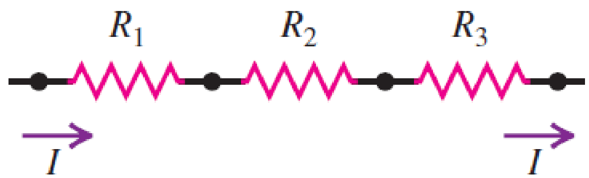
Chapter 26: Direct current circuits

- Resistors in series and parallel
- Kirchhoff's rules
- Electrical measuring instruments
- RC circuits
- Power distribution systems

Resistors in series



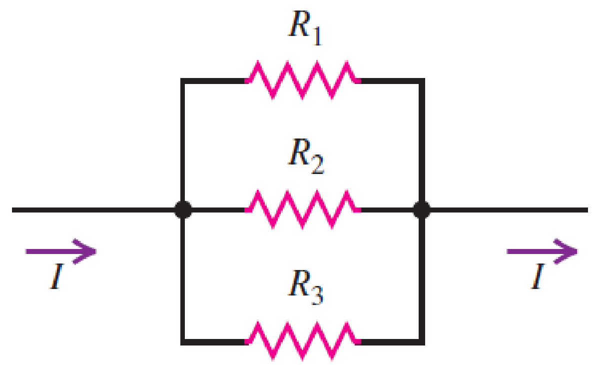
If the resistors are in series, then the current I must be the same in all of them. The potential difference across the entire combination of the resistors in series is the sum of the individual potential differences across each resistor,

$$\begin{aligned} V &= V_1 + V_2 + V_3 + \cdots \\ &= IR_1 + IR_2 + IR_3 + \cdots \\ &= I\left(R_1 + R_2 + R_3 + \cdots\right) \end{aligned}$$

The equation can be rewritten with an equivalent resistance, $V = IR_{\text{eq}}$, where

$$R_{\text{eq}} = \sum_{i=1}^{N} R_i$$

Resistors in parallel



If the resistors are in parallel, then the potential difference between the terminals of each resistor must be the same,

$$V = I_1 R_1\,,\ V = I_2 R_2\,,\ V = I_3 R_3\,,\ \cdots$$

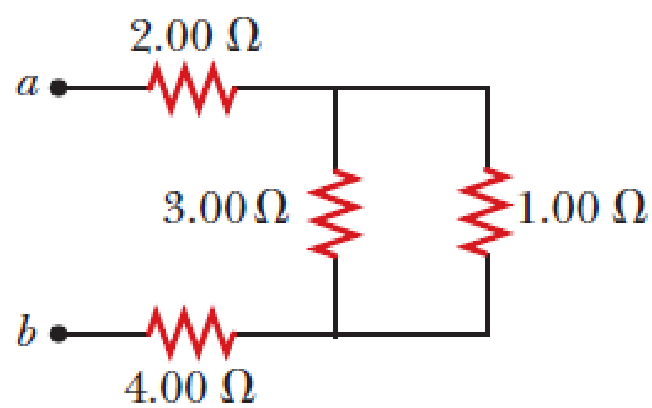
The total current I must equal the sum of the currents in the resistors,

$$I = I_1 + I_2 + I_3 + \cdots = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \cdots$$

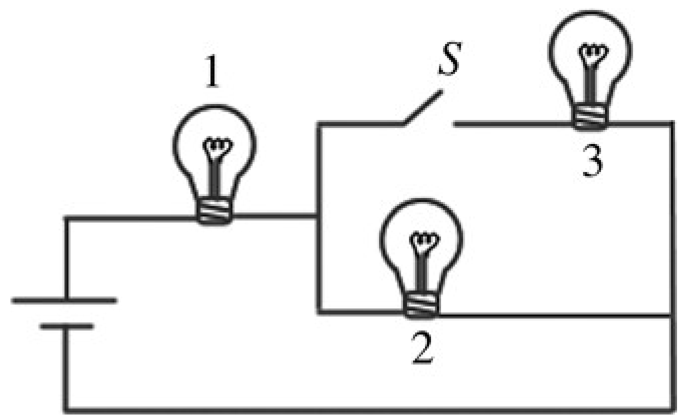
The equation can be rewritten with an equivalent resistance, $I = \dfrac{V}{R_{eq}}$, where

$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

Four resistors are connected as shown. Find the equivalent resistance between points *a* and *b*.



The figure shows three identical lightbulbs connected to a battery having a constant voltage across its terminals. What happens to the brightness of lightbulb 1 when the switch *S* is closed?



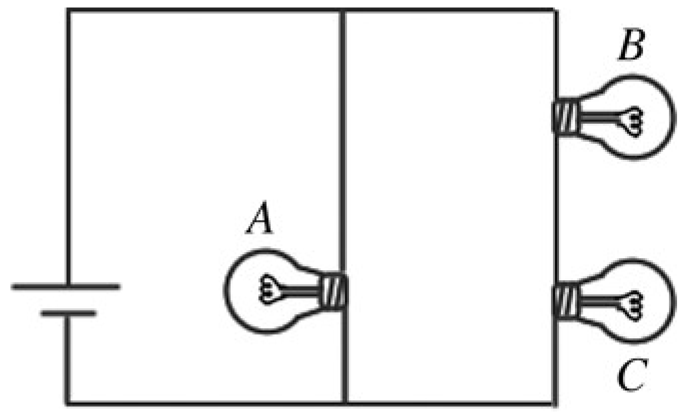
A. The brightness increases permanently.

