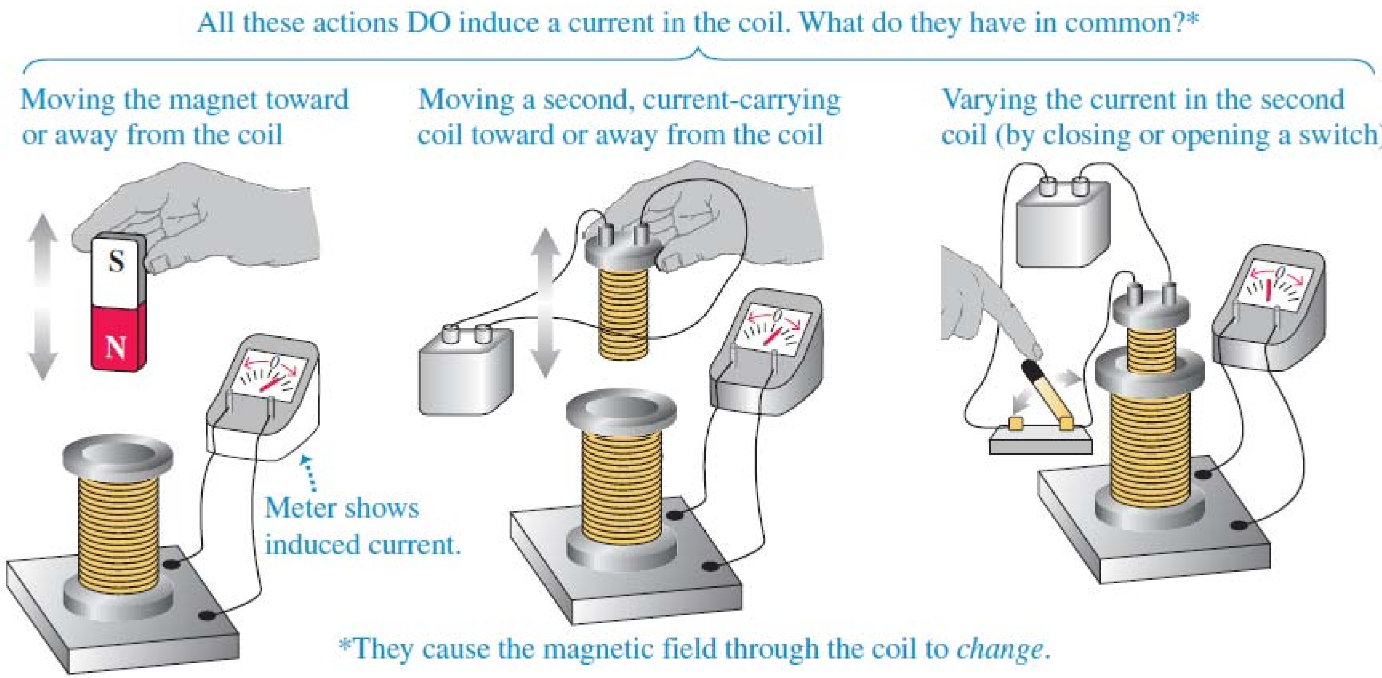
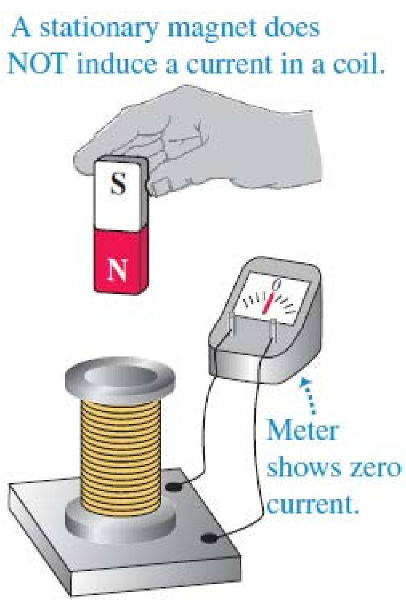
Chapter 29: Electromagnetic induction

- Induction experiments
- Faraday's law
- Lenz's law
- Motional electromotive force
- Induced electric fields
- Eddy currents
- Displacement current and Maxwell's equations
- Superconductivity

Induction experiments





If all these experiments are repeated precisely with a coil that has the same shape but different material and different resistance, the current in each case is inversely proportional to the total circuit resistance.

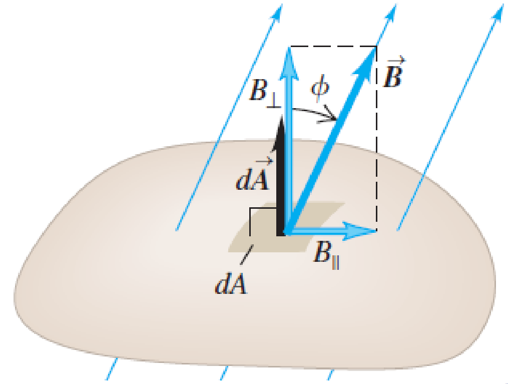
Magnetic flux

The common element in all induction effects is changing magnetic flux through a closed loop/circuit.

