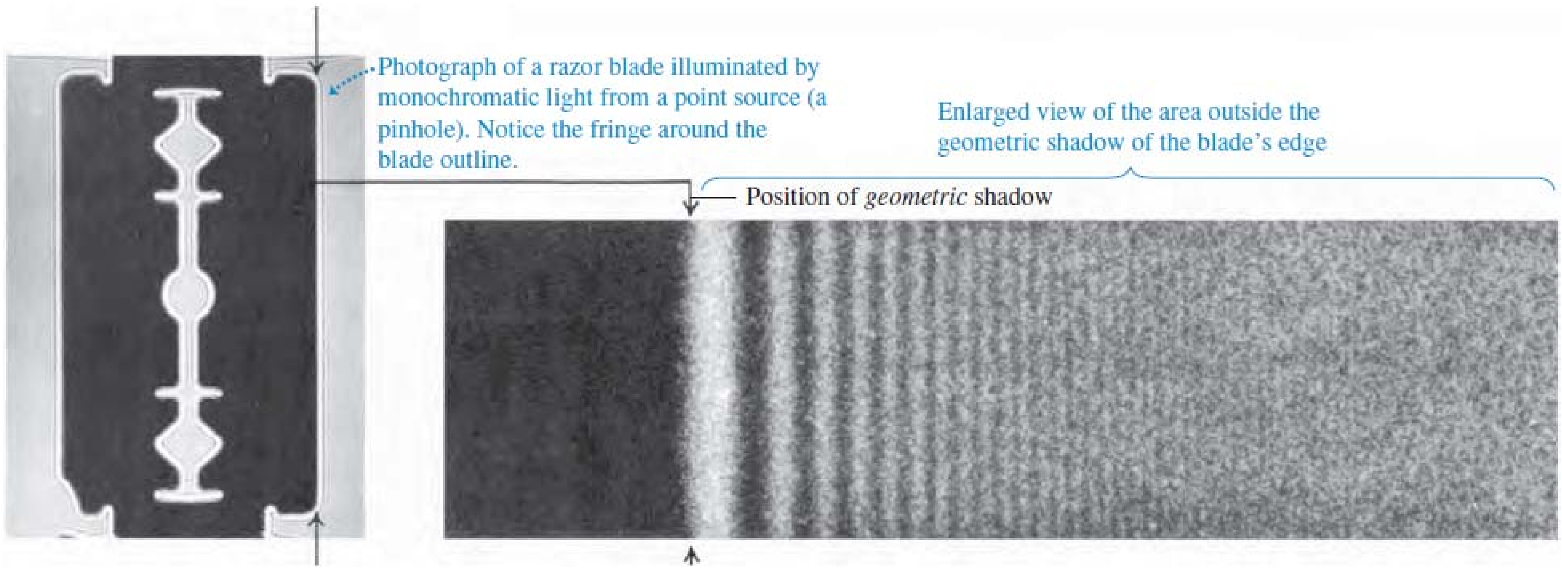
Chapter 36: diffraction

- Fresnel and Fraunhofer diffraction
- Diffraction from a single slit
- Intensity in the single slit pattern
- Multiple slits
- The Diffraction grating
- X-ray diffraction
- Circular apertures and resolving power
- Holography

Diffraction patterns

When light strikes an aperture or edge, an interference pattern occurs when projected onto a screen. This process is known as diffraction.

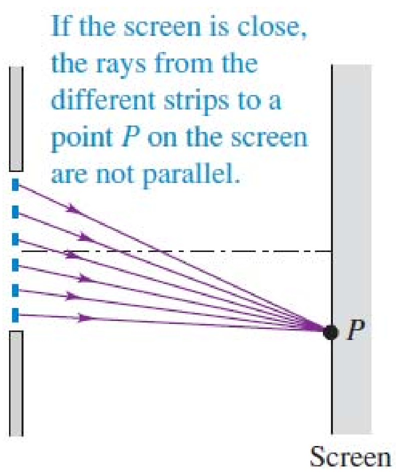


To help understand this phenomenon, we can consider every point of a wave front as a source of secondary wavelets, which spread out in all directions.

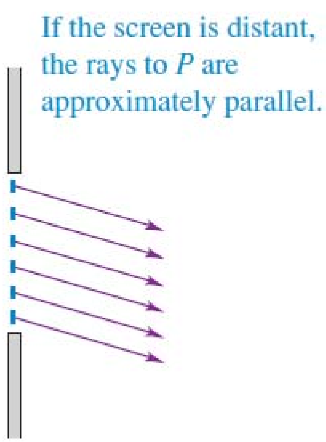
Individual displacements produced by these secondary waves can be combined using the superposition principle and can produce interference.

