Chapter 31: Alternating current

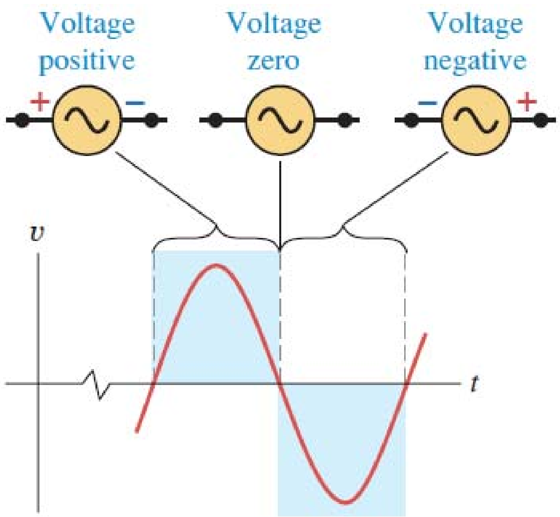
- Phasors and alternating current
- Resistance and reactance
- The LRC circuit
- Power in alternating-current circuits
- Resonance in alternating-current circuits
- Transformers

Alternating current

To supply an alternating current to a circuit, a source of alternating emf or voltage is required.

We use the term AC source for any device that supplies a sinusoidally varying voltage or current The circuit-diagram symbol for an AC source is

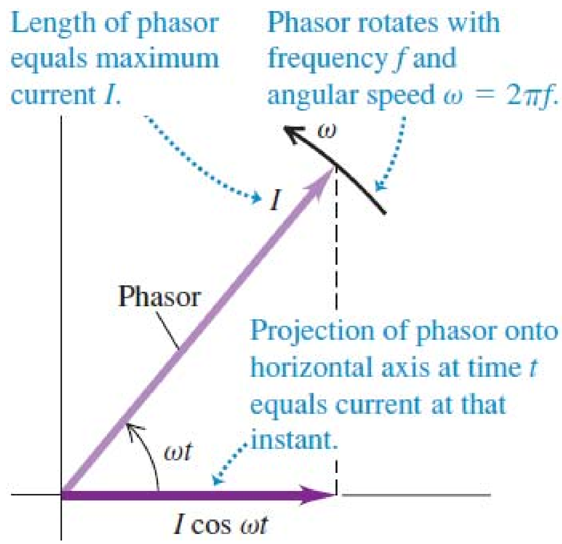
The sinusoidal voltage is given by $v = V_{\mathbf{peak}} \cos \omega t$, where V_{peak} is the amplitude of the oscillating voltage.



Phasor diagrams

We can use rotating vector diagrams called phasors to describe sinusoidally varying voltages and currents.

The vectors are called phasors and rotate counterclockwise with constant angular speed ω.



The vector rotates counterclockwise with constant angular speed ω.

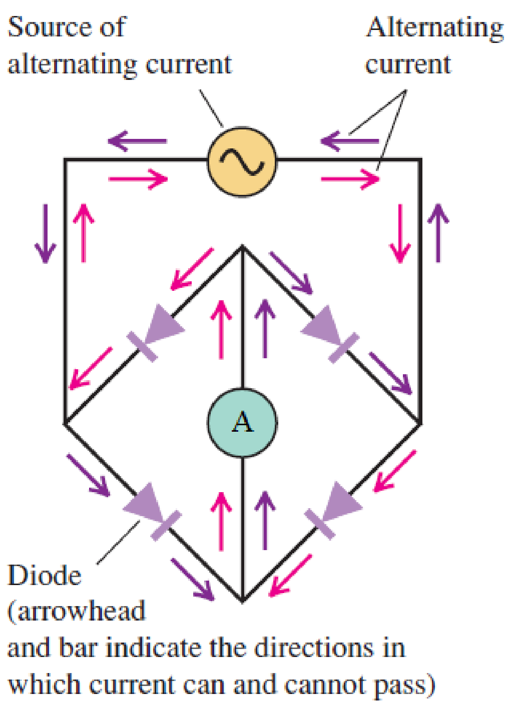
The projection of the phasor onto the horizontal axis at time is performed using the $\cos\omega t$ function.

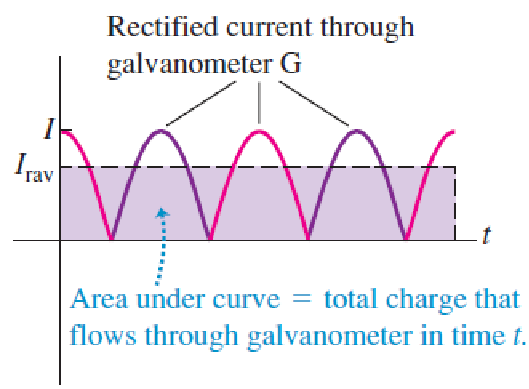
Full-wave rectifier

A rectifier uses diodes to change an AC current into a one-way current. A diode conducts current more readily in one direction (forward direction) than the other direction (backward direction). A diode has the symbol ▶|.