B. The brightness will decrease momentarily and then return.

C. The brightness remains the same as before the switch is closed.

D. The brightness decreases permanently.

In the circuit shown in the figure, all the lightbulbs are identical. Which of the following is the correct ranking of the brightness of the bulbs?



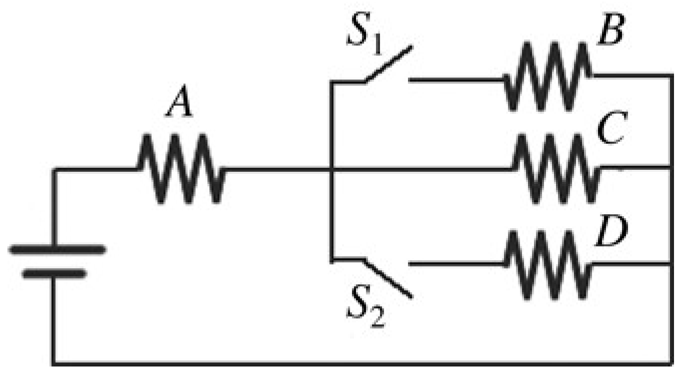
A. A is brightest, C is dimmest, and B is in between.

B. A is the brightest, and B and C have equal brightness but less than A.

C. A and B have equal brightness, and C is the dimmest.

D. B and C have equal brightness, and A is the dimmest.

In the circuit shown below, four identical resistors labeled A to D are connected to a battery. S_1 and S_2 are switches. Which of the following actions would result in the GREATEST amount of current through resistor A?



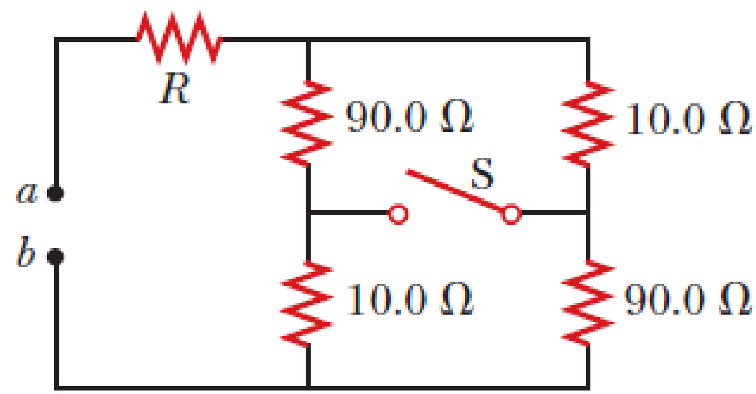
A. Closing S_2 only.

B. Closing both switches.

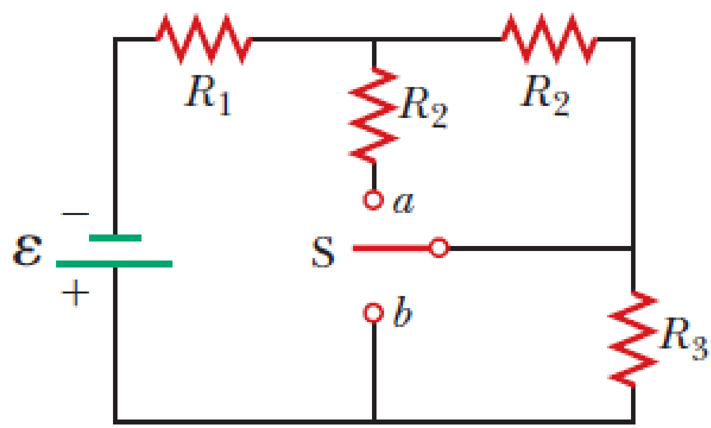
C. Closing S_1 only.

D. Leaving both switches open as shown.

For the circuit shown in the figure, assume the equivalent resistance drops by 50.0% when the switch is closed. Determine the value of *R*.

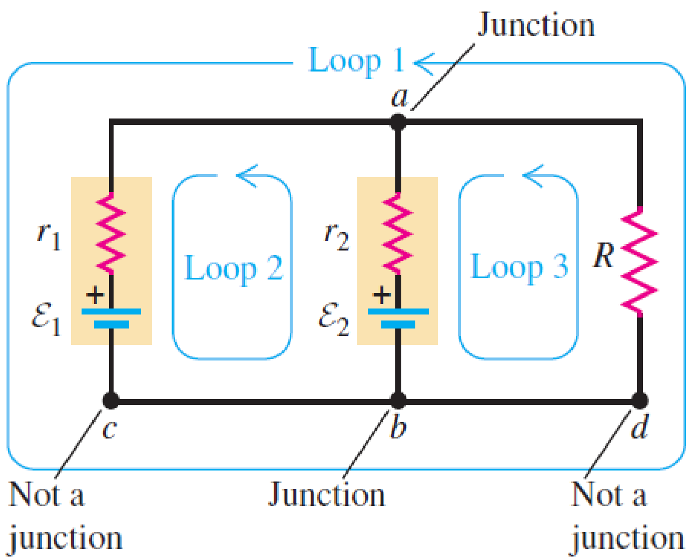


A battery with $\mathcal{E} = 6.00$ V and no internal resistance supplies current to the circuit shown in the figure. When the double-throw switch S is open as shown in the figure, the current in the battery is 1.00 mA. When the switch is closed in position *a*, the current in the battery is 1.20 mA. When the switch is closed in position *b*, the current in the battery is 1.80 mA. Find the resistances (a) R_1, (b) R_2, and (c) R_3.



Kirchhoff's rules

The German physicist Gustav Robert Kirchhoff developed techniques to calculate the current through the wires of a circuit.