For an infinitesimal area element dA in a magnetic field $\vec{B}$, the magnetic flux $d\Phi_B$ through the area is

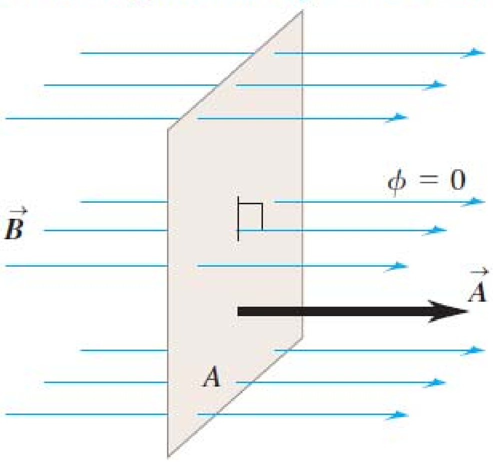
$$d\Phi_B = \vec{B} \cdot d\vec{A}$$

static $\Longrightarrow$ $\Phi_B = BA\cos\theta$

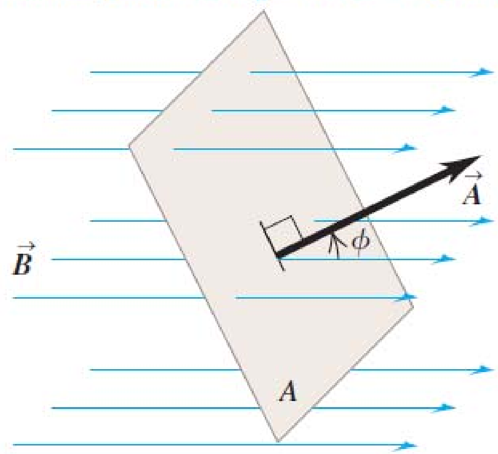


Magnetic flux through element of area $d\vec{A}$
$d\Phi_B = \vec{B} \cdot d\vec{A} = B_\perp dA = B\,dA\cos\phi$

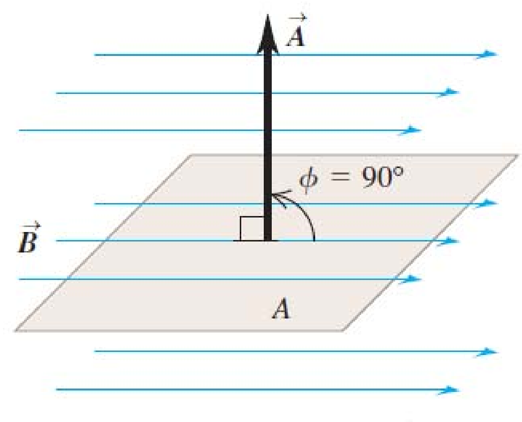
Surface is face-on to magnetic field:
- $\vec{B}$ and $\vec{A}$ are parallel (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 0$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA$.



Surface is tilted from a face-on orientation by an angle ϕ:
- The angle between $\vec{B}$ and $\vec{A}$ is ϕ.
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\phi$.



Surface is edge-on to magnetic field:
- $\vec{B}$ and $\vec{A}$ are perpendicular (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 90°$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos 90° = 0$.



Faraday's law

The induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

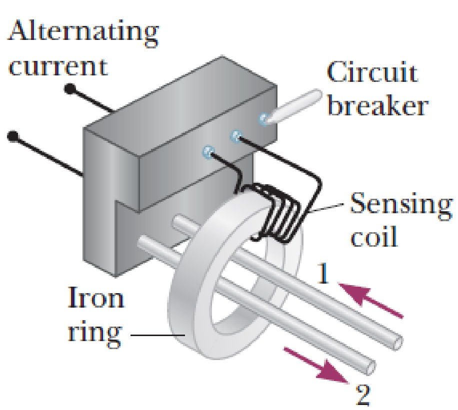
where the magnetic flux through an area contained by a closed loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

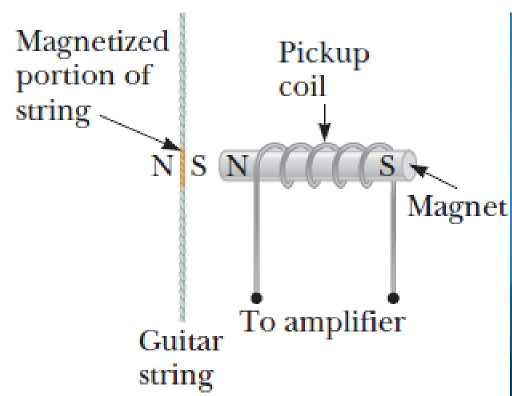
For a coil, the induced emf is given by $\mathcal{E}_{\text{coil}} = -N\dfrac{d\Phi_B}{dt}$.

<u>Some applications of Faraday's law</u>

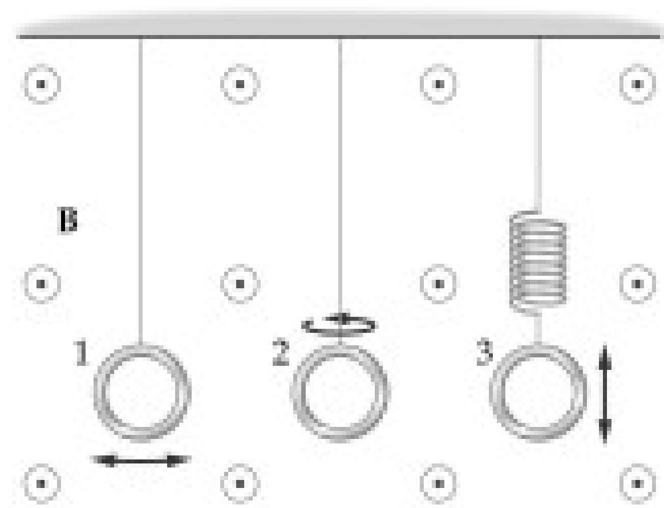
Ground fault circuit interrupter (GFCI)



Guitar pick-up coils



The three loops of wire shown in the figure are all subject to the same uniform magnetic field *B* that does not vary with time. Loop 1 oscillates back and forth as the bob in a pendulum, loop 2 rotates about a vertical axis, and loop 3 oscillates up and down at the end of a spring. Which loop, or loops, will have an emf induced in them?