The full-wave rectified average current running through the ammeter shown in the diagram is

$$I_{\mathrm{rav}} = \frac{2}{\pi} I_{\mathrm{peak}} \approx 0.637\, I_{\mathrm{peak}}$$





Root-mean-squared value

The sinusoidal voltage or current can be expessed in terms of its root-mean-squared value. We can square the current and then take the average $\langle \cdots \rangle$,

$$
\begin{aligned}
\langle i^2 \rangle &= \langle I_{\text{peak}}^2 \cos^2 \omega t \rangle \\
&= I_{\text{peak}}^2 \langle \cos^2 \omega t \rangle \\
&= \frac{1}{2} I_{\text{peak}}^2 \langle (1 + \cos 2\omega t) \rangle \\
&= \frac{1}{2} I_{\text{peak}}^2 + \frac{1}{2} I_{\text{peak}}^2 \cancelto{0}{\langle \cos 2\omega t \rangle} \\
&= \frac{1}{2} I_{\text{peak}}^2
\end{aligned}
$$

The root-mean-squared value is obtained by taking the square root,

$$
\begin{aligned}
I_{\text{rms}} &= \sqrt{\langle i^2 \rangle} \\
&= \frac{I_{\text{peak}}}{\sqrt{2}}
\end{aligned}
$$

The root-mean-squared value of the voltage is obtained in the same manner,

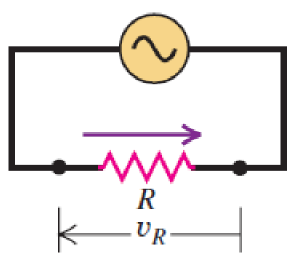
$$
V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}}
$$

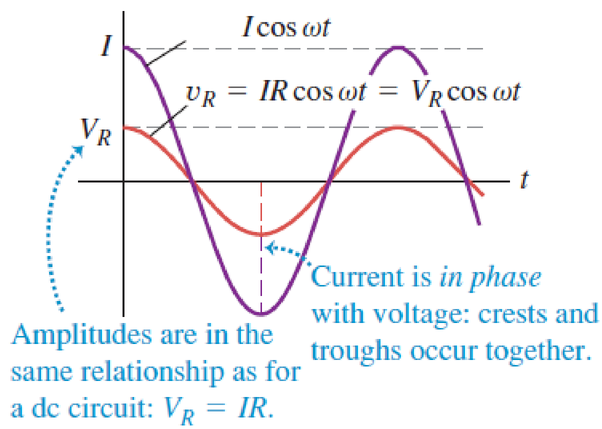
Resistance in AC circuits

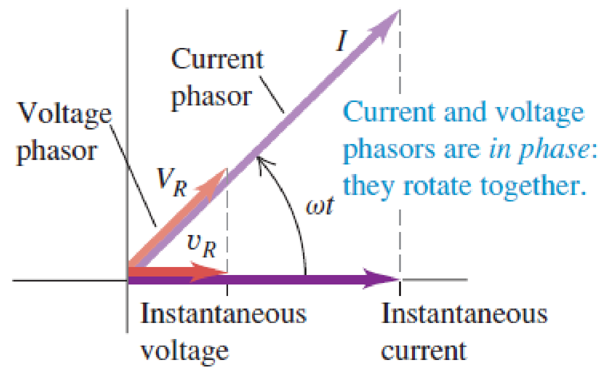
The voltage drop across the resistor is V_R, where the instantaneous voltage drop across the resistor is

$$v_R = V_R \cos \omega t$$
$$= I_{\text{peak}} R \cos \omega t$$

The resistance R is a real value and the voltage drop across the resistor is in phase with the current, i.e., the phasors for the current and voltage across the resistor point in the same direction.