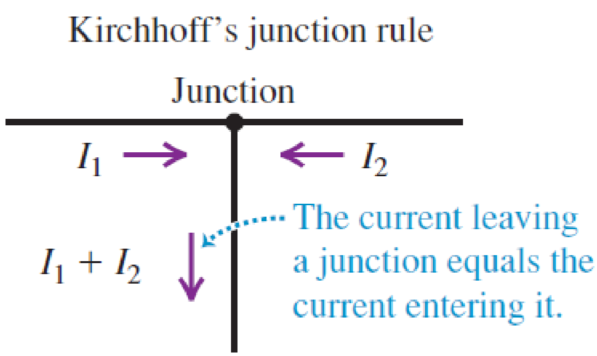
(1) **Junction rule**: the algebraic sum of the currents into any junction is zero.

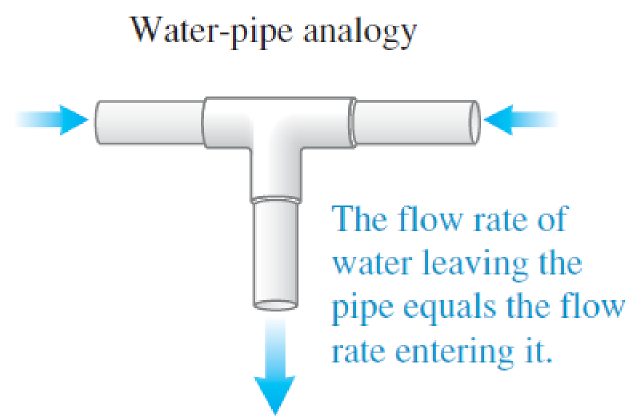
$$\sum I = 0$$ **(true at any junction)**

(2) **Loop rule**: the algebraic sum of the potential differences in any loop, including those associated with emfs and those of resistive elements, must equal zero.

$$\sum V = 0$$ **(true for any closed loop)**

The junction rule





The junction rule is based on conservation of electric charge. No charge can accumulate at a junction.

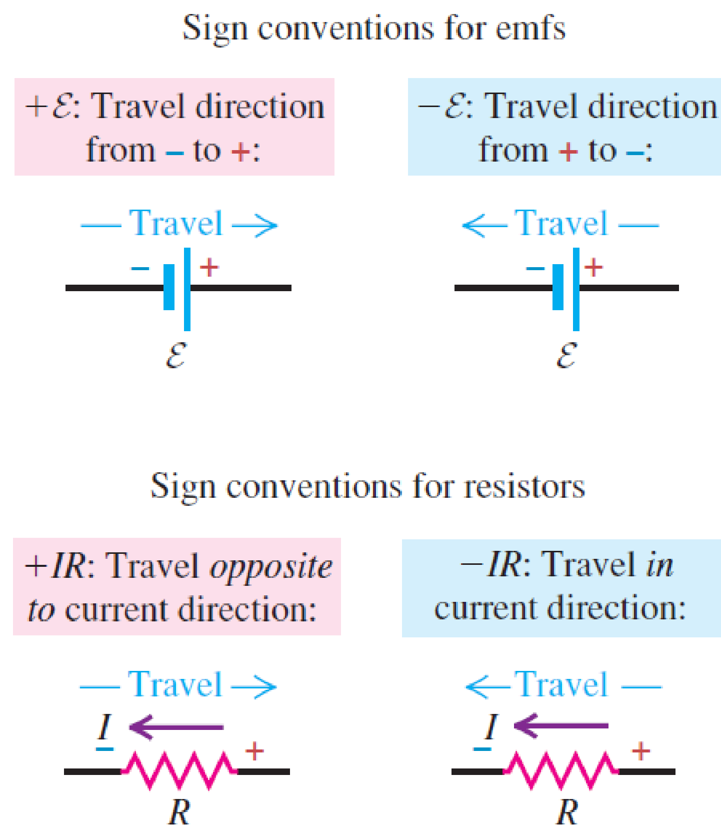
Therefore, the total charge entering the junction per unit time must equal the total charge leaving per unit time.

This rule is similar to the current at a water pipe junction, where there cannot be an accumulation of water at a junction and the mass of water must be conserved.

The loop rule

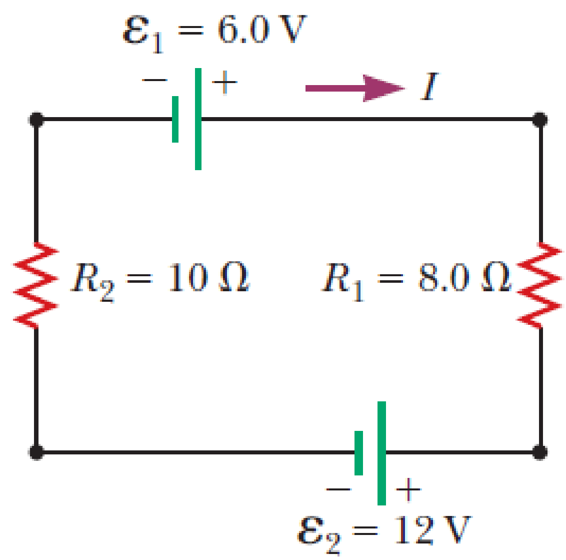
First assume a direction for the current in each branch of the circuit and mark it on a diagram of the circuit. Then, starting at any point in the circuit, add emfs and IR terms as you travel around the loop.

Follow the sign convention shown below to either add or subtract potential as you travel around a loop.



