Chapter 35: interference

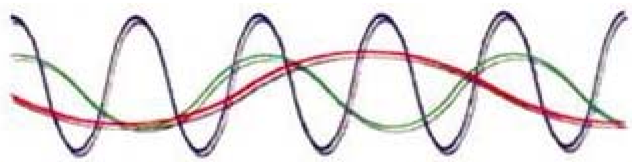
- Interference and coherent sources
- Two-source interference of light
- Intensity in interference patterns
- Interference in thin films
- The Michelson interferometer

Monochromatic and coherent light

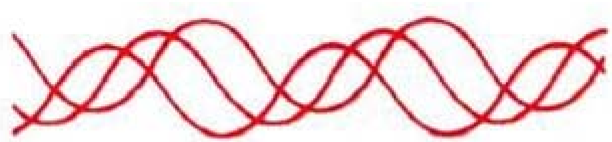
Monochromatic is a Greek word which translates to "one color." Monochromatic light consists of light waves all propagating with the same frequency.

If two wave sources have a *constant phase difference* at the *same frequency*, and they the *same waveform*, then the waves are said to be perfectly coherent.

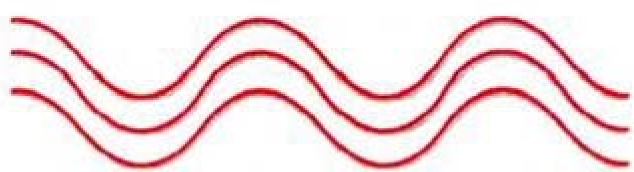
Truly perfect coherence never really occurs in nature, but many waves can often be approximated as coherent over a distance called the coherence length.



Sunlight (broad spectrum & very short coherence length)



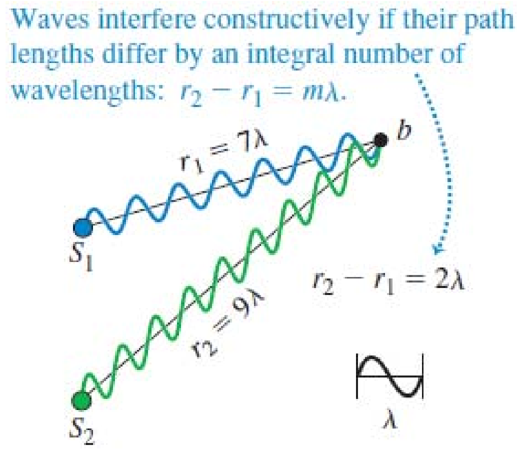
LED (narrow spectrum & short coherence length)



Laser (very narrow spectrum & long coherence length)

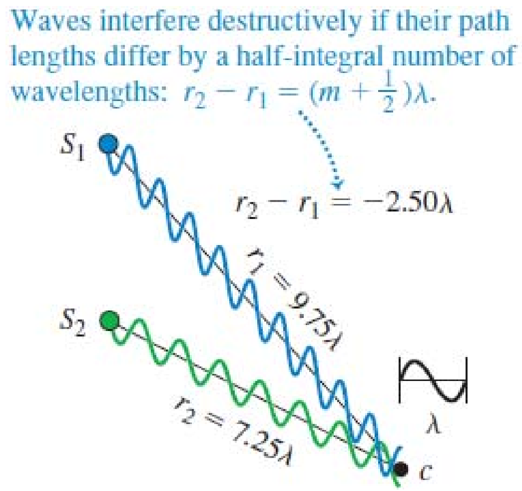
Interference

Constructive interference occurs when waves from two or more sources arrive at a point in phase such that they reinforce each other. Summation of the waves from the superposition principle results in a larger amplitude. The path difference $r_2 - r_1$ for the two sources must be an integral multiple of the wavelength λ,



$$r_2 - r_1 = m\lambda \quad \text{for} \quad m = 0, \pm 1, \pm 2, \ldots$$

Destructive interference occurs when waves from two or more sources arrive at a point in phase such that they oppose each other. Summation of the waves from the superposition principle results in a smaller amplitude. The path difference $r_2 - r_1$ for the two sources must be a half integral multiple of the wavelength λ,



$$r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda \quad \text{for} \quad m = 0,], \pm 1, \pm 2, \ldots$$

