source of emf has some resistance, which we call the internal resistance, *r*.

When a current is flowing through a source from the negative to positive terminal, the potential difference between the terminals (terminal voltage) is



$$V = \mathcal{E} - Ir$$

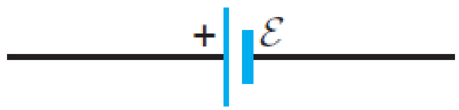
For a simple circuit with a resistor and a unideal emf source, the current flowing through the terminal is

$$I = \frac{\mathcal{E}}{r + R}$$

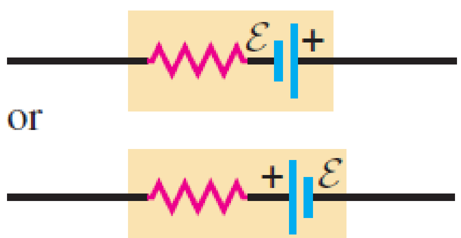
Some circuit diagram symbols

Conductor with negligible resistance

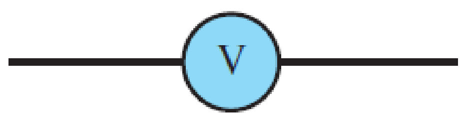
Resistor



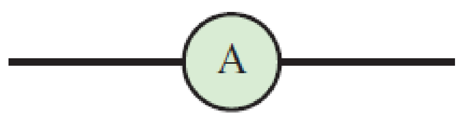
Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)



Source of emf with internal resistance *r* (*r* can be placed on either side)



Voltmeter (measures potential difference between its terminals)



Ammeter (measures current through it)

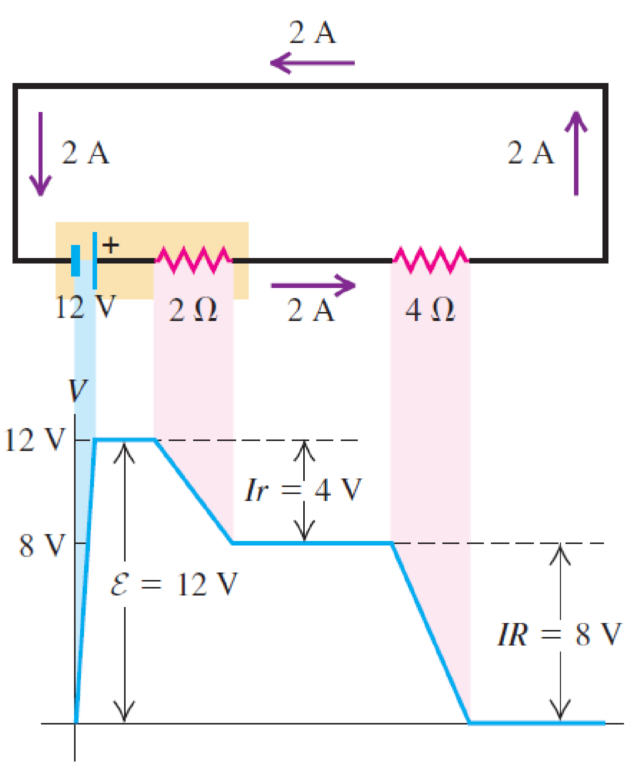
Potentials around circuits

The net change in potential energy for a charge q making a round trip around a complete circuit must be zero.

$$\mathcal{E} - Ir - IR = 0$$

$$\Longrightarrow \quad V - IR = 0$$

Potential rises and drops in a circuit.



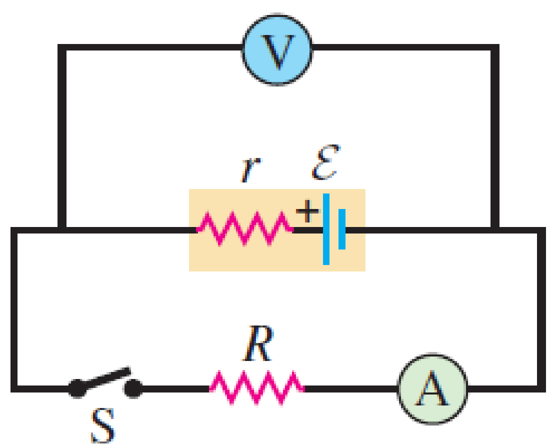
A potential gain of $\mathcal{E}$ is associated with the emf, and potential drops of Ir and IR are associated with the internal resistance of the source and the external circuit, respectively.