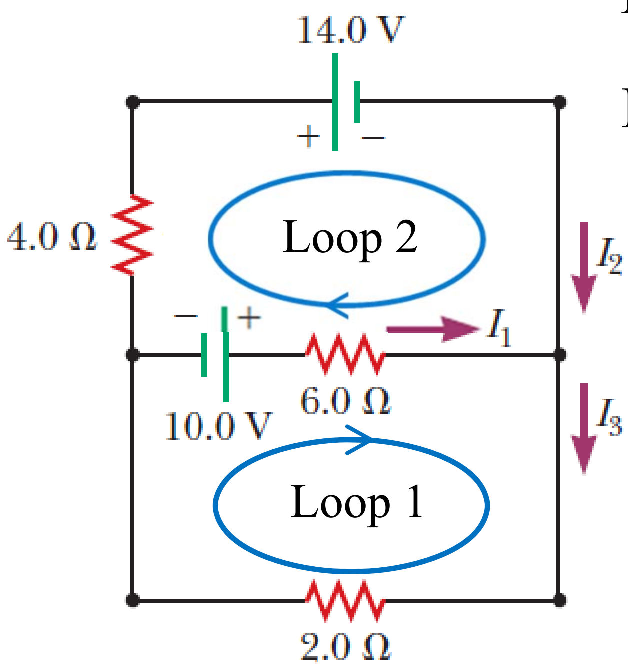
If you solve for a current and get a negative value, then it simply means you assumed the opposite direction for the current.

A single-loop circuit contains two resistors and two batteries as shown. Neglect the internal resistances of the batteries. Find the current in the circuit.



We cannot simplify the circuit by the rules associated with combining resistances in series and in parallel. (If the 10.0 V battery were removed and replaced by a wire from b to the 6.0-V resistor, we could reduce the remaining circuit.) Because the circuit is not a simple series and parallel combination of resistances, this problem is one in which we must use Kirchhoff's rules.

Junction rule: $I_1 + I_2 - I_3 = 0$



Loop 1: $10.0\,\text{V} - (6.0\,\Omega)\,I_1 - (2.0\,\Omega)\,I_3 = 0$

Loop 2: $-(4.0\,\Omega)\,I_2 - 14.0\,\text{V} + (6.0\,\Omega)\,I_1 - 10.0\,\text{V} = 0$

Three independent equations and three unknowns.

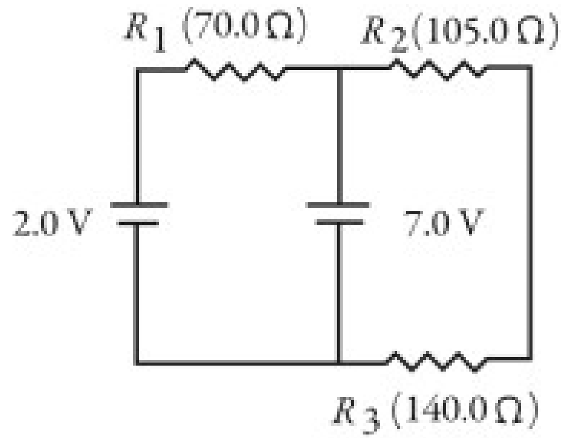
Solving for the currents gives

$$I_1 = 2.0\,\text{A}$$

$$I_2 = -3.0\,\text{A}$$

$$I_3 = -1.0\,\text{A}$$

For the circuit shown in the figure, what is the current through resistor R_1?



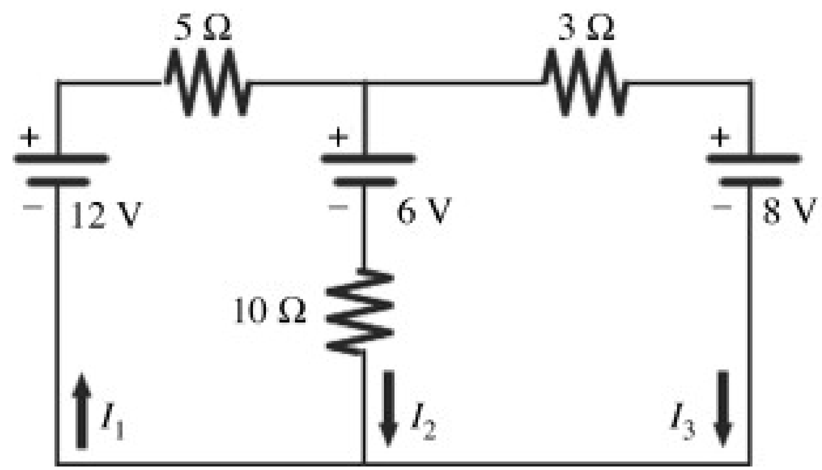
A. 0.071 A

B. 0.016 A

C. 0.130 A

D. 0.029 A

For the circuit shown in the figure, all quantities are accurate to 2 significant figures. What is the value of the current I_1?



A. 0.32 A

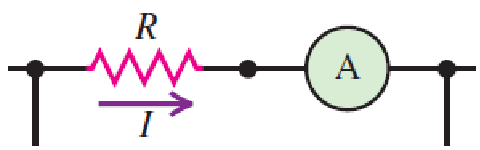
B. 0.11 A

C. 0.61 A

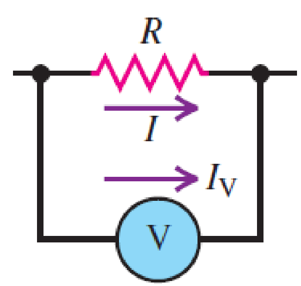
D. 0.89 A

Electric measuring instruments

A current-measuring instrument is called an **ammeter**. An ammeter measures the current passing through it. An ammeter is placed in a series configuration when measuring the current in a wire. Ammeters should have as little internal resistance as possible.