Interference from multiple point sources

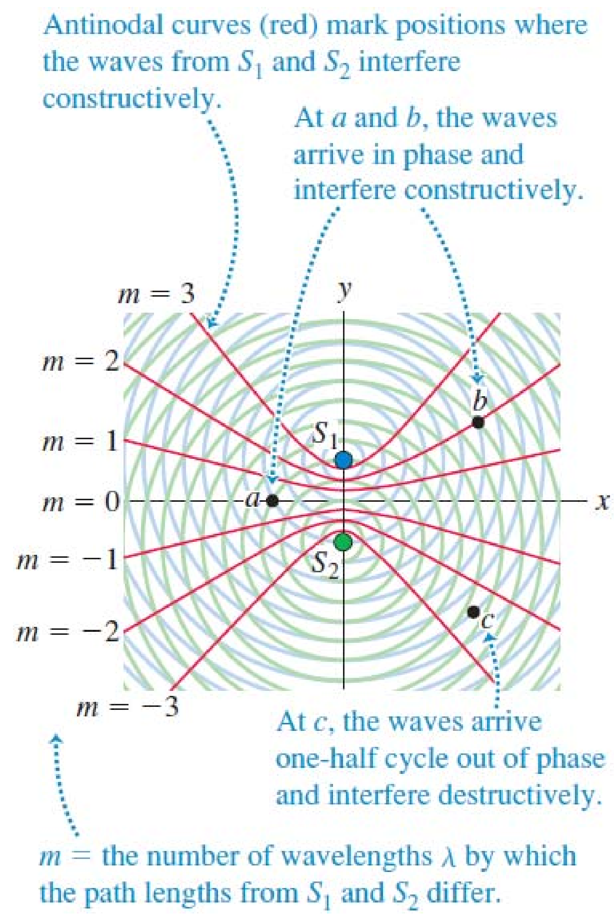
Multiple point-source light emitters placed at different locations cause interference patterns at difference locations in space.

To study this concept of interference by different sources, we should consider just two sources for simplicity.

For two sources separated by a small distance, and each emitting monochromatic light outward in all three spatial dimensions, the waves fronts interfer both constructively and destructively at different spatial locations.

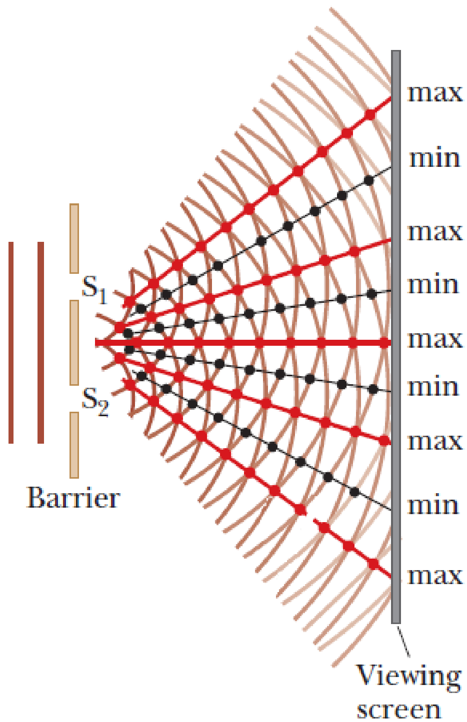
To the right is an *antinodal* curve, which shows the locations for *constructive* interference.

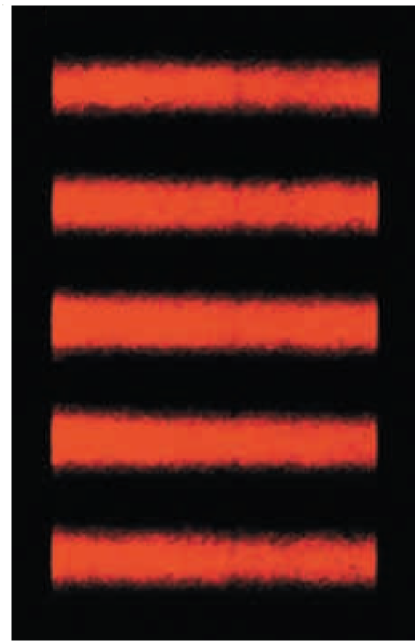
Nodal curves can also be created that show where destructive interference occurs.