A. Loop 1 only

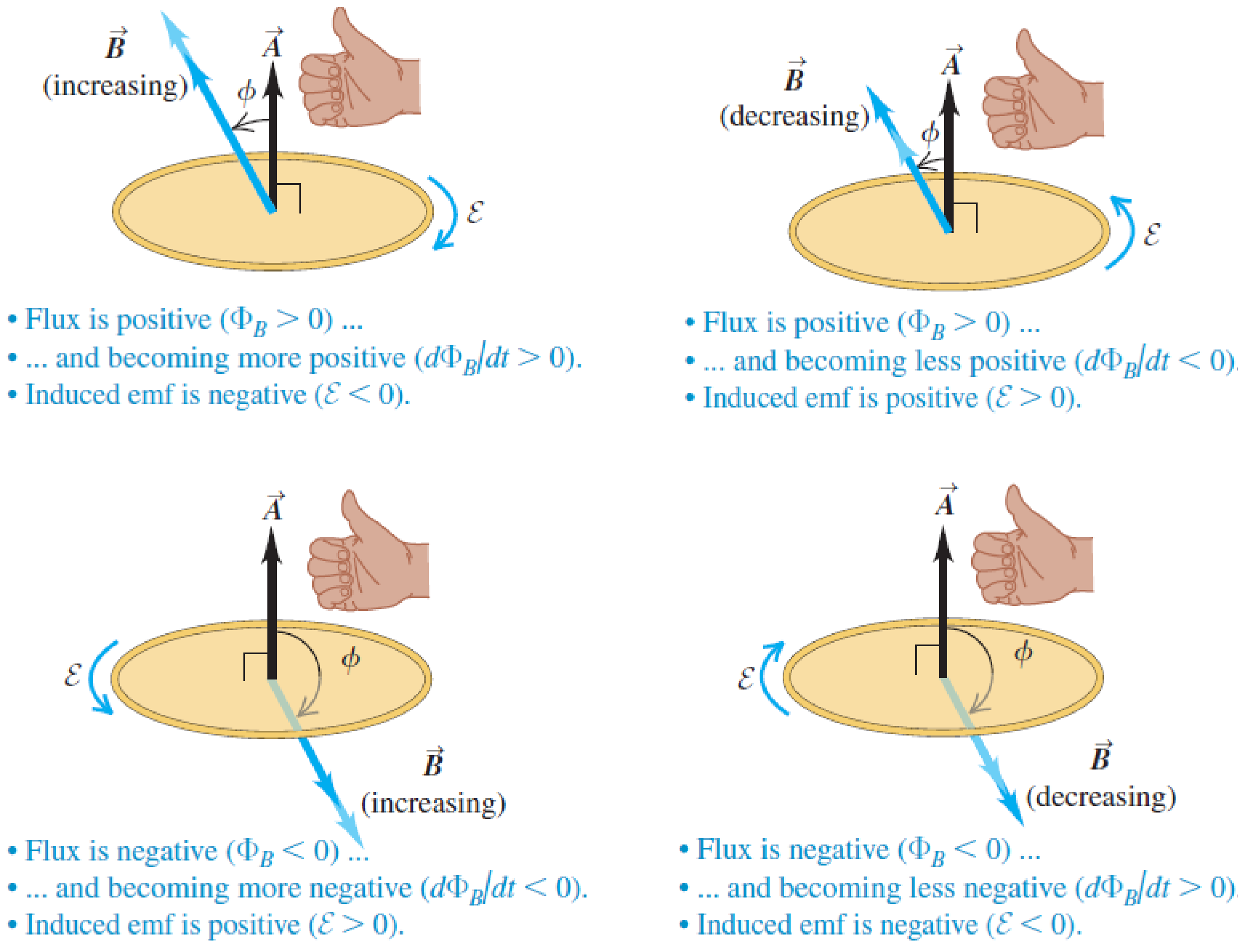
B. Loop 2 only

C. Loop 3 only

D. Loops 1 and 2

Direction of induced emf/current

We can find the direction of an induced emf or current by using the right-hand-rule and keeping track of the sign/direction.



A circular loop of wire lies in the plane of the paper. An increasing magnetic field points out of the paper. What is the direction of the induced current in the loop?

A. Counter-clockwise
B. Clockwise
C. Counter-clockwise then clockwise
D. Clockwise then counter-clockwise

A coil lies flat on a tabletop in a region where the magnetic field vector points straight up. The magnetic field vanishes suddenly. When viewed from above, what is the direction of the induced current in this coil as the field fades?

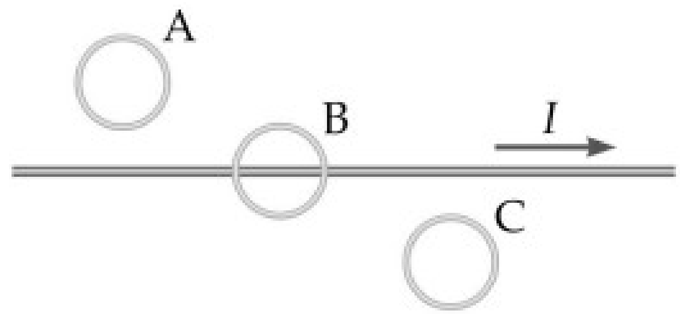
A. Counter-clockwise

B. Clockwise

C. Counter-clockwise then clockwise

D. Clockwise then counter-clockwise

The long straight wire in the figure carries a current I that is decreasing with time at a constant rate. The circular loops *A*, *B*, and *C* all lie in a plane containing the wire. The induced emf in each of the loops *A*, *B*, and *C* is such that



A. no emf is induced in any of the loops.

B. loop *A* has a counter-clockwise emf, loop *B* has no induced emf, and loop *C* has a clockwise emf.

C. loop *A* has a clockwise emf, loop *B* has no induced emf, and loop *C* has a counterclockwise emf.

D. loop *A* has a counter-clockwise emf, loops *B* and *C* have clockwise emfs.

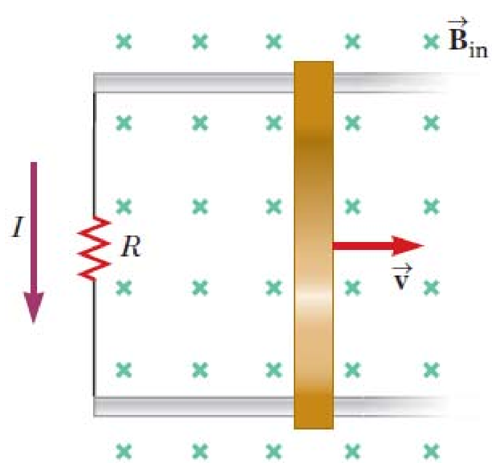
Lenz's law

Lenz's law is an alternative method for determining the direction of an induced current or emf.