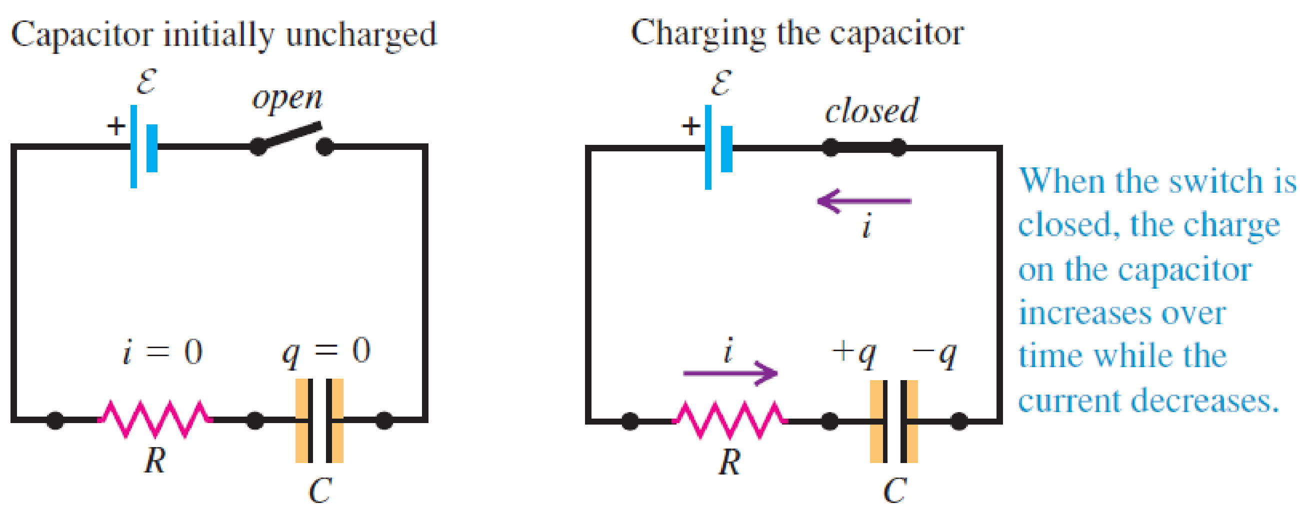
A **voltmeter** measures the electric potential difference between two points. A voltmeter is placed in a parallel configuration when measuring the voltage between two points in a circuit. Voltmeters should have a large internal resistance such that the change in the current(s) through the circuit caused by the presence of the voltmeter is negligible.



Simple RC circuit

Circuits can have both resistors and capacitors in them. Thus, the current in wires and potential differences across resistors can change with time.

A simple RC circuit (resistor-capacitor circuit) has a single loop with a capacitor in series with a resistor. A switch is initially open and the capacitor is charged. When the switch closes, the capacitor begins to charge.



Charging a capacitor in a simple RC circuit

After the switch is closed, the instantaneous potential differences across the resistor and capacitor given in terms of the instantaneous current *i* and charge *q* are

$$v_R = iR \quad \text{and} \quad v_C = \frac{q}{C}$$

Kirschoff's loop rule gives $\mathcal{E} - iR - \frac{q}{C} = 0$. We can rewrite this equation as

$$i = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

The current in the loop changes as the capacitor begins to charge, $i = \frac{dq}{dt}$. Thus, we have the first-order differential equation,

$$\frac{dq}{dt} = -\frac{1}{RC}(q - C\mathcal{E})$$

This is a separable equation, where $\int_0^q \frac{dq'}{q' - C\mathcal{E}} = -\int_0^t \frac{dt'}{RC}$. The solution is

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

$q(t)$, $i(t)$, & $v(t)$ for a charging capacitor

The full charge on a capacitor is $Q_f = C\mathcal{E}$. Therefore, using the results from the previous slide, we may express the charge as a function of time as

$$q(t) = Q_f\left(1 - e^{-t/RC}\right)$$

Recalling that $i = \frac{dq}{dt}$ and defining the initial current immediately after the switch was closed to be $I_0 = \frac{\mathcal{E}}{R}$, the current as a function of time is

$$i(t) = I_0 e^{-t/RC}$$

The potential difference across the capacitor plates as a function of time is found by dividing the $q(t)$ equation by the capacitance C, which gives

$$v(t) = \mathcal{E}\left(1 - e^{-t/RC}\right)$$

Note that the RC time constant is sometimes denoted by τ, where $\tau = RC$.

A resistor and a capacitor are connected in series across an ideal battery having a constant voltage across its terminals. At the moment contact is made with the battery the voltage across the capacitor C is

A. equal to the battery's terminal voltage.

B. less than the battery's terminal voltage, but greater than zero.

C. zero.

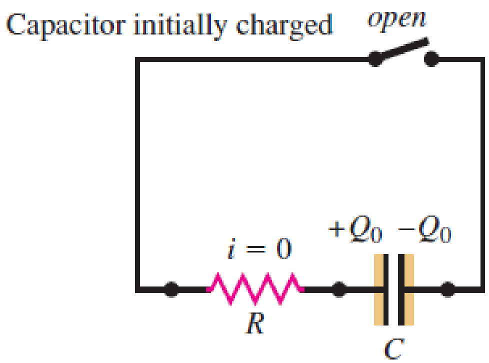
A resistor and a capacitor are connected in series across an ideal battery having a constant voltage across its terminals. At the moment contact is made with the battery the voltage across the resistor R is

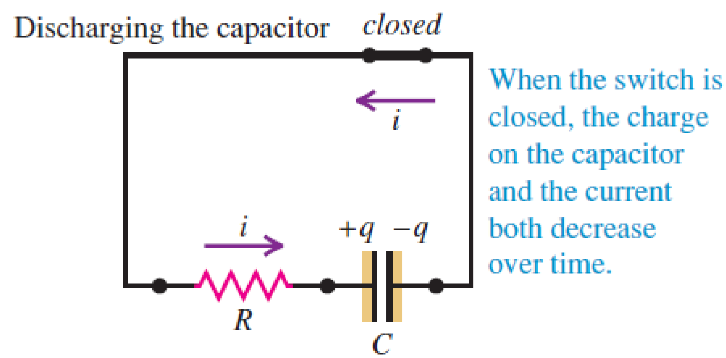
A. equal to the battery's terminal voltage.