Young’s double slit experiment

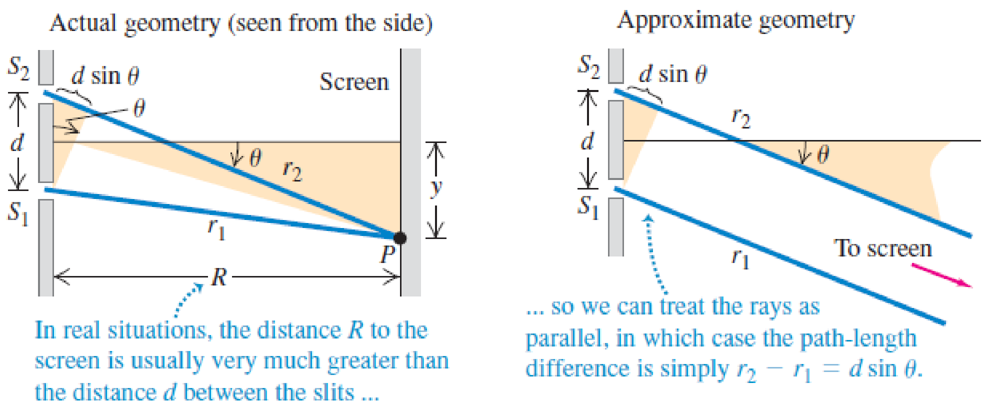
Light waves passing through two slits causes an interference profile at on a screen some distance away from the screen.





Slits S_1 and S_2 behave as coherent sources of light waves that produce an interference pattern on the viewing screen. A pattern of bright and dark areas appears on the viewing screen.

Analysis of double slit interference



The bright regions on the viewing screen (constructive interference) occur at angles for which

$$d \sin\theta = m\lambda \quad \text{for} \quad m = 0, \pm 1, \pm 2, \ldots$$

The dark regions on the viewing screen (destructive interference) occur at angles for which

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad \text{for} \quad m = 0, \pm 1, \pm 2, \ldots$$

Interference fringes are the succession of bright and dark bands. For *small angles only*, we can write down the position of the bright and dark fringes y_m as a function of the parameters shown in the above diagram, where

$$y_m^{\text{bright}} \approx R\frac{m\lambda}{d} \qquad y_m^{\text{dark}} \approx R\frac{(m + 1/2)\,\lambda}{d}$$

Amplitude of two source interference

Assuming the two waves have the same amplitude E_P, we can write the magnitude of the electric field at point P due to each wave separately as

$$E_1 = E_P \sin \omega t \quad \text{and} \quad E_2 = E_P \sin(\omega t + \phi)$$

Although the waves are in phase at the slits, their phase difference ϕ at point P depends on the path difference $r_2 - r_1 = d \sin \theta$.

A path difference of λ (for constructive interference) corresponds to a phase difference of 2π. A path difference of $r_2 - r_1$ is the same fraction of λ as the phase difference ϕ is of 2π. We can describe this fraction mathematically with the ratio

$$\frac{r_2 - r_1}{\lambda} = \frac{\phi}{2\pi}$$

This relationship allows us to write $\phi = 2\pi \frac{r_2 - r_1}{\lambda} = 2\pi \frac{d}{\lambda} \sin \theta$. Using the superposition principal, $E = E_1 + E_2$, and using the trigonometric identity,

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

We find the relationship, $E = 2E_P \sin\left(\omega t + \frac{\phi}{2}\right) \cos \frac{\phi}{2}$.

Intensity of two source interference

The intensity of a wave is proportional to the square of the resultant electric field magnitude, $I \propto E_P^2$. The squared electric field of the sinusoidal wave is given by

$$E^2 = 4E_P^2 \sin^2\left(\omega t + \frac{\phi}{2}\right)\cos^2\frac{\phi}{2}$$

Taking the time average of the sinusoidal wave, $\left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right)\right\rangle = \frac{1}{2}$, gives us the average of the electric field squared,

$$\left(E^2\right)_{\text{avg}} = 2E_P^2\cos^2\frac{\phi}{2}$$

The intensity follows as $I = \sqrt{\frac{\epsilon_0}{\mu_0}}\left(E^2\right)_{\text{avg}} = 2\sqrt{\frac{\epsilon_0}{\mu_0}}E_P^2\cos^2\frac{\phi}{2}. = I_0\cos^2\frac{\phi}{2}$

Remember that the phase difference was $\phi = \frac{2\pi}{\lambda}(r_2 - r_1)$, where $k = \frac{2\pi}{\lambda}$ and $r_2 - r_1 = d\sin\theta$. We can substitute this expression into our intensity relationship to get

$$I = I_0\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) \approx I_0\cos^2\left(\frac{\pi d y}{\lambda R}\right)$$

A light source emits visible light of two wavelengths $\lambda = 430$ nm and $\lambda' = 510$ nm. The source is used in a double-slit interference experiment in which $L = 1.50$ m and $d = 0.0250$ mm.