<u>Lenz's law</u>: the direction of any magnetic induction effect is such as to oppose the cause of the effect.

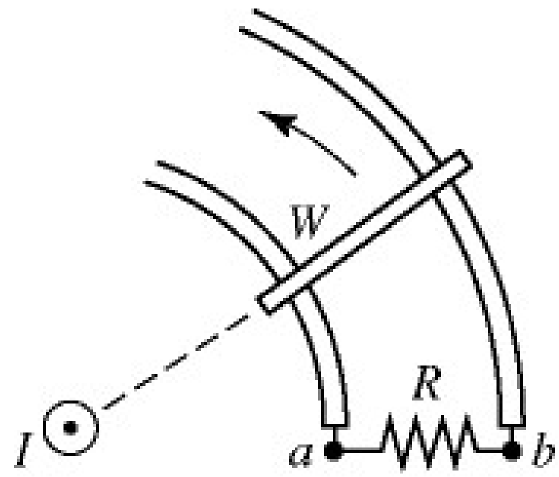
Within the area bounded by the circuit, this field is <u>opposite</u> to the original field if the original field is *increasing*, but this field is in the <u>same</u> direction as the original field if the original field is <u>decreasing</u>. That is, the induced current **opposes the change in flux** through the circuit (not the flux itself).

As the conducting bar slides to the right, the magnetic flux due to the external magnetic field into the page through the area enclosed by the loop increases in time.



By Lenz's law, the induced current must be counterclockwise to produce a counteracting magnetic field directed out of the page.

In the figure, a straight wire carries a steady current *I* perpendicular to the plane of the page. A bar is in contact with a pair of circular rails, and rotates about the straight wire. The direction of the induced current through the resistor *R* is

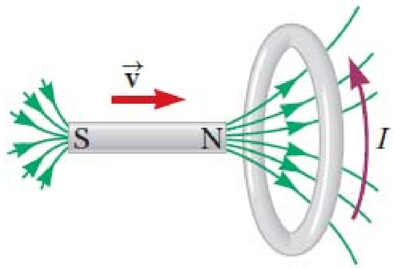


A. from *a* to *b*.

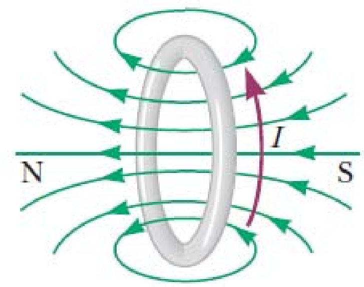
B. from *b* to *a*.

C. There is no induced current through the resistor.

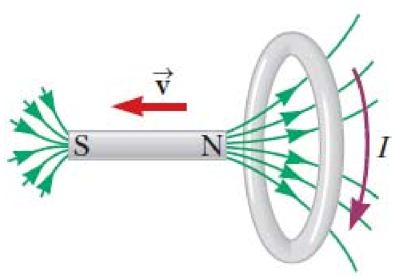
Lenz’s law using bar magnets & loops



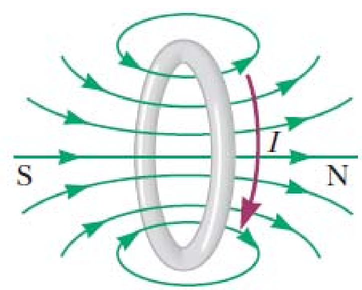
When the magnet is moved toward the stationary conducting loop, a current is induced in the direction shown. The magnetic field lines are due to the bar magnet.



This induced current produces its own magnetic field directed to the left that counteracts the increasing external flux.

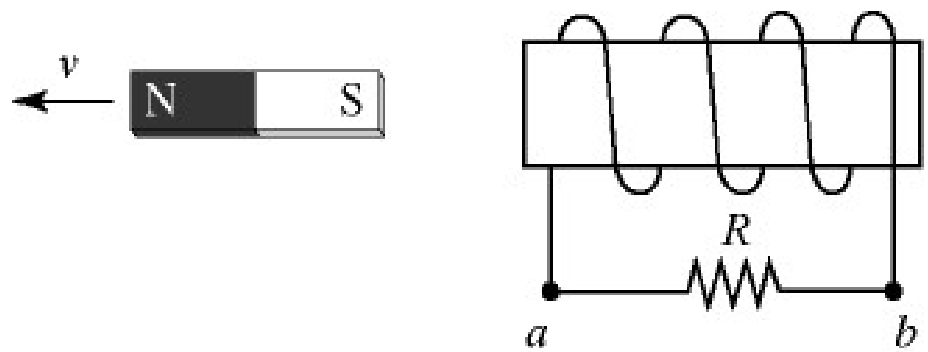


When the magnet is moved away from the stationary conducting loop, a current is induced in the direction shown.



This induced current produces a magnetic field directed to the right and so counteracts the decreasing external flux.

In the figure, a bar magnet moves away from the solenoid. The induced current through the resistor *R* is

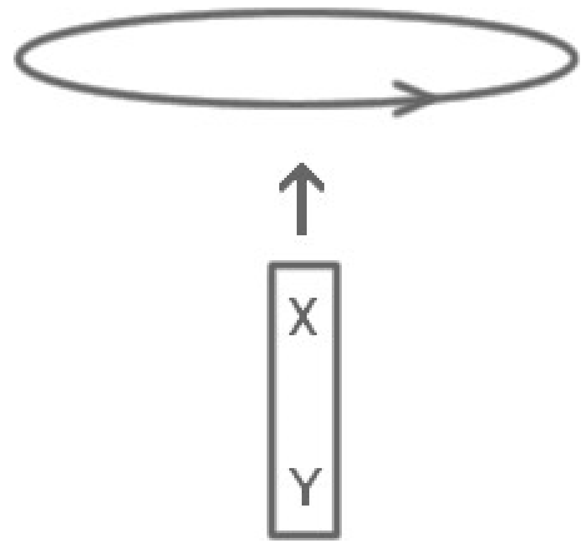


A. from *a* to *b*.

B. from *b* to *a*.

C. There is no induced current through the resistor.

The figure shows a bar magnet moving vertically upward toward a horizontal coil. The poles of the bar magnets are labeled X and Y. As the bar magnet approaches the coil it induces an electric current in the direction indicated on the figure (counter-clockwise as viewed from above). What are the correct polarities of the magnet?



