Chapter 28: Sources of the magnetic field

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- Magnetic field of a straight current carrying conductor
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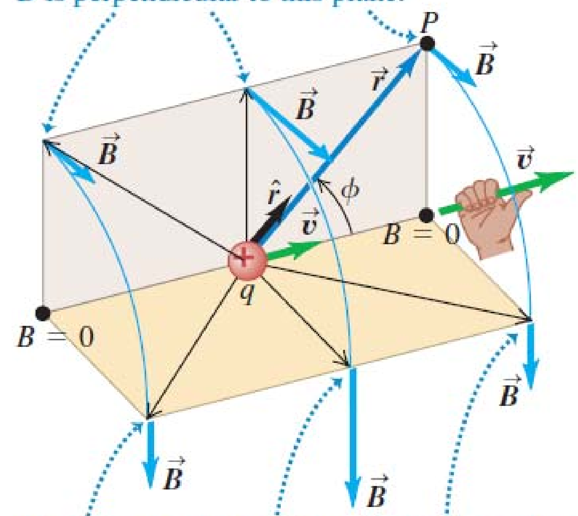
Magnetic field of a moving charge



We call the location of the moving charge at a given instant the <u>source point</u> and the point *P* where we want to find the field the <u>field point</u>.

The magnitude of the magnetic field at point *P* for a single charge moving in space is given by

$$B = \frac{\mu_0}{4\pi}\frac{|q|\,v}{r^2}\sin\phi$$

where μ_0 is the permeability of free space, $\mu_0 = 4\pi\times10^{-7}$ N/A^2.

The direction of the magnetic field wraps around the moving charge's trajectory according to the right-hand-rule. The angle ϕ is made between the trajectory and the shortest line between the source point and point *P* and defined in the range 0° to 180°.

The vector representation of the magnetic field in this case is $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$.

Magnetic field of a current-carrying element

The total magnetic field caused by several moving charges is the vector sum of the fields caused by the individual charges.

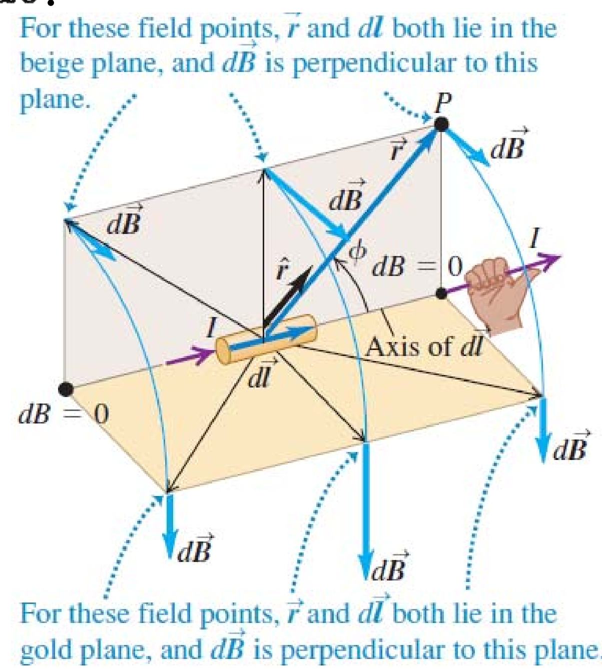
This concept can be applied to many charge carriers creating a current I.

If there are n moving charged particles per unit volume, each of charge q, the total moving charge dQ in the segment is $dQ = nqA\,d\ell$.

The magnitude of the magnetic field $d\vec{B}$ follows as

$$dB = \frac{\mu_0}{4\pi}\frac{|dQ|\ v_\mathrm{d}}{r^2}\sin\phi$$

$$= \frac{\mu_0}{4\pi}\frac{n\,|q|\,v_\mathrm{d} A\,d\ell}{r^2}\sin\phi$$



The current is given by $I = n|q|v_\mathrm{d}A$, which gives

$$dB = \frac{\mu_0}{4\pi}\frac{I\,d\ell}{r^2}\sin\phi$$

The vector representation of the magnetic field follows as

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell}\times\hat{r}}{r^2}.$$

Magnetic field from a cylinder with current

The last equation on the previous slide is known as the Biot-Savart law.

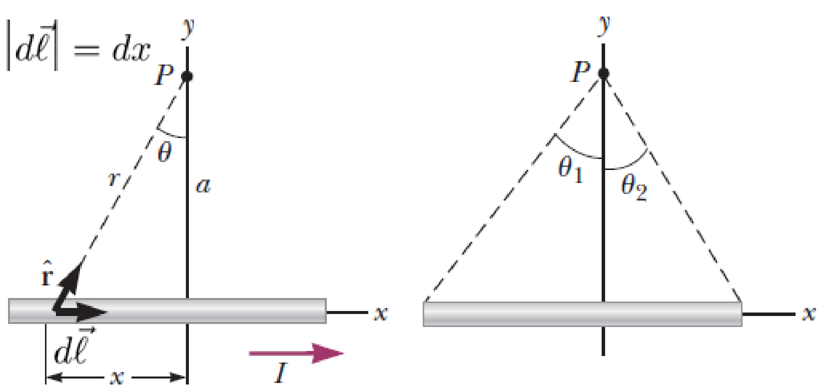
We can integrate both sides of the equation to get the following expression

$$\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$

Problem

A thin, straight wire carrying a constant current I is placed along the x axis.