Inductor reactance

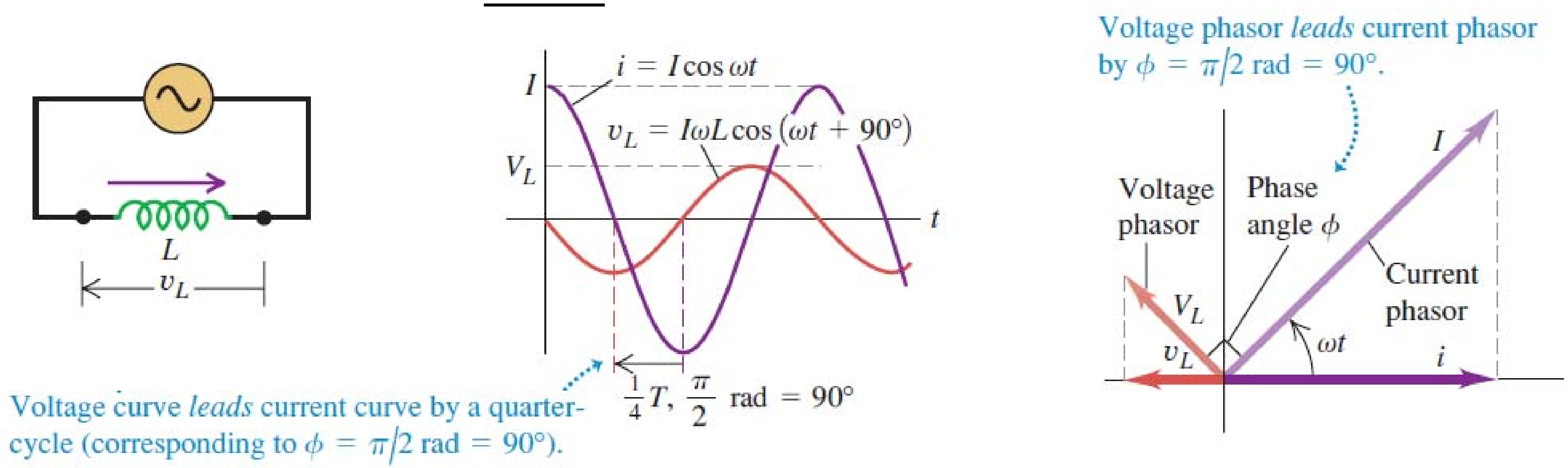
Although there is no real resistance in an ideal inductor, there is an instantaneous voltage v_L across the inductor.

For the current $i = I_{peak} \cos \omega t$, we have the voltage across an ideal inductor,

$$\begin{aligned} v_L &= L\frac{di}{dt} \\ &= -I_{peak} L\omega \sin \omega t \\ &= -V_L \sin \omega t \end{aligned}$$

where $V_L = I_{peak} L\omega$. Therefore the inductive reactance is given as $X_L = L\omega$.

The voltage across an ideal inductor (no resistance from the coil's wire) is phase shifted by 90° in the counter-clockwise direction. It is said that the voltage across the inductor <u>leads</u> the current.



Capacitor reactance

The voltage across an ideal capacitor is given by $v_C = \dfrac{q}{C}$.

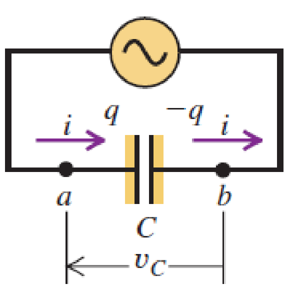
For the current $\dfrac{dq}{dt} = i = I_{\text{peak}} \cos \omega t$, we can integrate with respect to time to find the charge,

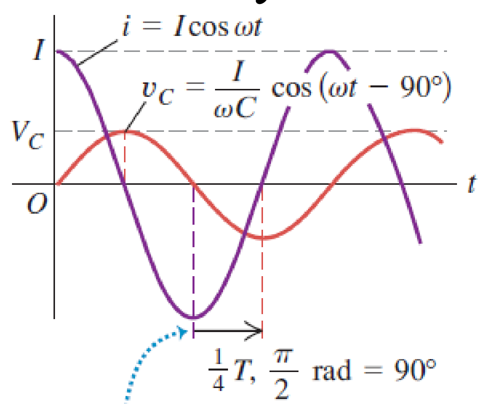
$$\int_0^q dq' = \int_0^t I_{\text{peak}} \cos \omega t \, dt'$$

$$\Longrightarrow \quad q = \frac{I_{\text{peak}}}{\omega} \sin \omega t$$

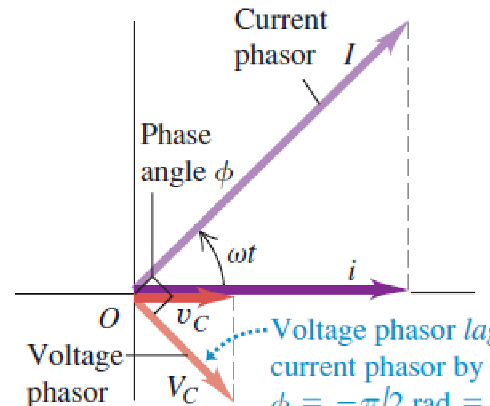
$$\Longrightarrow \quad v_c = \frac{I_{\text{peak}}}{\omega C} \sin \omega t$$

where the capacitance reactance is $X_C = \dfrac{1}{\omega C}$. The phase shift is 90° in the clockwise direction and commonly said that it lags the current.