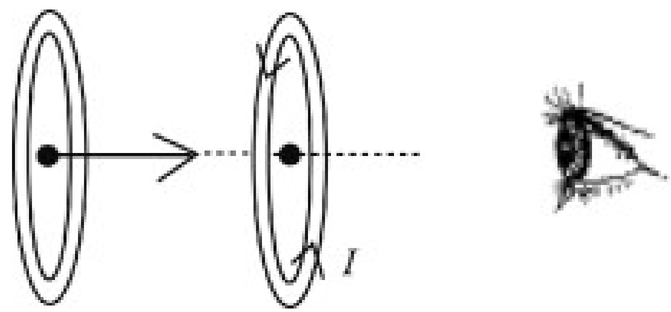
A. X is a south pole, Y is a north pole.

B. X is a north pole, Y is a south pole.

C. Both X and Y are south poles.

D. Both X and Y are north poles.

A closed, circular loop has a counter-clockwise current flowing through it as viewed by a person on the right, as shown in the figure. If a second closed circular loop with the same radius approaches this loop with constant velocity along a common axis as shown, in what direction will a current flow in the approaching loop as viewed by the person on the right?



A. clockwise.

B. counter-clockwise.

C. No current will be induced because the velocity of approach is constant.

Motional electromotive force

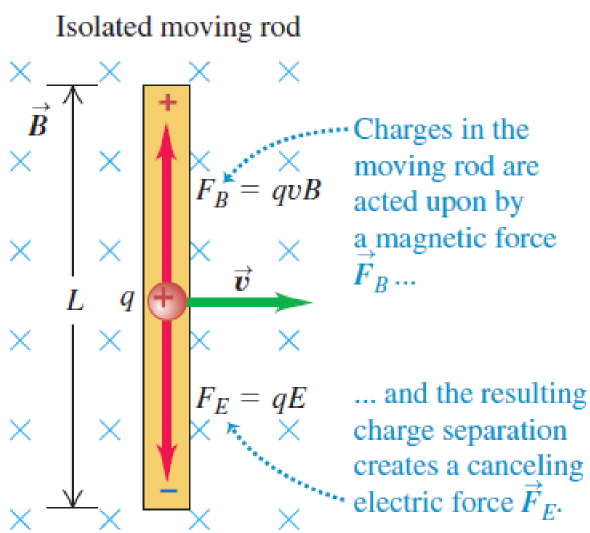
The motional emf for a conductor with any shape, moving in any static magnetic field, uniform or not, is given by

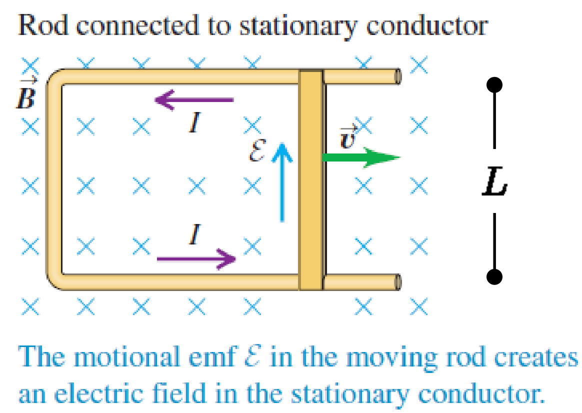
$$d\mathcal{E} = \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell}$$

where $d\ell$ is the conductor length element and $d\mathcal{E}$ is the emf in the length element.

For a closed loop moving through the magnetic field, we may find the circuit emf by integration,

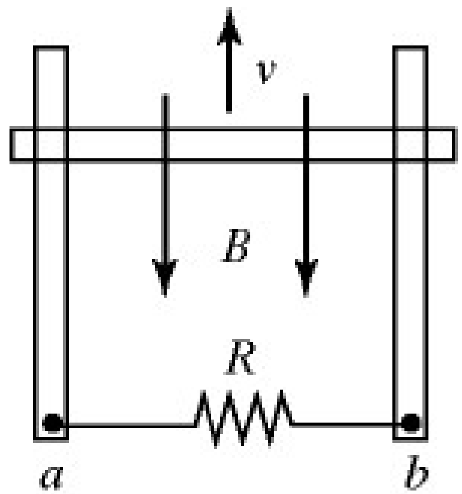
$$\mathcal{E} = \oint \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell}$$





$$\mathcal{E}_{\text{rod}} = EL = vBL$$

A copper bar is in contact with a pair of parallel metal rails and is in motion with velocity v. A uniform magnetic field is present pointing downward, as shown. The bar, the rails, and the resistor R are all in the same plane. The induced current through the resistor R is

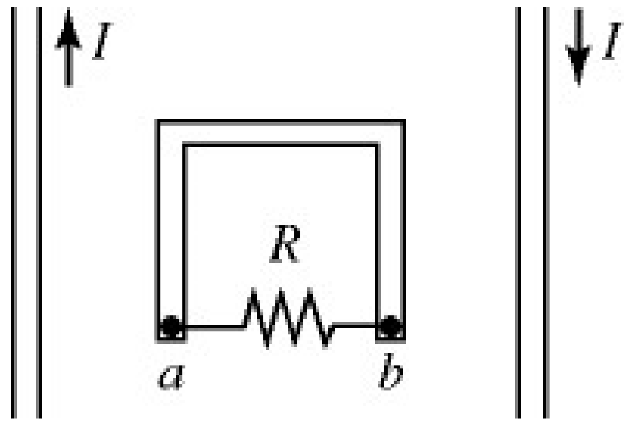


A. from *a* to *b*.

B. from *b* to *a*.

C. There is no induced current through the resistor.

Two parallel wires carry currents of magnitude I in opposite directions as shown. A rectangular loop is midway between the wires. The current I is decreasing with time. The induced current through the resistor R is