B. less than the battery's terminal voltage, but greater than zero.

C. zero.

Discharging a capacitor





For an initially charged capacitor with initial charge Q_0, the instantaneous potential differences across the resistor and capacitor given in terms of the instantaneous current i and charge q after the switch is closed are

$$v_R = iR \quad \text{and} \quad v_C = \frac{q}{C}$$

Kirschoff's loop rule gives $iR + \frac{q}{C} = 0 \implies i = -\frac{q}{RC} = \frac{dq}{dt}$.

This is a separable equation, where $\int_{Q_0}^{q} \frac{dq'}{q'} = -\int_0^t \frac{dt'}{RC}$. The solution is

$$\ln\left(\frac{q}{Q_0}\right) = -\frac{t}{RC}$$

$q(t)$, $i(t)$, & $v(t)$ for a discharging capacitor

Using the results from the previous slide, we may express the charge as a function of time for a discharging capacitor as

$$q(t) = Q_0 e^{-t/RC}$$

Recalling that $i = \frac{dq}{dt}$ and defining the initial current to be $I_0 = -\frac{Q_0}{RC}$, the current as a function of time for a discharging capacitor is

$$i(t) = I_0 e^{-t/RC}$$

The potential difference across the capacitor plates as a function of time for a discharging capacitor is

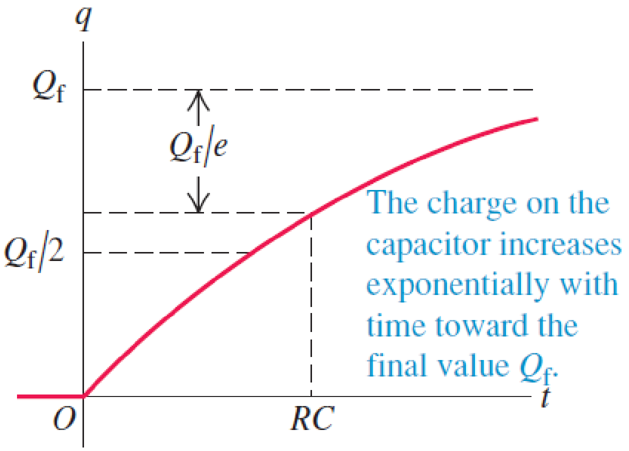
$$v(t) = V_0 e^{-t/RC}$$

where the initial potential difference across the capacitor plates is $V_0 = \frac{Q_0}{C}$.

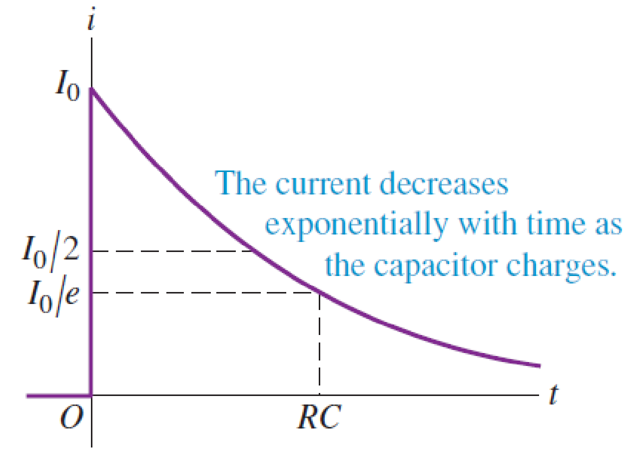
Graphs of charging and discharging capacitors