Determine the magnitude and direction of the magnetic field at point P.



Solution

First, let us note that

$$d\vec{\ell} \times \hat{r} = \left| d\vec{\ell} \times \hat{r} \right| \hat{k} = dx \sin\left(\frac{\pi}{2} - \theta\right) \hat{k} = dx\, \cos\theta\, \hat{k}$$

Then the Biot-Savart law gives $d\vec{B} = \dfrac{\mu_0 I}{4\pi} \dfrac{dx \cos\theta}{r^2} \hat{k}.$

Also, $\cos\theta = \dfrac{a}{r}$ and $x = -a \tan\theta.$ ⟹ $dx = -a \sec^2\theta\, d\theta$

Therefore, $d\vec{B} = -\dfrac{\mu_0 I}{4\pi} \left(a \sec^2\theta\, d\theta\right) \left(\dfrac{\cos^2\theta}{a^2}\right) \cos\theta \hat{k}.$

$$\Longrightarrow \quad \vec{B} = -\int_{\theta_1}^{\theta_2} \frac{\mu_0 I}{4\pi} \frac{\cos\theta}{a} d\theta\, \hat{k}$$

Therefore, $\boxed{\vec{B} = \dfrac{\mu_0 I}{4\pi a} \left(\sin\theta_1 - \sin\theta_2\right) \hat{k}.}$

A vertical wire carries a current straight down. To the east of this wire, the magnetic field points

A. toward the west.

B. toward the south.

C. toward the east.

D. toward the north.

A horizontal wire carries a current straight toward you. From your point of view, the magnetic field at a point directly below the wire points

A. directly away from you.

B. vertically upward.

C. to the right.

D. to the left.

Two long, parallel wires placed side-by-side on a horizontal table carry identical size currents in opposite directions. The wire on your right carries current toward you, and the wire on your left carries current away from you. From your point of view, the magnetic field at the point exactly midway between the two wires

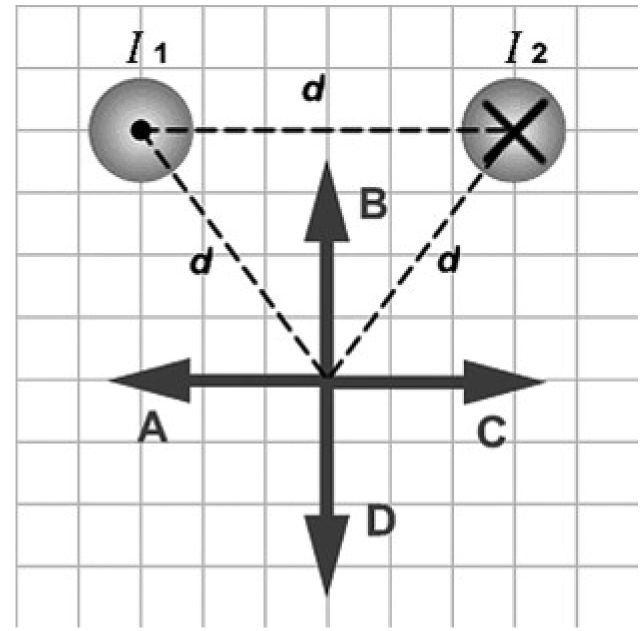
A. points upward.

B. points toward you.

C. is zero.

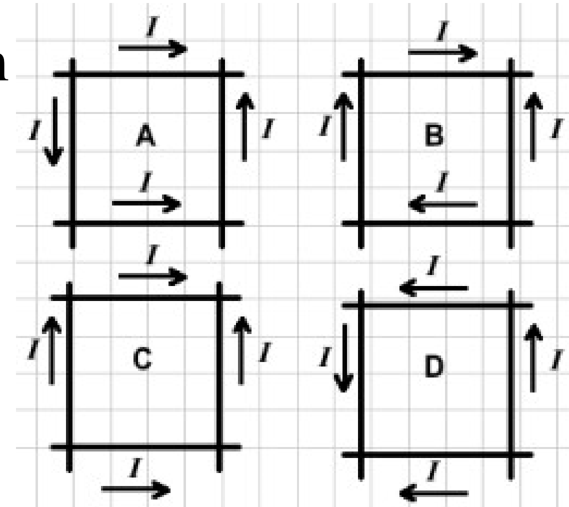
D. points downward.

The figure shows two long wires carrying equal currents I_1 and I_2 flowing in opposite directions. Which of the arrows labeled *A* through *D* correctly represents the direction of the magnetic field due to the wires at a point located at an equal distance *d* from each wire?