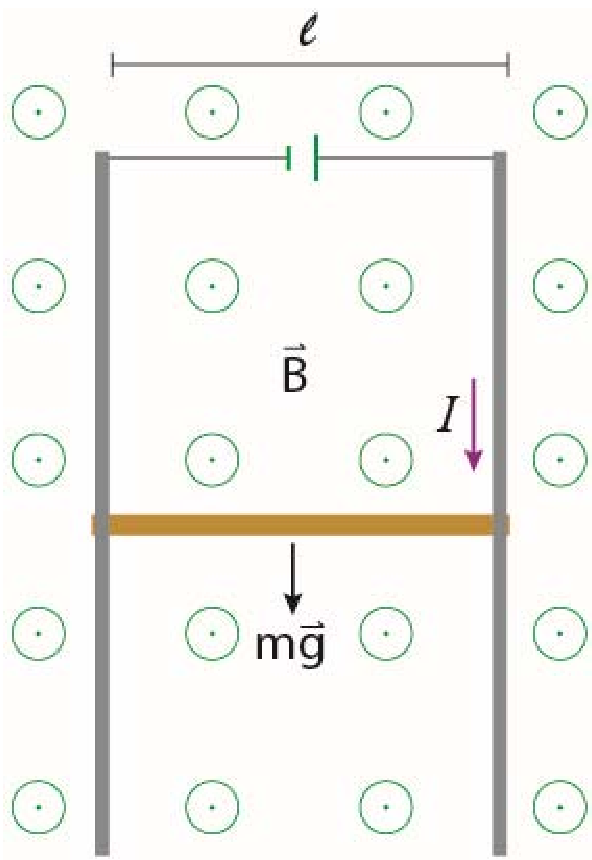


A. from a to b.

B. from b to a.

C. There is no induced current through the resistor.

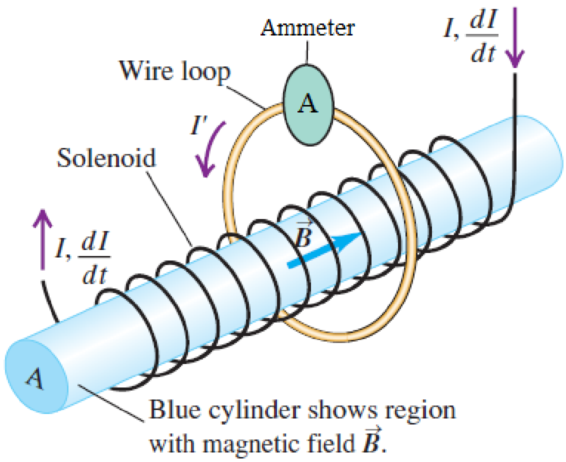
A non-ideal conducting bar of mass $m = 0.250$ kg, which carries a resistance of $R = 1.30\ \Omega$ slides without friction on two perfectly conducting rails that are connected at one end by a power supply. The two rails are separated by a distance $\ell = 75.0$ cm. A magnetic field is perpendicular to the plane that the bar and rails make and has a magnitude of $B = 1.50$ T. The bar is allowed to slide along the vertical direction in a gravitational field.

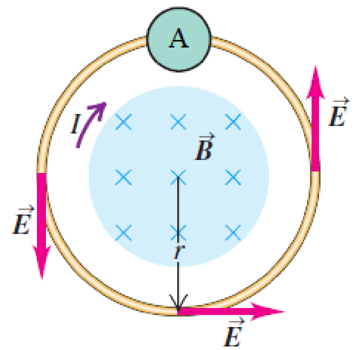


(a) If the initial velocity is zero, then what must the potential difference across the power supply be for the bar to stay at its current location?

(b) Let us assume that we remove the power supply and reclose the circuit at the top. What is the maximum velocity that the bar can achieve?

Induced electric field in a conducting loop





We can change the magnetic field inside the loop by placing a solenoid at its center. A time changing current through the solenoid's wire will cause a time changing $\vec{B}$, which will induce an emf in the wire loop.

Taking the area vector $\vec{A}$ to point in the same direction as $\vec{B}$, the magnetic flux through the loop is

$$\Phi_B = BA = \mu_0 NIA$$

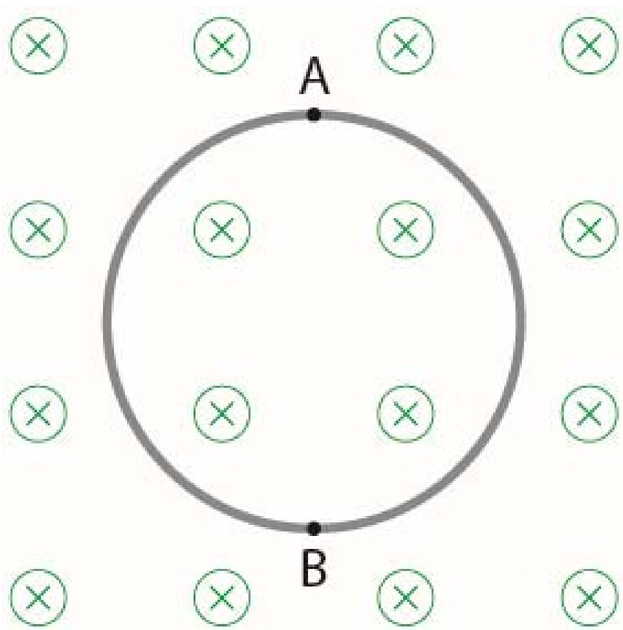
When the current in the solenoid $\vec{I}$ changes, the magnetic flux changes,