Charging

Graph of capacitor charge versus time for a charging capacitor

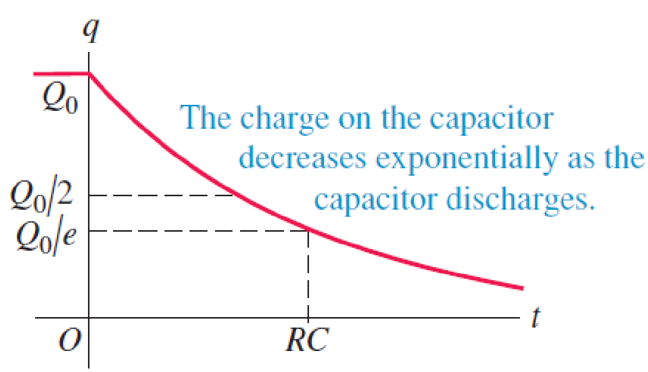


Graph of current versus time for a charging capacitor

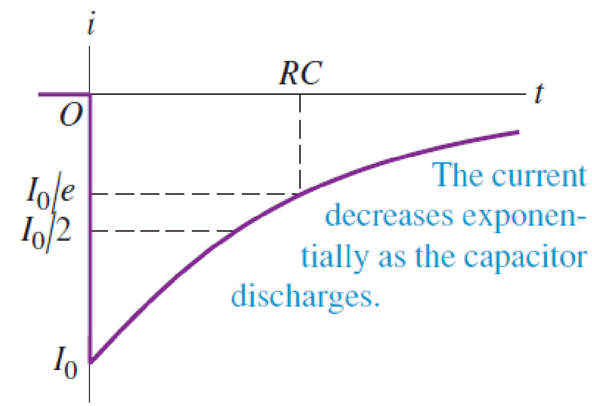


Discharging

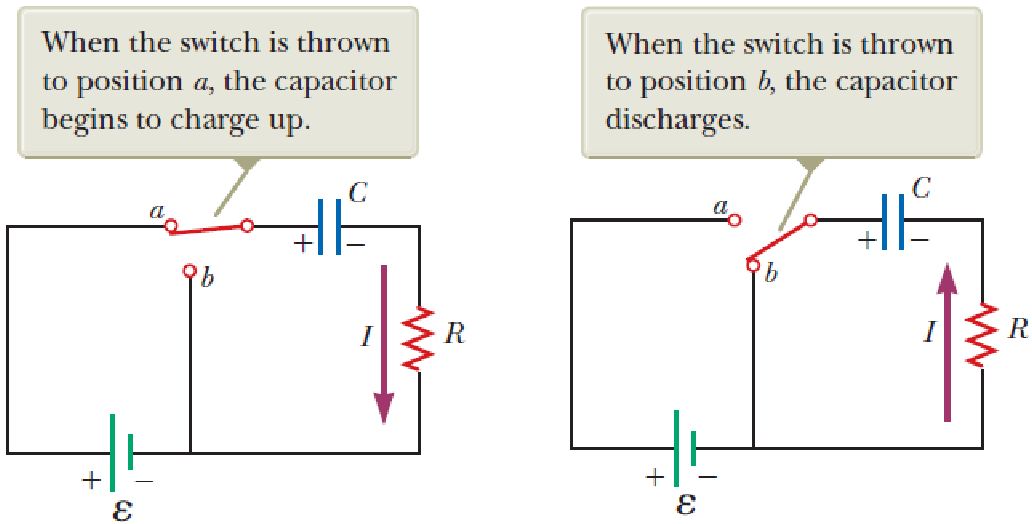
Graph of capacitor charge versus time for a discharging capacitor



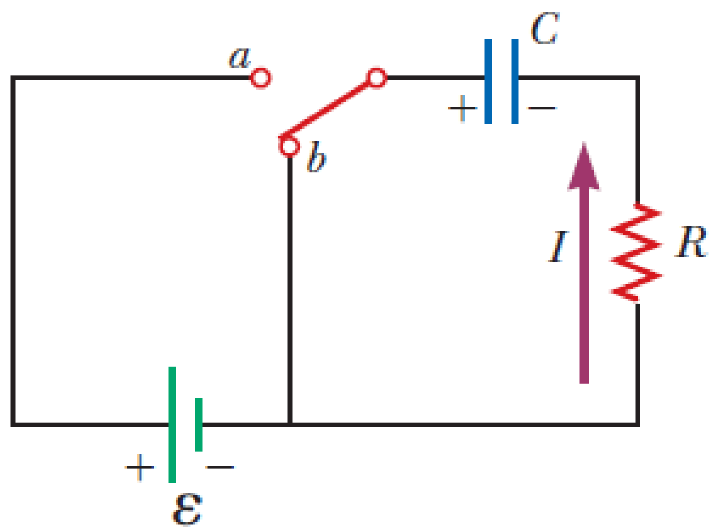
Graph of current versus time for a discharging capacitor



An uncharged capacitor and a resistor are connected in series to a battery as shown, where $\mathcal{E} = 12.0$ V, $C = 5.00$ mF, and $R = 8.00 \times 10^5\ \Omega$. The switch is thrown to position *a*. Find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.



Consider a capacitor of capacitance C that is being discharged through a resistor of resistance R as shown in the figure. After how many time constants is the charge on the capacitor one-fourth its initial value?



A 5.00 μF capacitor is charged to a potential difference of 800 V and then discharged through a resistor. How much energy is delivered to the resistor in the time interval required to fully discharge the capacitor?

A 5.5 mF capacitor is discharged through a 2.1 kΩ resistor. How long will it take for the capacitor to lose half its initial stored energy?