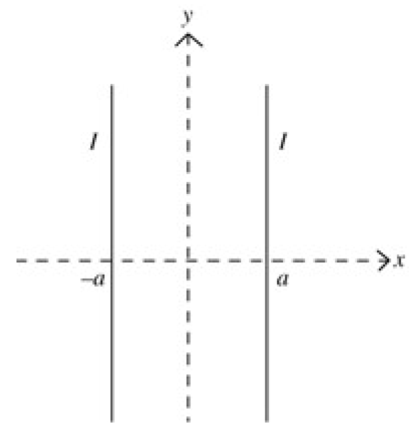
1. A
2. B
3. C
4. D

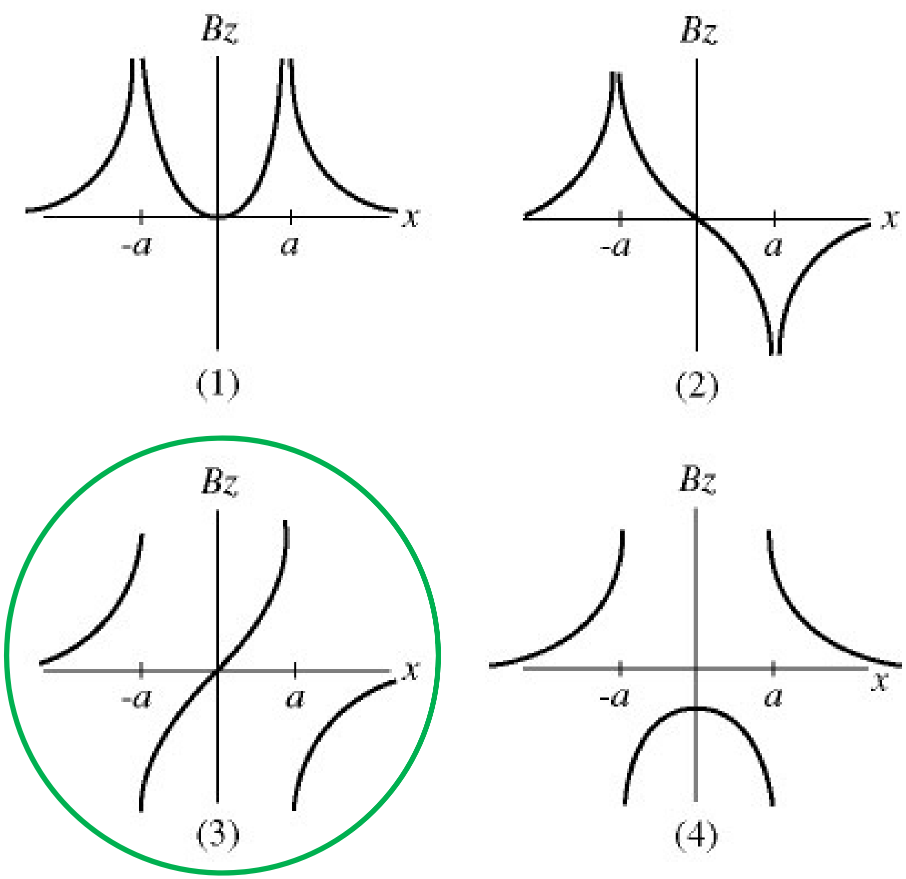
The figures show sets of insulated wires that cross each other at right angles without actually making electrical contact. The magnitude of the current is the same in all the wires, and the directions of current flow are as indicated. For which configuration will the magnetic field at the center of the square formed by the wires be equal to zero?



1. A
2. B
3. C
4. D

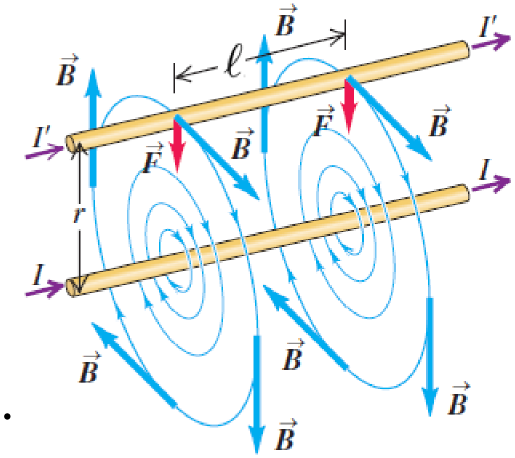
Two very long parallel wires in the *xy*-plane, a distance 2*a* apart, are parallel to the *y*-axis and carry equal currents *I*. The +z direction points perpendicular to the *xy*-plane in a right-handed coordinate system. If both currents flow in the +*y* direction, which one of the graphs best represents the *z* component of the net magnetic field B_z, in the *xy*-plane, as a function of *x*?





Force between parallel conductors

Consider two long, straight, parallel wires separated by a distance *a* and carrying currents I_1 and I_2 in the same direction.



The direction of $\vec{B}_2$ is perpendicular to wire 1. The magnetic force on a length ℓ, of wire 1 is $\vec{F}_1 = I_1 \vec{\ell} \times \vec{B}_2$.

Because the direction of $\vec{B}_2$ is perpendicular to wire direction, the magnitude of the magnetic force on wire 1 is simply $F_1 = I_1 \ell B_2$.

We know that the magnitude of the magnetic field from wire 2 is $B_2 = \dfrac{\mu_0 I_2}{2\pi r}$.

Therefore, the magnitude of the magnetic force per unit length on wire 1 may be written as

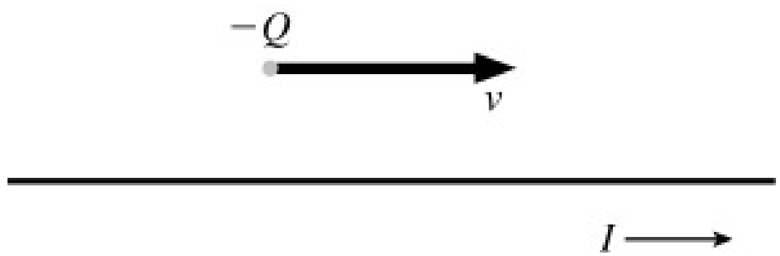
$$\boxed{\frac{F_1}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

The force per unit length on wire 2 is opposite in direction and equal in magnitude,

$$\boxed{\frac{F_2}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

Note that wires with parallel currents attract, and wires with antiparallel currents repel!!!