(a) Find the separation distance between the third-order bright fringes for the two wavelengths.

(b) Are there any locations on the screen where the bright fringes from the two wavelengths overlap exactly?

Light from a monochromatic source shines through a double slit onto a screen 5.00 m away. The slits are 0.180 mm apart. The dark bands on the screen are measured to be 1.70 cm apart. What is the wavelength of the incident light?

A. 392 nm

B. 457 nm

C. 612 nm

D. 784 nm

In a double slit experiment, the slit separation is constructed to be exactly 4 times the wavelength of the light passing through the slits. At what angles from the center of the pattern will the third bright fringes on both sides of the central fringe occur?

A. ±36.9°

B. ±48.6°

C. ±67.5°

D. ±75.0°

At most, how many bright fringes can be formed on each side of the central bright fringe (not counting the central bright fringe) when light of 625 nm falls on a double slit whose spacing is 3.12×10^{-6} m?

A. 3

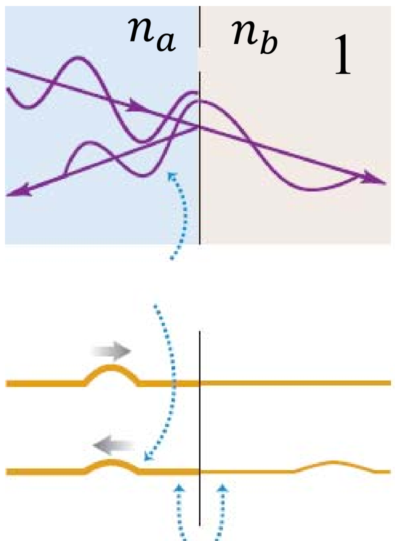
B. 4

C. 5

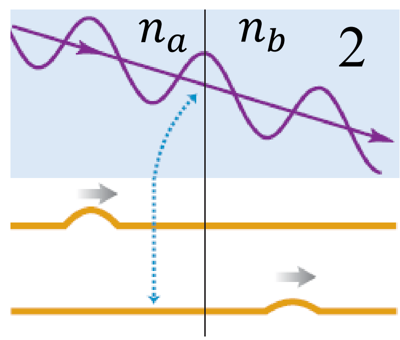
D. 6

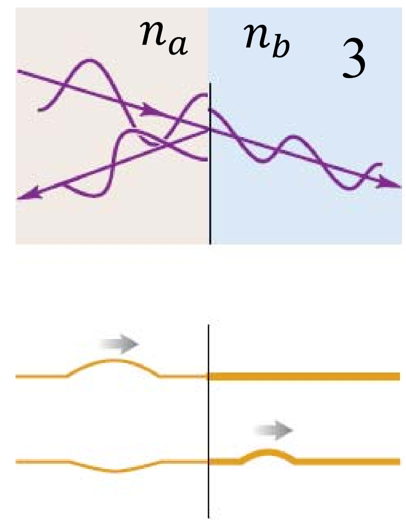
Phase of a reflected wave at an interface

At normal incidence, the reflected electric field amplitude, E_r, is determined by the incident electric field amplitude, E_i, and the refractive indices, n_a and n_b,



$$E_r = \frac{n_a - n_b}{n_a + n_b} E_i$$





1. When $n_a > n_b$ light travels more slowly in the first material than in the second. In this case, E_r and E_i have the same sign, and the phase shift of the reflected wave relative to the incident wave is zero.

2. When $n_a = n_b$ the amplitude of the reflected wave is zero.

3. When $n_a < n_b$ light travels more slowly in the second material than in the first. In this case, E_r and E_i have opposite signs, and the phase shift of the reflected wave relative to the incident wave is π.

Thin film interference

Light waves are reflected from the front and back surfaces of thin films. The difference between the paths of each light ray can cause a phase difference.

The thickness, t, and wavelength of light in the material, $\lambda_n = \lambda/n$ (where λ is the wavelength in free space), will determine the phase difference between the front and rear reflected waves.