When switch S in the figure is open, the voltmeter V of the battery reads 3.08 V. When the switch is closed, the voltmeter reading drops to 2.97 V, and the ammeter A reads 1.65 A. Assume that the two meters are ideal, so they don't affect the circuit.

(a) Determine the emf.

(b) Determine the internal resistance of the battery.

(c) Determine the circuit resistance *R*.



When a potential difference of 10 V is placed across a certain solid cylindrical resistor, the current through it is 2 A. If the diameter of this resistor is now tripled, the current will be

A. 3 A.

B. 2/3 A.

C. 2/9 A.

D. 18 A.

If the voltage across a circuit of constant resistance is doubled, how is the current in the circuit affected?

A. The current is quadrupled.

B. The current is reduced by a factor of 2.

C. The current is reduced by a factor of 4.

D. The current is doubled.

If the resistance in a circuit connected to a constant current is halved, how is the voltage in the circuit affected?

A. The voltage is increased by a factor of 2.

B. The voltage remains constant.

C. The voltage is reduced by a factor of 2.

D. The voltage is quadrupled.

Energy and power in electric circuits

As an amount of charge q passes through the circuit element, there is a change in potential energy equal to qV.

If the change in energy is **negative** (electrons moving from the negative terminal to the positive terminal), then energy is being transferred out of the circuit in the form of heat and/or work.

If the change in energy is **positive** (electrons moving from the positive terminal to the negative terminal), then energy is being transferred **into** the circuit, *e.g.*, charging a battery.

If the current through the element is $dQ = I\,dt$, then in a time interval an amount of charge passes through the element. Multiplying by the potential difference then gives, $VdQ = IV\,dt$.

For a constant potential difference and current, dividing by the change in time gives us an expression for the power,

$$P = V\frac{dQ}{dt} = IV \quad \Longrightarrow \quad P = I^2 R = \frac{V^2}{R}$$

A circuit maintains a constant resistance. If the current in the circuit is doubled, what is the effect on the power dissipated by the circuit?

A. The power dissipated is quadrupled.

B. The power dissipated remains constant.

C. The power dissipated is doubled.

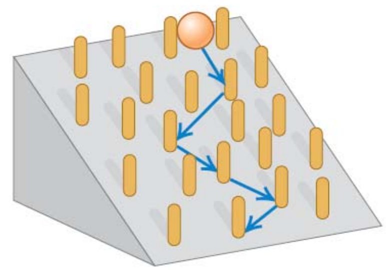
D. The power dissipated is reduced by a factor of 2.

Theory of metallic conduction

From Newton's 2nd law, we may write the acceleration of a charged particle in an electric field as

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

The average time between collisions is denoted by $t = \tau$. For an electron that has zero initial velocity, the velocity just before a collision is $\vec{v} = \vec{v}_0 + \vec{a}\tau$.



The average velocity is the sum of the zero initial velocity and the final velocity after the electron is accelerated,

$$\vec{v}_{\mathrm{avg}} = \vec{a}\tau = \frac{q\vec{E}}{m}\tau$$

The average velocity is equal to the drift velocity, $\vec{v}_{\mathrm{avg}} = \vec{v}_{\mathrm{d}}$, and therefore we may use our earlier expression for the current density to write

$$\vec{J} = nqv_{\mathrm{d}} = \frac{nq^2\tau\vec{E}}{m} = \frac{\vec{E}}{\rho}$$

Substituting $-e = q$ and solving the above equation for ρ gives $\boxed{\rho = \dfrac{m}{ne^2\tau}}$

An electric heater is constructed by applying a potential difference of 120 V across a Nichrome wire that has a total resistance of 8.00 Ω.

(a) Determine the current carried by the wire.

(b) Determine the power rating of the heater.

An immersion heater must increase the temperature of 1.50 kg of water from 10.0 °C to 50.0 °C in 10.0 min while operating at 110 V. The heat capacitance of liquid water at 1 atm of pressure C_w is 4.184 kJ kg^{-1} $°C^{-1}$. What is the required resistance of the heater?