A negatively charged particle is moving to the right, directly above a wire having a current flowing to the right, as shown in the figure. In which direction is the magnetic force exerted on the particle?



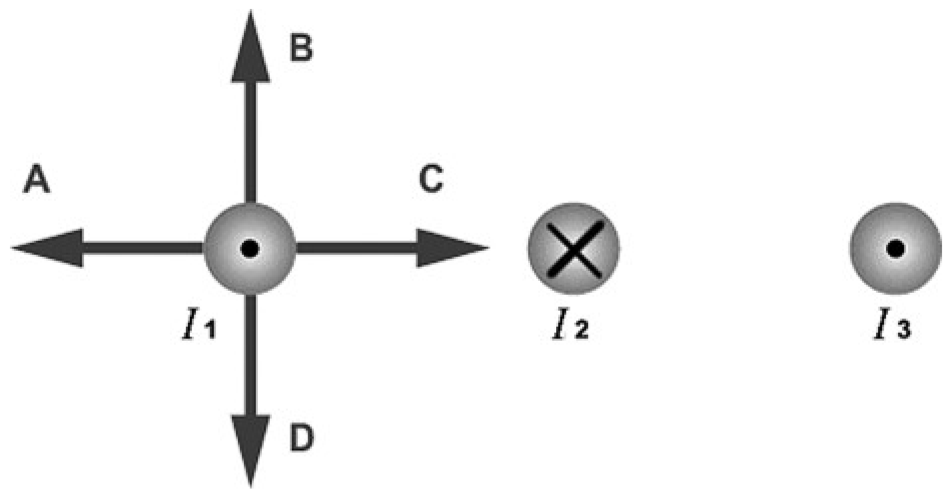
A. Upward.

B. Out of the page.

C. Into the page.

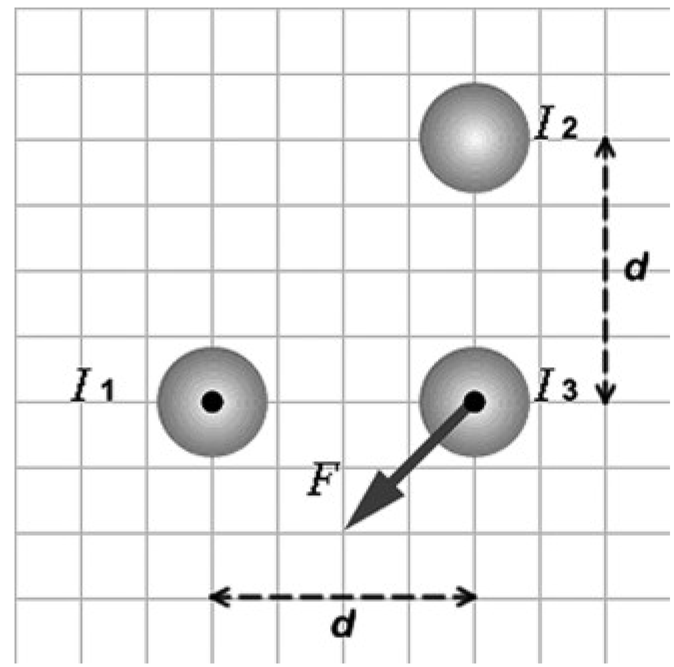
D. Downward.

The figure shows three long, parallel current-carrying wires. The magnitudes of the currents are equal and their directions are indicated in the figure. Which of the arrows drawn near the wire carrying current 1 correctly indicates the direction of the magnetic force acting on that wire?



1. A
2. B
3. C
4. D

The figure shows three long, parallel, current-carrying wires. The current directions are indicated for currents I_1 and I_3. The arrow labeled F represents the net magnetic force acting on current I_3. The three currents have equal magnitudes. What is the direction of the current I_2?



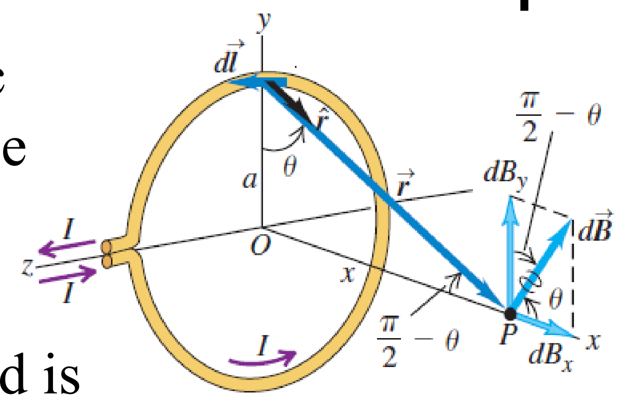
A. Vertically upward.

B. Vertically downward.

C. Out of the picture.

D. Into the picture.

Magnetic field on the axis of a circular loop



We can use the Biot-Savart law to find the magnetic field at a point P on the axis of the loop, at a distance x from the center.