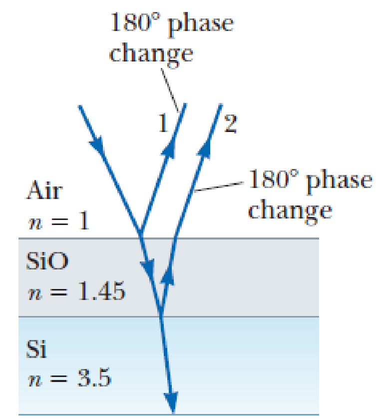
If neither or both of the reflected waves from the two surfaces have a π phase shift, the conditions for constructive and destructive interference are

Constructive

$$2t = m\lambda_n$$

Destructive

$$2t = \left(m + \frac{1}{2}\right)\lambda_n$$



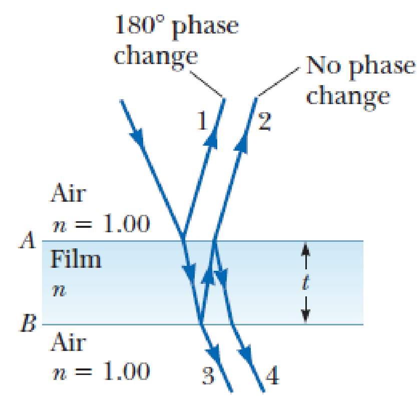
If only one of the reflected waves from the two surfaces has a π phase shift, the conditions for constructive and destructive interference are

Constructive

$$2t = \left(m + \frac{1}{2}\right)\lambda_n$$

Destructive

$$2t = m\lambda_n$$



A puddle of water has a thin film of gasoline floating on it. A beam of light is shining perpendicular on the film. If the wavelength of light incident on the film is 560 nm and the indices of refraction of gasoline and water are 1.40 and 1.33, respectively, what is the minimum thickness of the film to see a bright reflection?

A. 100 nm
B. 200 nm
C. 300 nm
D. 400 nm

Light of frequency 6.00×10^{14} Hz illuminates a soap film ($n = 1.33$) having air on both sides of it. When viewing the film by reflected light, what is the minimum thickness of the film that will give an interference maximum when the light is incident normally on it? ($c = 3.00 \times 10^{8}$ m/s)

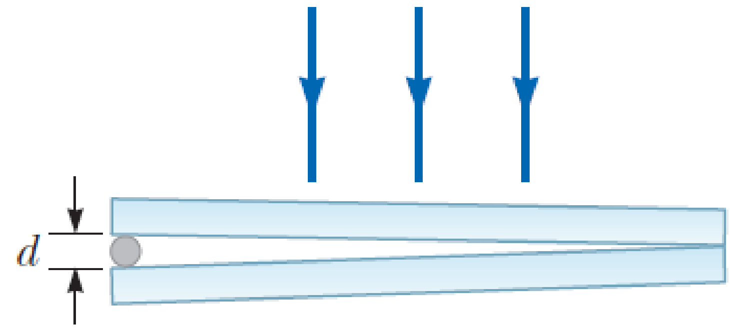
A. 24.0 nm

B. 94.0 nm

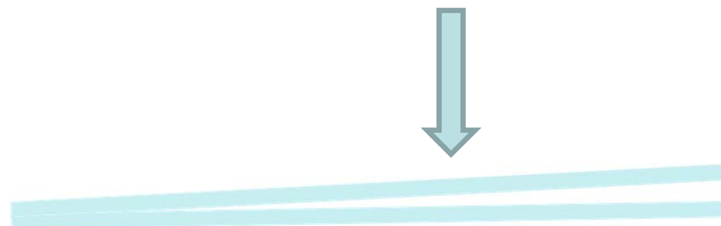
C. 188 nm

D. 279 nm

An air wedge is formed between two glass plates separated at one edge by a very fine wire of circular cross section as shown. When the wedge is illuminated from above by 600 nm light and viewed from above, 30 dark fringes are observed. Calculate the diameter *d* of the wire.



Two optically flat glass plates, 16.0 cm long, are in contact at one end and separated by 0.0200 mm at the other end. The space between the plates is occupied by oil with index of refraction 1.45. The index of refraction of the glass plates is 1.55. The plates are illuminated at normal incidence with monochromatic light, and fringes are observed. If the dark fringes are spaced 2.00 mm apart, what is the wavelength of the monochromatic light?