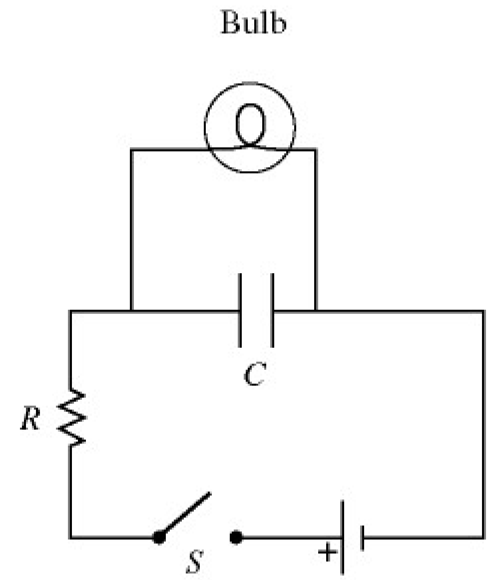
A. 11.6 s

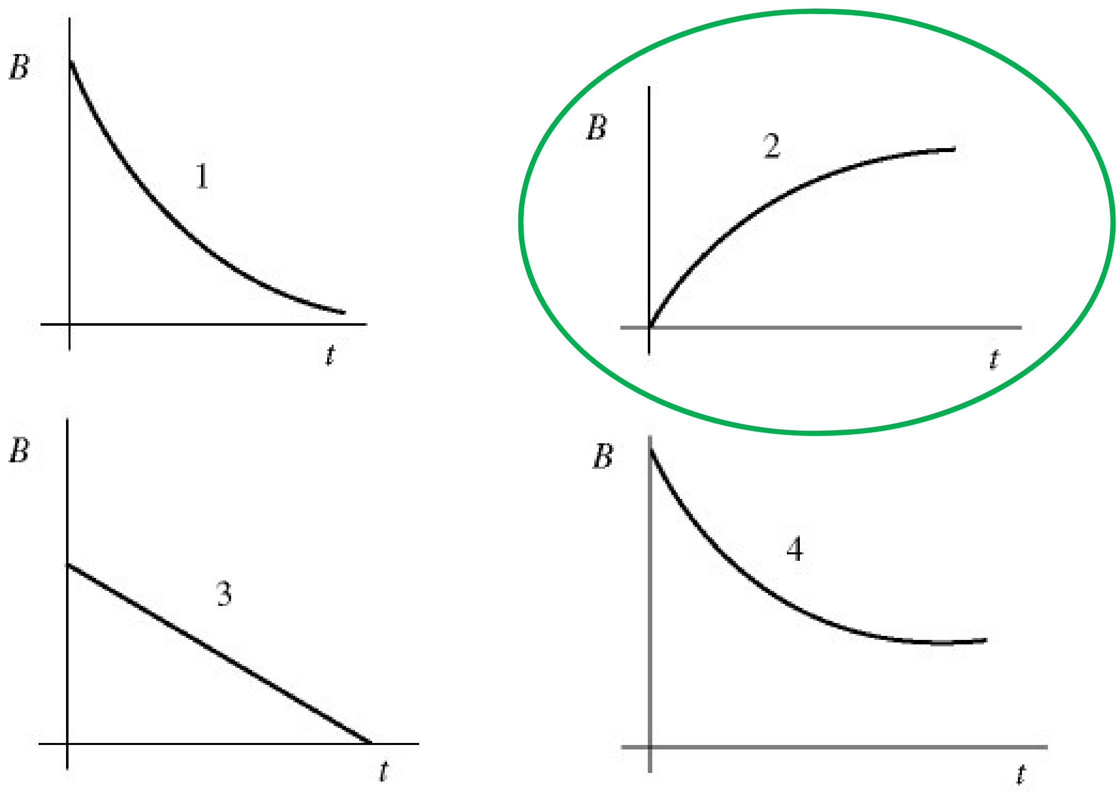
B. 8.01 s

C. 5.78 s

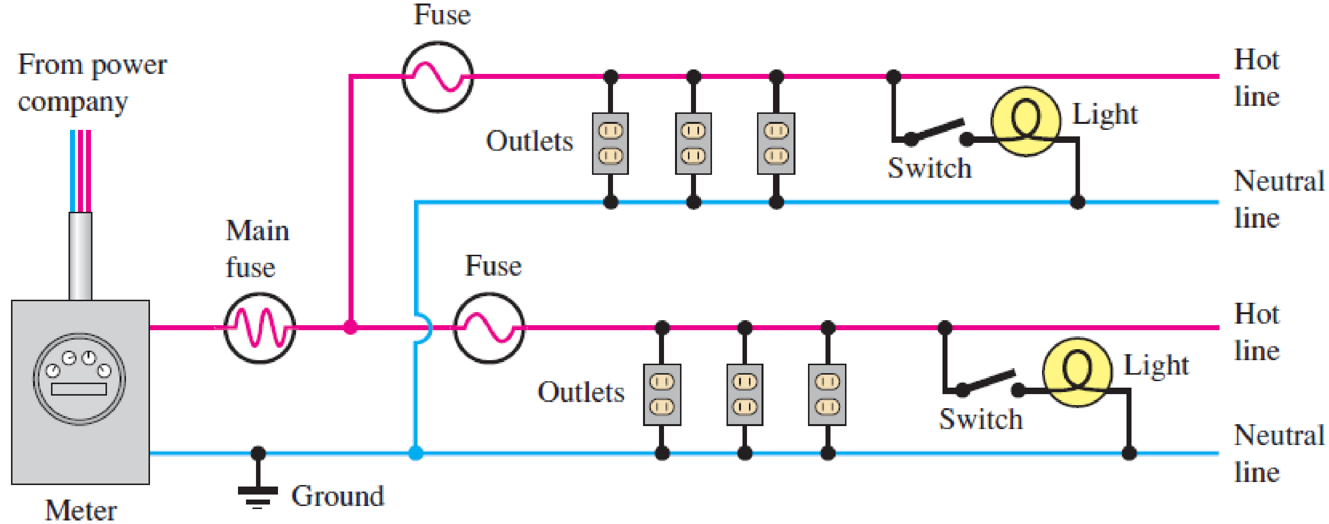
D. 4.00 s

A light bulb is connected in the circuit shown in the figure with the switch S open and the capacitor uncharged. The battery has no appreciable internal resistance. Which one of the following graphs best describes the brightness B of the bulb as a function of time t after closing the switch?





Household wiring



The power company provides three conductors. One is neutral; the other two are both at V_{rms} = 120 V with respect to the neutral but with opposite polarity (180° out of phase), giving a voltage between them of V_{rms} = 240 V.

The various lights and appliances to be operated are always connected in *parallel* to the power source.

A *fuse* or *circuit breaker* limits the amount of current through wire severing the closed loop when the current exceeds threshold.

For houses, *ground* is an actual electrode driven into the earth.