We first note that $d\vec{\ell}$ and $\hat{r}$ are perpendicular. Also, $r^2 = a^2 + x^2$ and the magnitude of the magnetic field is

$$dB = \frac{\mu_0 I}{4\pi} \frac{d\ell}{a^2 + x^2}$$

For a point *P* <u>on</u> the *x* axis, <u>only</u> the *x*-component of the magnetic field is nonzero. In other words, all components pointing radially outwards from the *x* axis are cancelled by field components pointed in the opposite direction caused by current running through the opposite side of the current loop.

Thus, we need only consider the x-component, which gives

$$B_x = \int \frac{\mu_0 I}{4\pi} \frac{d\ell}{a^2 + x^2} \cos\theta = \int \frac{\mu_0 I}{4\pi} \frac{a\, d\ell}{(a^2 + x^2)^{3/2}} = \int_0^{2\pi} \frac{\mu_0 I}{4\pi} \frac{a\,(a\, d\theta)}{(a^2 + x^2)^{3/2}}$$

Therefore, the magnetic field at point *P* is $\boxed{\vec{B} = \frac{1}{2}\mu_0 I \frac{a^2}{(a^2 + x^2)^{3/2}} \hat{i}.}$

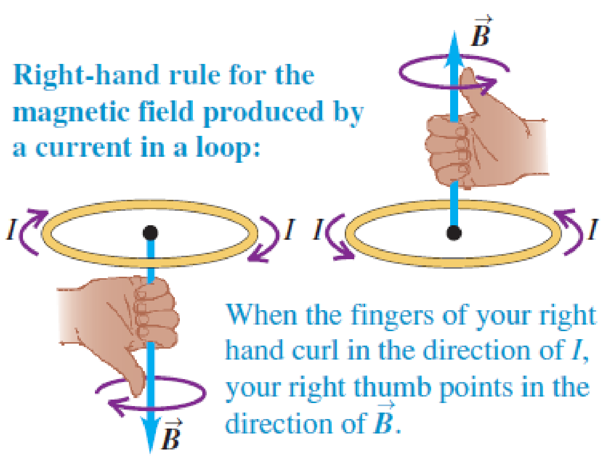
Magnetic field on the axis of a coil

Suppose we have a coil consisting of N loops, all with the same radius.

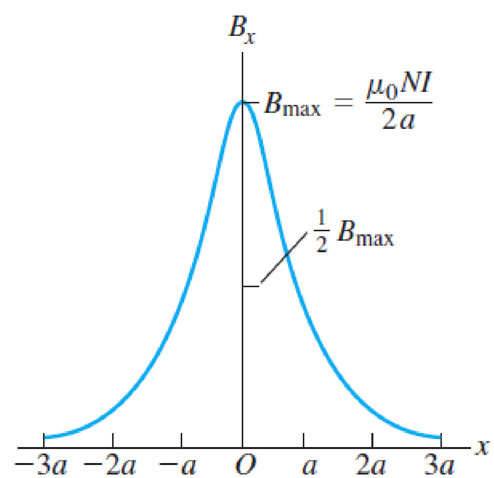
The loops are closely spaced so that the plane of each loop is essentially the same distance x from the field point P. Then the total field is N times the field of a single loop,

$$B = \frac{1}{2}\mu_0 N I \frac{a^2}{(a^2 + x^2)^{3/2}}$$

The direction of the magnetic field on the axis of a loop or coil can be determined using the right-hand-rule.



The magnitude of the magnetic field is maximum at the center of the loop/coil and it reduces as the point P moves away from the center along its principal axis.



A current carrying loop of wire lies flat on a table top. When viewed from above, the current moves around the loop in a counter-clockwise sense. For points <u>outside</u> the loop, the magnetic field caused by this current

A. circles the loop in a counter-clockwise direction.

B. circles the loop in a clockwise direction.

C. points straight up.

D. points straight down.

A current carrying loop of wire lies flat on a table top. When viewed from above, the current moves around the loop in a counter-clockwise sense. For points inside the loop, the magnetic field caused by this current

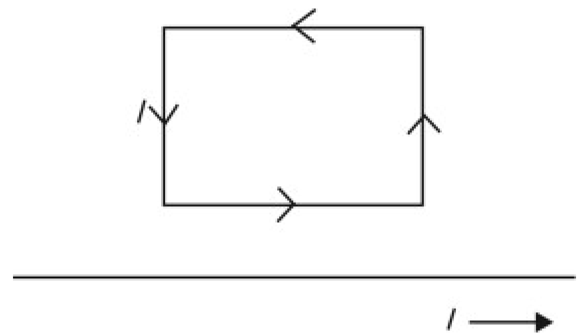
A. circles the loop in a counter-clockwise direction.

B. circles the loop in a clockwise direction.

C. points straight up.

D. points straight down.

A long straight conductor has a constant current flowing to the right. A wire rectangle is situated above the wire, and also has a constant current flowing through it. Which of the following statements is true?