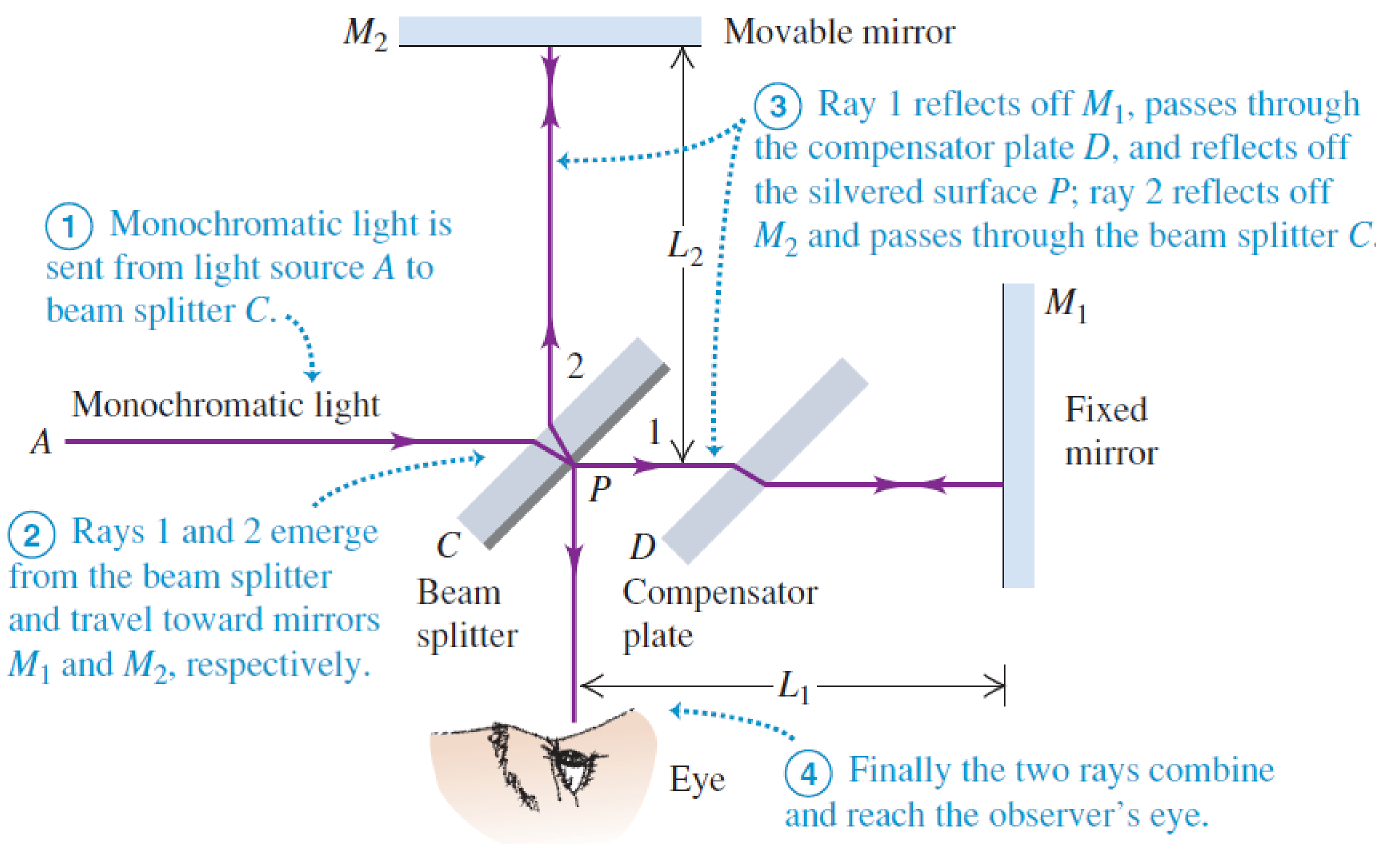
A. 425 nm

B. 525 nm

C. 675 nm

D. 725 nm

The Michelson interferometer



If M_2 is moved either backward or forward a distance of $\lambda/2$, then the difference in path length between rays 1 and 2 changes by a distance of λ.

Light of wavelength 550.5 nm is used to calibrate a Michelson interferometer. How many dark fringes are counted as the mirror is moved 0.180mm?