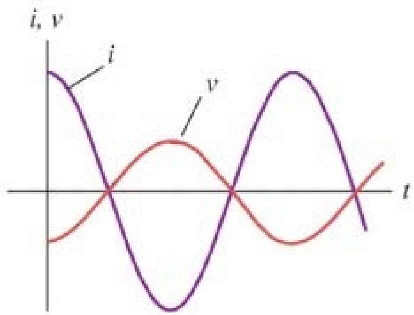


Voltage curve *lags* current curve by a quarter-cycle (corresponding to $\phi = -\pi/2$ rad $= -90°$).

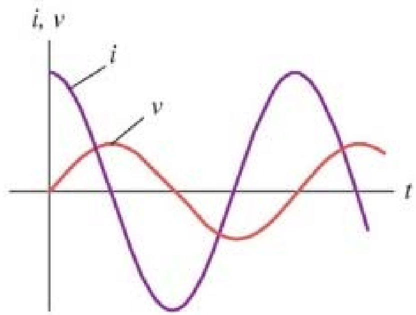


Voltage phasor *lags* current phasor by $\phi = -\pi/2$ rad $= -90°$.

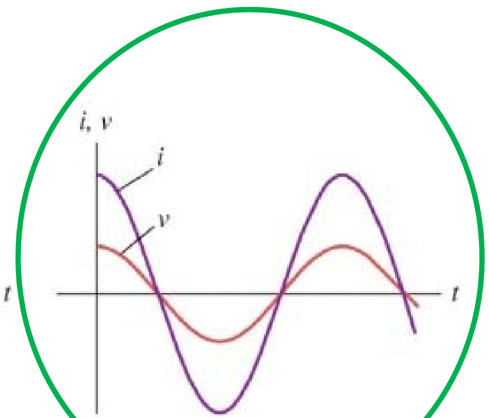
A resistor is connected across an AC source. Which of the following graphs shows the instantaneous current through the resistor and the instantaneous voltage across the resistor?



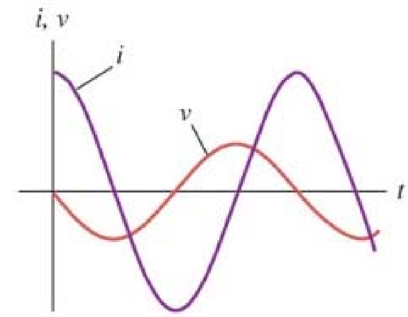
A.



B.

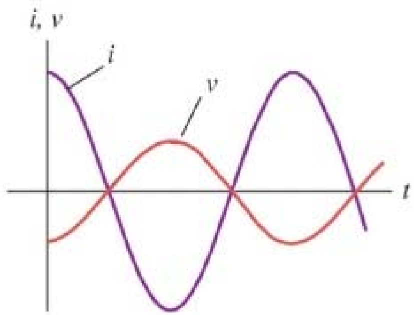


C.

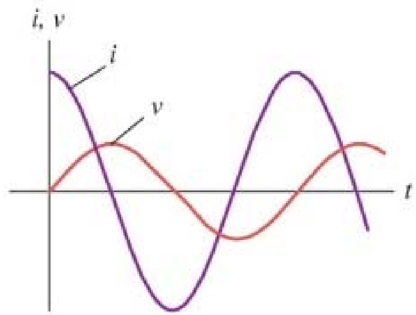


D.

An inductor is connected across an AC source. Which of the following graphs shows the instantaneous current through the inductor and the instantaneous voltage across the inductor?



A.



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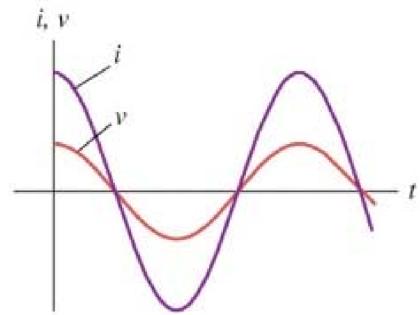
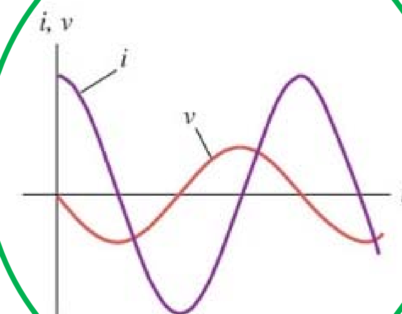
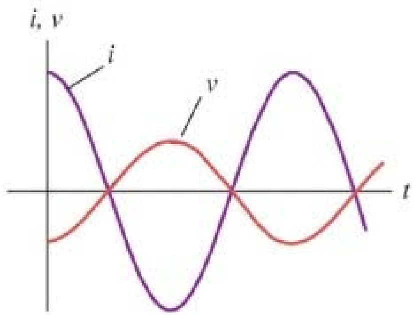


C.