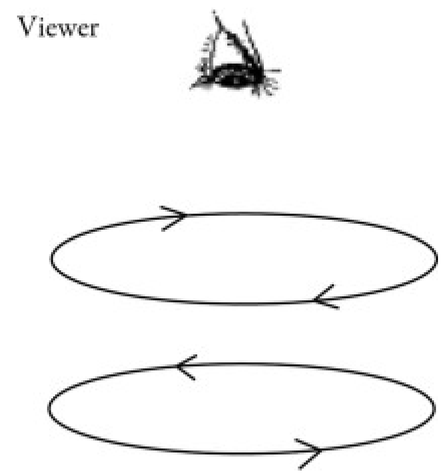
A. The net magnetic force on the wire rectangle is downward, and the net torque on it is zero.

B. The net magnetic force on the wire rectangle is upward, and there is also a net torque on it.

C. The net magnetic force on the wire rectangle is zero, but there is a net torque on it.

D. The net magnetic force on the wire rectangle is downward, and there is also a net torque on it.

A ring with a clockwise current (as seen from above the ring) is situated with its center directly above another ring, which has a counter-clockwise current. In what direction is the net magnetic force exerted on the top ring?



A. To the left

B. Downward

C. To the right

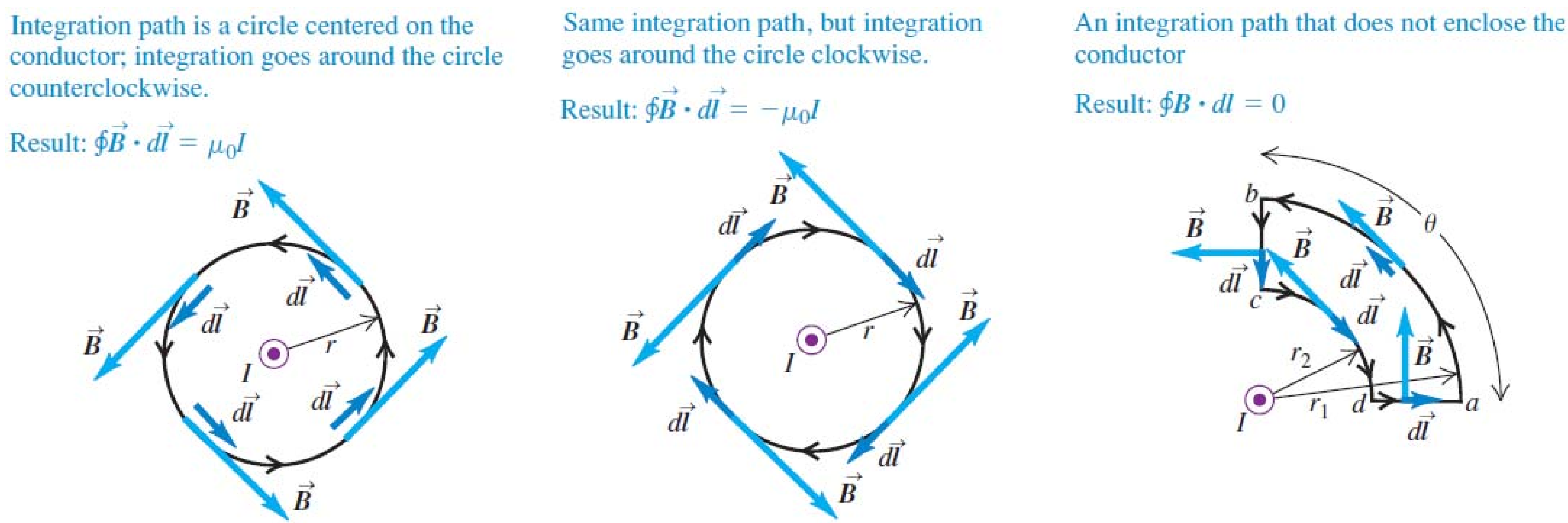
D. Upward

Ampere's law

Ampere's law is formulated not in terms of magnetic flux, but rather in terms of the line integral of $\vec{B}$ around a closed path. The line integral is equal to the permeability of free space multiplied by the magnitude of the current,

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

We typically use Ampere's law to draw a closed path around a current carrying conductor.



Consider a solenoid of length *L*, *N* windings, and radius *b* (*L* is much longer than *b*). A current *I* is flowing through the wire. If the radius of the solenoid were doubled (becoming 2*b*), and all other quantities remained the same, the magnetic field inside the solenoid would

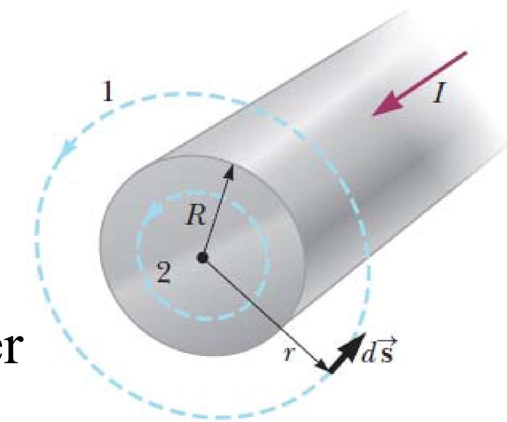
A. become twice as strong.

B. become one half as strong.

C. remain the same.

Application of Ampere's law to an infinitely long, straight wire

An infinitely long, straight wire of radius *R* carries a steady current *I* that is uniformly distributed through the cross section of the wire.



Calculate the magnetic field a distance *r* from the center of the wire in the regions $r < R$ and $r \geq R$.