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \mu_0 NA\frac{dI}{dt}$$

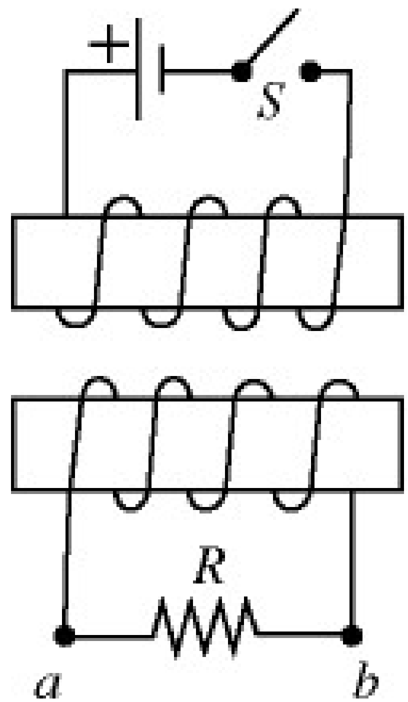
The emf is caused by an induced electric field in the wire due to the changing flux, where $\mathcal{E} = \int \vec{E} \cdot d\vec{\ell}$. For this case we may write Faraday's law as

$$\int \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

A loop of flexible conducting material has a radius of $r = 40.0$ cm, where it is placed in a magnetic field of magnitude $B = 0.300$ T as illustrated above. The loop is grasped at points A and B and pulled tight to close the loop. If it takes 0.100 s to close the loop, then what is the magnitude of the average induced emf in it during that time interval?



In the figure, two solenoids are side-by-side. The switch *S*, initially open, is closed. The induced current through the resistor *R* is



A. from *a* to *b*.

B. from *b* to *a*.

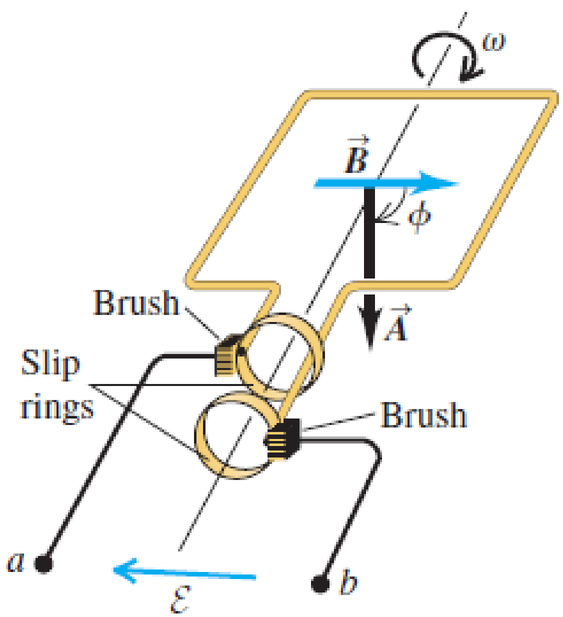
C. There is no induced current through the resistor.

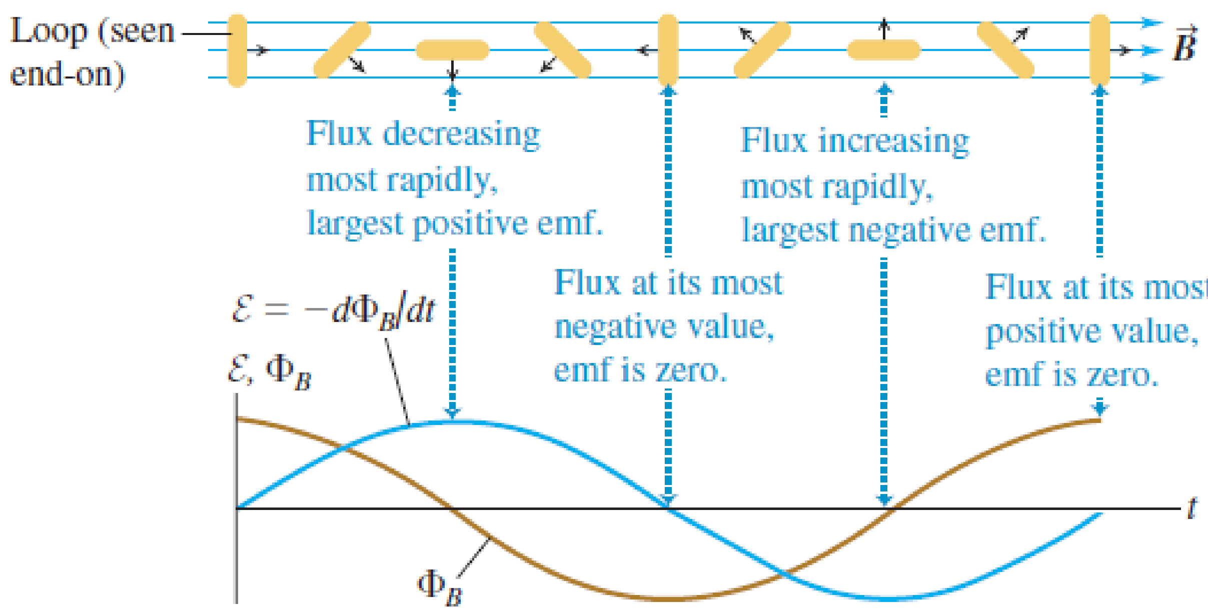
Generators

A conducting loop rotates in a magnetic field, producing an emf.

Connections from each end of the loop to the external circuit are made by means of that end's slip ring.

For an ideal AC generator, the induced emf lags the magnetic flux by 90°.