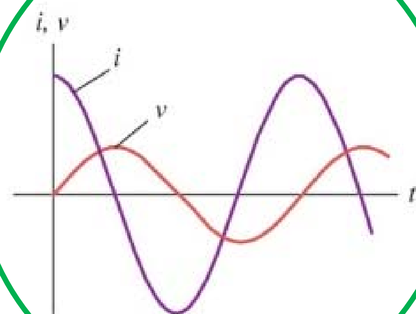


D.

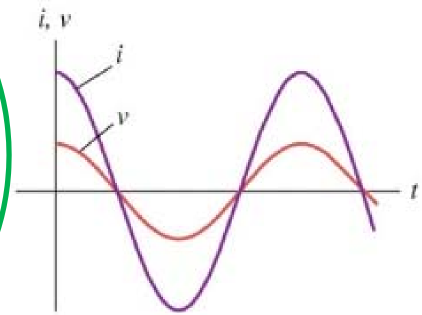
A capacitor is connected across an AC source. Which of the following graphs shows the instantaneous current through the capacitor and the instantaneous voltage across the capacitor?



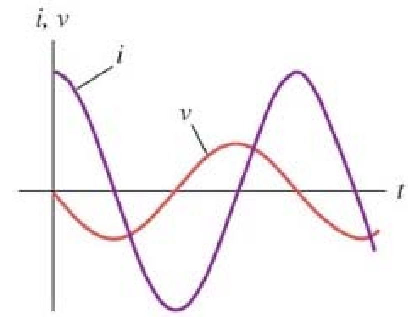
A.



B.



C.



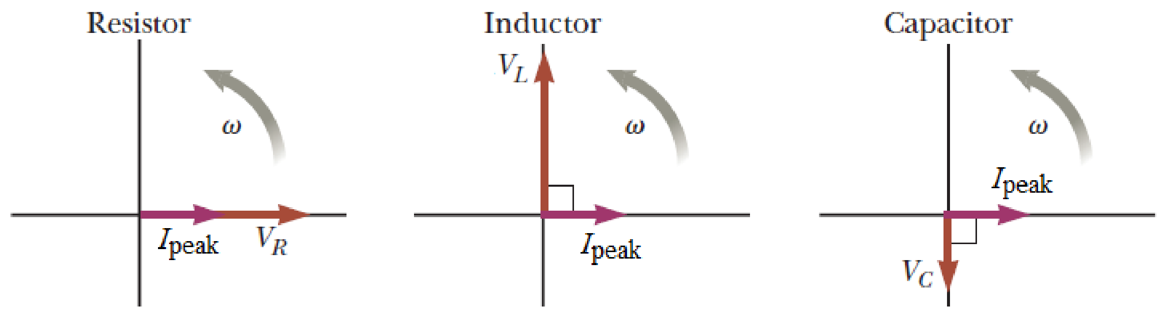
D.

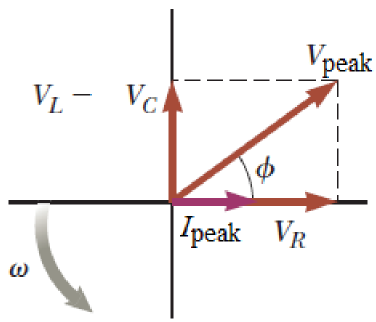
Impedance

An ideal LRC circuit has three different voltage phasors with magnitudes, V_R, V_L, and V_C. From the previous slides, these magnitudes were found to be

$$V_R = RI_{\mathbf{peak}} \qquad V_L = X_L I_{\mathbf{peak}} \qquad V_C = X_C I_{\mathbf{peak}}$$

The voltages across the inductor and capacitor are antiparallel and they are both orthogonal to the voltage across the resistor.





We can take the difference between the two magnitudes V_L and V_C, and then use Pythagorean's theorem to find the magnitude of the total voltage drop,

$$V_{\mathbf{peak}} = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= I_{\mathbf{peak}}\sqrt{R^2 + (X_L - X_C)^2}$$

The <u>impedance</u> is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $V_{\mathbf{peak}} = I_{\mathbf{peak}} Z$.

A resistor, an inductor, and a capacitor are connected in series to an AC source. Under what conditions will the impedance of the circuit increase as the inductive reactance increases?

A. When the inductive reactance is greater than or equal to the capacitive reactance.

B. When the inductive reactance is less than or equal to the capacitive reactance.

C. All conditions.

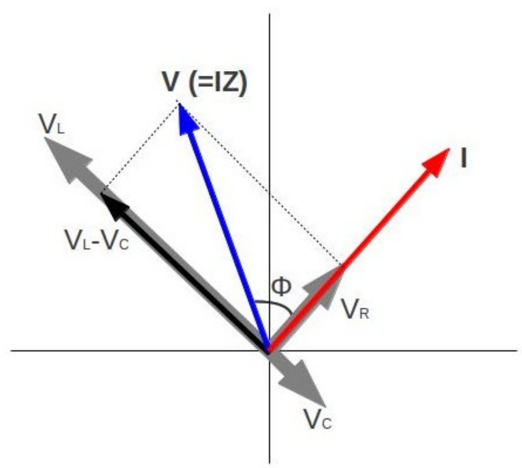
D. No conditions.