For $r < R$

The current has the ratio $\frac{I_{\text{in}}}{I} = \frac{A_{\text{in}}}{A_{\text{wire}}} = \frac{\pi r^2}{\pi R^2}$.

$$\Longrightarrow \oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{in}} = \mu_0 I \frac{r^2}{R^2}$$

$$\Longrightarrow \oint \vec{B} \cdot d\vec{\ell} = B \int_0^{2\pi} r\, d\phi = B\,(2\pi r)$$

$$\Longrightarrow B\,(2\pi r) = \mu_0 I \frac{r^2}{R^2}$$

$$\Longrightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

For $r \geq R$

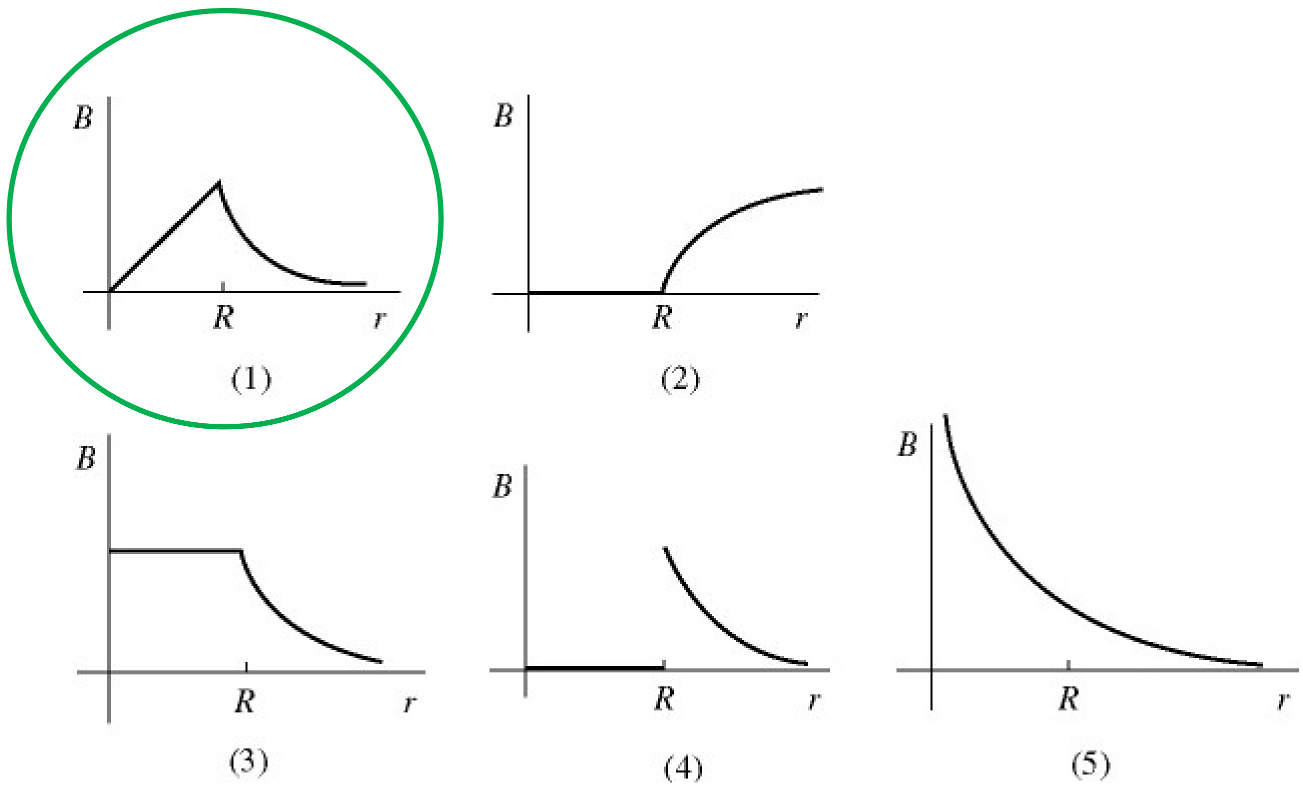
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

$$\Longrightarrow \oint \vec{B} \cdot d\vec{\ell} = B \int_0^{2\pi} r\, d\phi = B\,(2\pi r)$$