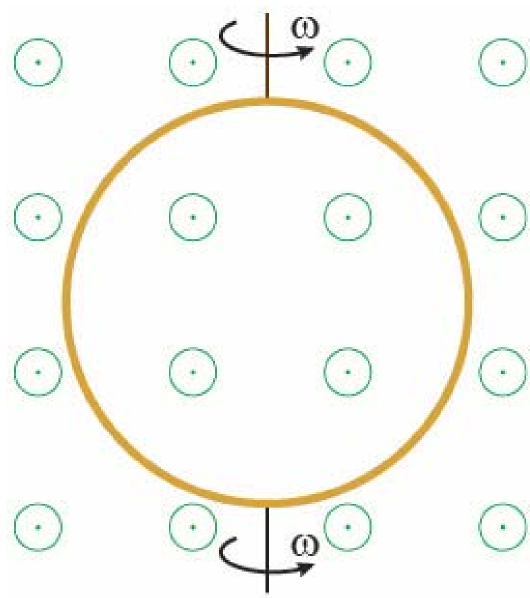
A loop of radius $r = 60.0$ cm is rotating in a magnetic field of magnitude $B = 0.500$ T. The loop has an angular frequency of $\omega = 12.0$ rad/s and carries a resistance of $R = 0.700\ \Omega$. At $t = 0$, the magnetic flux is maximum through the loop.

(a) Calculate the peak emf.

(b) Calculate the maximum current through the loop while rotating.

(c) At $t = 2.40$ s, determine is the magnitude of the emf.

(d) At $t = 2.40$ s, determine is the magnitude of the current.

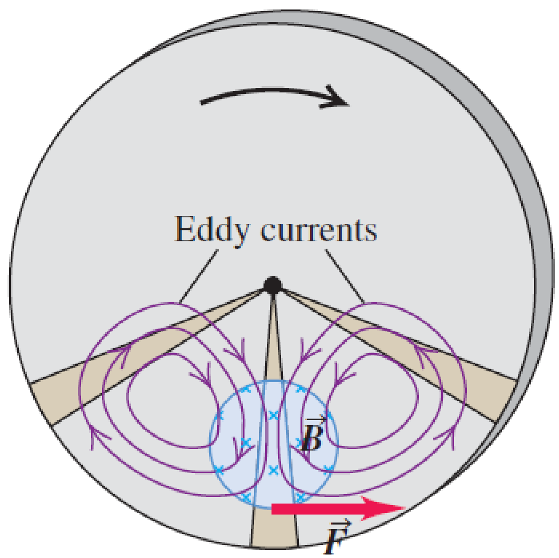


Eddy currents

Many parts of electrical equipment contain masses of metal moving in magnetic fields or located in changing magnetic fields.

The induced currents that circulate throughout the volume of large chunks of conducting materials resemble swirling eddies in a river. Thus, these electrical currents are often referred to as eddy currents.

Rotating disk in a magnetic field.



The interaction between the eddy currents and the field causes a braking action on the disk.

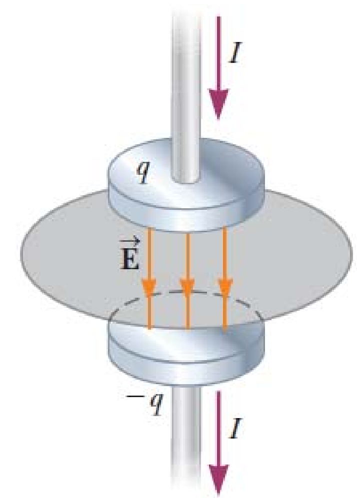
Generalized Ampere's law

Ampere's law is <u>incomplete</u> when in the form $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$.

Consider charging a capacitor; conducting wires lead a conduction current $\vec{I}_C$ into one plate and out of the other. No current runs between the capacitor plates, but there is an increasing electric field caused by the charge accumulation on the plates,

$$q = Cv = \frac{\epsilon A}{d} Ed = \epsilon AE = \epsilon \Phi_E$$

where the electric flux is noted as Φ_E.



Taking the time derivative of the accumulating charge gives the displacement current,

$$I_D = \epsilon \frac{d\Phi_E}{dt}$$

It is really the time changing electric flux that caused the time changing magnetic field. The displacement current is a fictitious current, but easier to visualize.

Thus, the generalized Ampere's law is $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (I + I_D) = \mu_0 I + \mu_0 \epsilon \frac{d\Phi_E}{dt}$.

A capacitor is charging in a simple RC circuit with a dc battery. Which one of the following statements about this capacitor is accurate?

A. There is a magnetic field between the capacitor plates because charge travels between the plates by jumping from one plate to the other.

B. There is no magnetic field between the capacitor plates because no charge travels between the plates.

C. There is a magnetic field between the capacitor plates, even though no charge travels between them, because the magnetic flux between the plates is changing.

D. There is a magnetic field between the capacitor plates, even though no charge travels between them, because the electric flux between the plates is changing.

Maxwell's equations (integral form)

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{\mathbf{encl}}}{\epsilon_0}$$ (Gauss's law for electric fields)

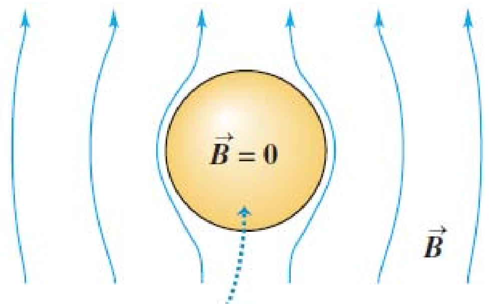
$$\oiint \vec{B} \cdot d\vec{A} = 0$$ (Gauss's law for magnetic fields)

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left(I + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\mathbf{encl}}$$ (Ampere's law)

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$ (Faraday's law)

Superconductivity

Superconductivity is the sudden disappearance of all electrical resistance when the material is cooled below a temperature called the critical temperature, T_C. This critical temperature is reduced when the material is placed in an external magnetic field.



Magnetic flux is expelled from the material, and the field inside it is zero (Meissner effect).

The magnetic flux through a superconducting material is zero. This zero magnetic flux through the superconductor is called the Meissner effect.

Superconducting materials are perfectly diamagnetic below their critical temperature due to the expulsion of the magnetic flux. Therefore, superconducting materials can levitate in a magnetic field.