Phase angle

In addition to finding the magnitude (impedance), we can also find the phase angle ϕ. The resistance is orthogonal to the inductor's and capacitor's reactance. Defining the phase angle to be positive when counter-clockwise to the current phasor,

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

If the current is $i = I_{\mathrm{peak}} \cos \omega t$, then the source voltage is $v = V_{\mathrm{peak}} \cos(\omega t + \phi)$.



A resistor, an inductor, and a capacitor are connected in series to an AC source. What is the phase angle between the voltages of the inductor and capacitor in this RLC circuit?

A. 0°

B. 90°

C. 180°

D. 270°

A series AC circuit contains a resistor, an inductor $L = 150$ mH, a capacitor $C = 4.7$ μF, and a source with $V_{rms} = 240$ V operating at 60.0 Hz. The root-mean-squared current in the circuit is 90.0 mA.

(a) Determine the inductive reactance.

(b) Determine the capacitive reactance.

(c) Determine the impedance.

(d) Determine the resistance.

(d) Determine the phase angle between the current and the source voltage.

Power in AC circuits

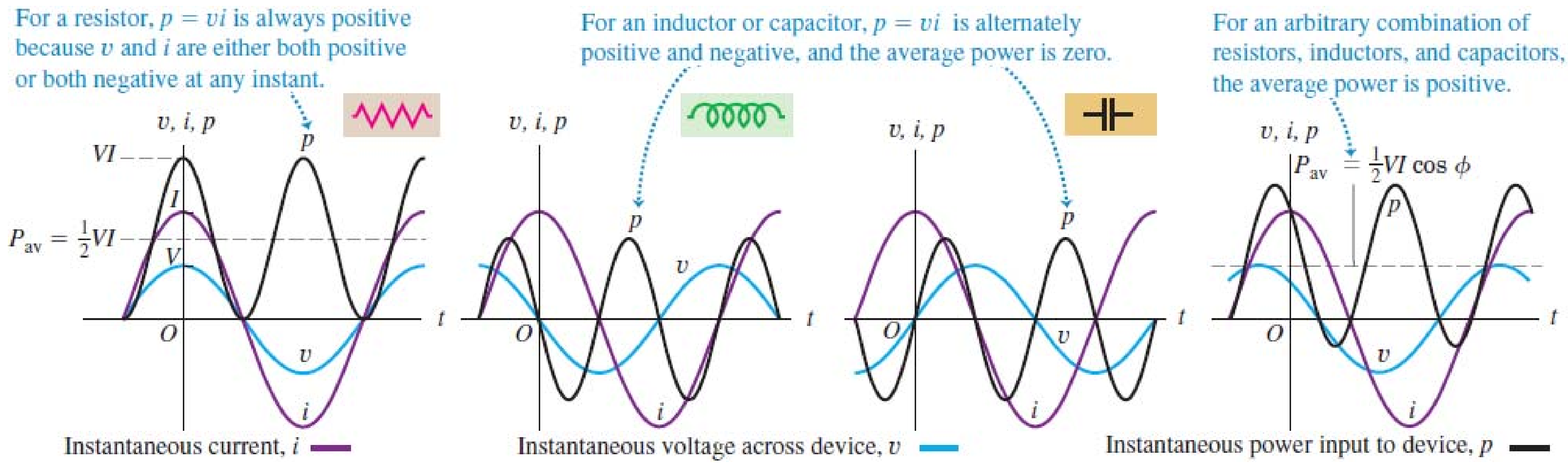
The instantaneous power delivered to a component in a circuit is $p = iv$.

For AC circuits, it is useful to look at the time averaged power.

The average power delivered to a resistor is

$$\begin{aligned} P_{\mathbf{avg}} &= \langle i v_R \rangle \\ &= \langle (I_{\mathbf{peak}} \cos \omega t)\,(V_R \cos \omega t) \rangle \\ &= \frac{1}{2} I_{\mathbf{peak}} V_R \\ &= I_{\mathbf{rms}} V_{R,\mathbf{rms}} \end{aligned}$$

An ideal inductor and ideal capacitor are ±90° out of phase. We also know that $\langle \cos \omega t \sin \omega t \rangle = 0$. Therefore, $P_{\mathbf{avg}} = 0$ for inductors and capacitors.