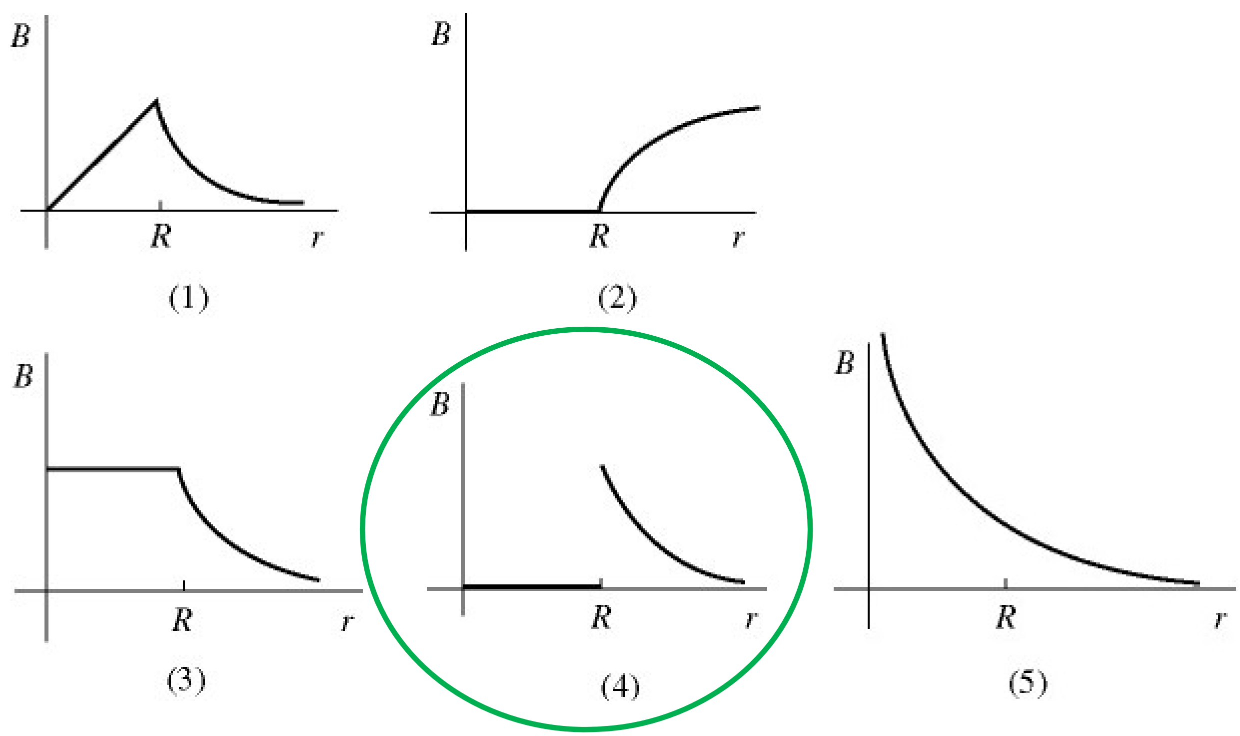
$$\Longrightarrow B\,(2\pi r) = \mu_0 I$$

$$\Longrightarrow B = \frac{\mu_0 I}{2\pi r}$$

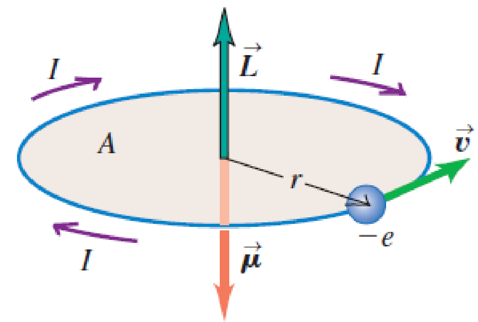
A very long, solid, conducting cylinder of radius *R* carries a current along its length uniformly distributed throughout the cylinder. Which one of the graphs shown in the figure most accurately describes the magnitude *B* of the magnetic field produced by this current as a function of the distance *r* from the central axis?



A very long, hollow, thin-walled conducting cylindrical shell (like a pipe) of radius R carries a current along its length uniformly distributed throughout the thin shell. Which one of the graphs shown in the figure most accurately describes the magnitude B of the magnetic field produced by this current as a function of the distance r from the central axis?



Bohr Magneton



In the Bohr model of an atom, an electron moving with speed in a circular orbit of radius r has an angular momentum $|\vec{L}| = mvr$ and an oppositely directed orbital magnetic dipole moment $|\vec{\mu}| = IA$, where $A = \pi r^2$. It also has a spin angular momentum and an oppositely directed spin magnetic dipole moment.

The equivalent current I is the total charge e passing any point on the orbit per unit time $T = 2\pi r/v$,

$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

The magnetic dipole moment follows as $\mu = IA = \frac{1}{2}evr = \frac{eL}{2m}$.

The angular momentum is quantized and is an integer n multiple of $h/2\pi = \hbar$, where $h = 6.626\times10^{-34}$ kg m^2/s is the Planck constant and $\hbar$ is the reduced Planck constant.

Therefore, $\mu = \frac{ne\hbar}{2m}$, where $\mu_B = \frac{e\hbar}{2m} = 9.274 \times 10^{-24}\ \mathrm{A\,m^2}$ is known as the Bohr magneton.

Magnetic materials

The magnetization of a material is given in terms of the magnetic moment μ and the volume V, $\vec{M} = \frac{\vec{\mu}}{V}$. The total magnetic field is $\vec{B}_{\mathbf{inside}} = \vec{B}_{\mathbf{outside}} + \mu_0 \vec{M}$.

Ferromagnetic substances contain permanent atomic magnetic moments that tend to align parallel to each other even in a weak external magnetic field. Once the moments are aligned, the substance remains magnetized after the external field is removed.

Paramagnetic substances have a weak magnetism resulting from atoms that have permanent magnetic moments. These moments interact only weakly with one another and are randomly oriented in the absence of an external magnetic field. When a paramagnetic substance is placed in an external magnetic field, its atomic moments tend to line up with the field.

Diamagnetic substances have atoms/molecules with no permanent magnetic moments. When an external magnetic field is applied, there is an induced magnetic moment in the opposite direction of the external field. Diamagnetism is much weaker than paramagnetism.

When a material is magnetized, the permeability is $\mu = K_m \mu_0$, where the relative permeability K_m is given in terms of the magnetic susceptibility χ_m, $K_m = 1 + \chi_m$.