An AC source of period T and voltage amplitude V_{peak} is connected to a single unknown ideal element that is either a resistor, an inductor, or a capacitor. At time $t = 0$ the voltage is zero. At time $t = T/4$ the current in the unknown element is equal to zero and the voltage is $V = +V_{peak}$. At time $t = T/2$ the current is $I = -I_{peak}$, where I_{peak} is the current amplitude. What is the unknown element?

A. Capacitor

B. Resistor

C. Inductor

D. Inductor or capacitor

Resonance in a circuit

As the angular frequency ω varies, the current amplitude $I_{\mathbf{peak}} = \frac{V_{\mathbf{peak}}}{Z}$ varies.

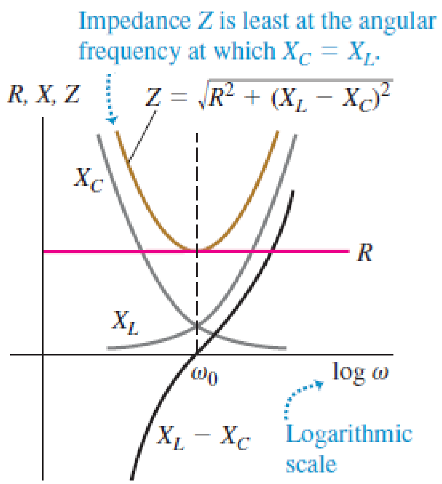
The current amplitude I_{peak} is maximized when the impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is minimized. If we cannot change the real resistance R, then we are left with reducing the out-of-phase terms,

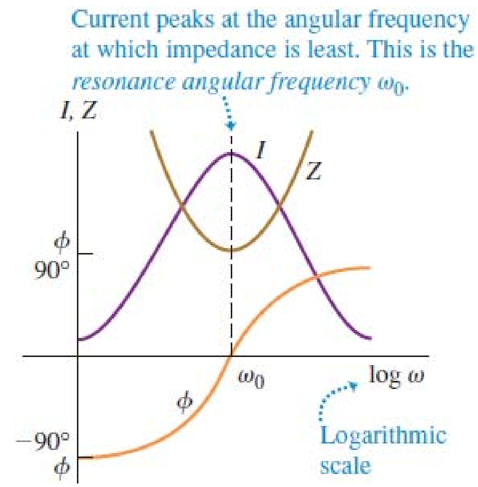
$$X_L - X_C = 0$$

$$\Rightarrow \omega L - \frac{1}{\omega C} = 0$$

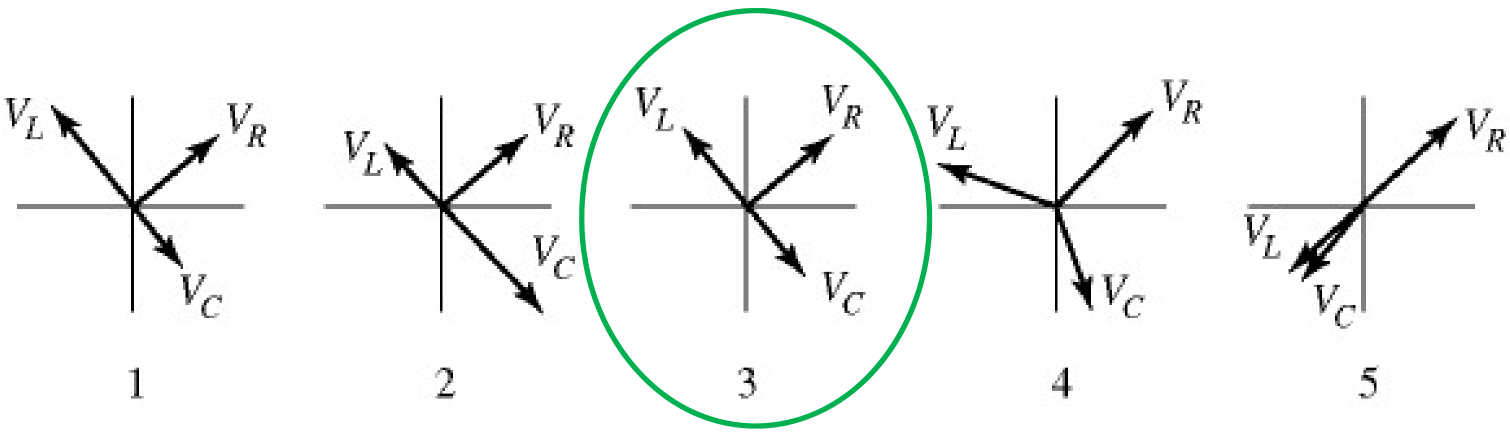
Solving for the <u>resonant</u> angular frequency (ω that gives the lowest impedance) gives

$$\omega = \frac{1}{\sqrt{LC}}$$





Which one of the phasor diagrams shown below best represents an ideal series LRC circuit driven at resonance?



When an LRC series circuit is driven at resonance, which of the following statements about the circuit is correct?

A. The impedance of the circuit is zero.

B. The inductive reactance and the capacitive reactance are both zero.

C. The impedance of the circuit has its maximum value.

D. The inductive reactance and the capacitive reactance are exactly equal to each other.

In a series LRC circuit, the frequency at which the circuit is at resonance is f_0. If you double the resistance, the inductance, the capacitance, and the voltage amplitude of the ac source, what is the new resonance frequency?

A. $f_0/4$

B. $f_0/2$

C. $2f_0$

D. $4f_0$

Consider a series RLC circuit for which $R = 150\ \Omega$, $L = 20.0$ mH, $V_{rms} = 20.0$ V, and $\omega = 5$ kHz. Determine the value of the capacitance for which the current is a maximum.

When an LRC series circuit is at resonance, which one of the following statements about that circuit is accurate?

A. The impedance has its maximum value.

B. The reactance of the inductor is zero.

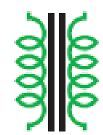
C. The current amplitude is a maximum.

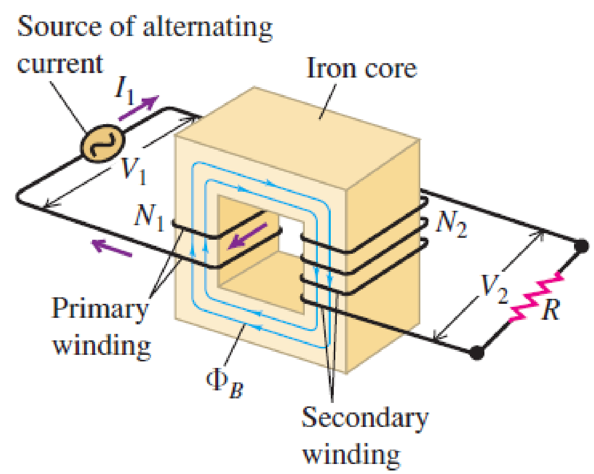
D. The reactance of the capacitor is zero.

Transformers

The key components of the transformer are two coils or windings, electrically insulated from each other but wound on the same core. The winding to which power is supplied is called the primary; the winding from which power is delivered is called the secondary.

The core is typically made of a material high relative permeability K_m material such as iron. The symbol for a transformer is



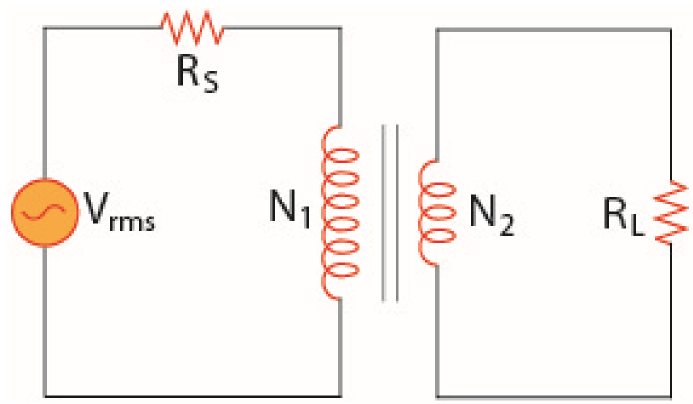


The ratio of the secondary emf $\mathcal{E}_2$ to the primary emf $\mathcal{E}_1$ is equal at any instant to the ratio of secondary to primary turns

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$$

If the windings have zero resistance, the induced emfs are equal to the terminal voltages,

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$



In the ideal transformer circuit shown, the load resistance is $R_L = 50.0\ \Omega$. The turns ratio is $N_1/N_2 = 2.50$, and the rms source voltage is $V_{rms} = 80.0$ V. If a voltmeter across the load resistance measures an rms voltage of 25.0 V, then what is the source resistance, R_S? $_{1}^{2}$

Which of the following statements is true regarding transformers?

A. A transformer is used to increase or decrease a direct current voltage.

B. In a transformer, the power input is less than the power output.

C. In a transformer, if the secondary coil contains more loops than the primary coil, then it is a step-up transformer.

D. A transformer converts mechanical energy into electrical energy.

The primary coil of a transformer has 100 turns and its secondary coil has 400 turns. If the ac voltage applied to the primary coil is 120 V, what voltage is present in its secondary coil?

A. 70 V

B. 280 V

C. 480 V

D. 630 V

An power plant needs to deliver energy at a rate of 20 MW to a city 1.0 km away. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission.

(a) If the resistance of the wires is 2.0 Ω and the energy costs are about 11 cents/kWh, estimate the cost of the energy converted to internal energy in the wires during one day.

(b) Repeat the calculation for the situation in which the power plant delivers the energy at its original voltage of 22 kV.