Types of diffraction

Fresnel diffraction occurs when both the source and the screen are relatively close (near field) to the obstacle that forms the diffraction pattern. The near-field situation can be complicated to analyze.

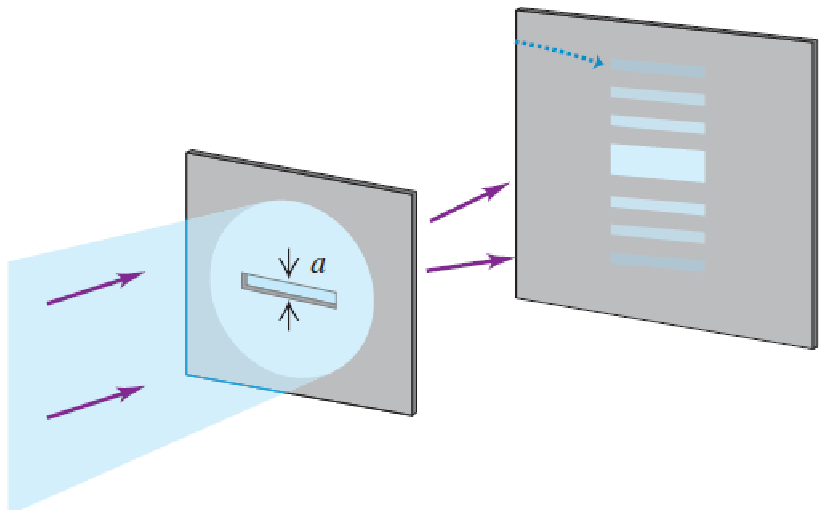


Fraunhofer diffraction occurs when both the source and the screen are relatively far away (far field) from the obstacle. We can approximate all lines from the source to the obstacle as well as the obstacle to the viewing screen to be parallel, which makes the far-field case much simpler to analyze.



Single slit diffraction

The diffraction pattern consists of a central bright band bordered by alternating dark and bright bands with rapidly decreasing intensity.

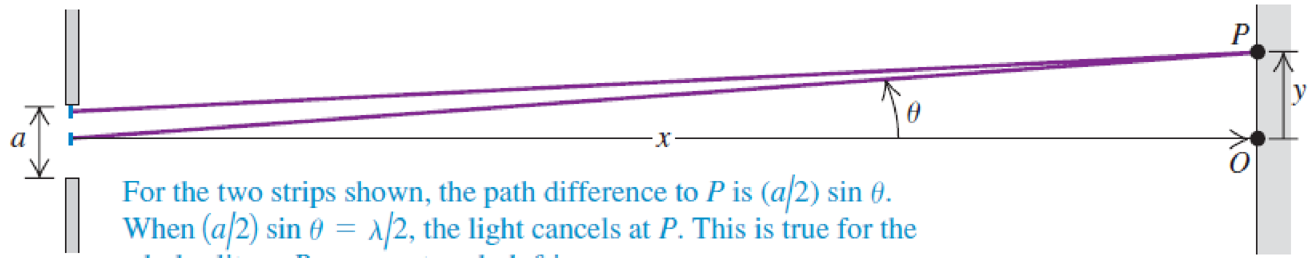


Remember that in the double slit experiment, the *maxima* were counted from the center.

In the single slit experiment, the *minima* are counted from the center.

Minima of a single slit diffraction pattern

When the distance x to the screen is much greater than the slit width a, the rays from a distance $a/2$ apart may be considered parallel.



For the two strips shown, the path difference to P is $(a/2)\sin\theta$. When $(a/2)\sin\theta = \lambda/2$, the light cancels at P. This is true for the whole slit, so P represents a dark fringe.

A dark fringe (minimum) occurs when light from every strip in the top half of the slit cancels out the light from a corresponding strip in the bottom half.

The condition for a minimum on the viewing screen is

$$a\sin\theta = m\lambda \qquad \text{for} \qquad m = \pm1, \pm2, \pm3, \ldots$$

For very small angles we may use the approximation, $y/x \approx \tan\theta \approx \theta \approx \sin\theta$, which allows us to write location of the minima on the viewing screen,

$$y \approx m\lambda\frac{x}{a}$$

A single-slit diffraction pattern is formed on a distant screen. Assuming the angles involved are small, by what factor will the width of the central bright spot on the screen change if the slit width is doubled?

A. It will be cut to one-quarter its original size.

B. It will be cut in half.

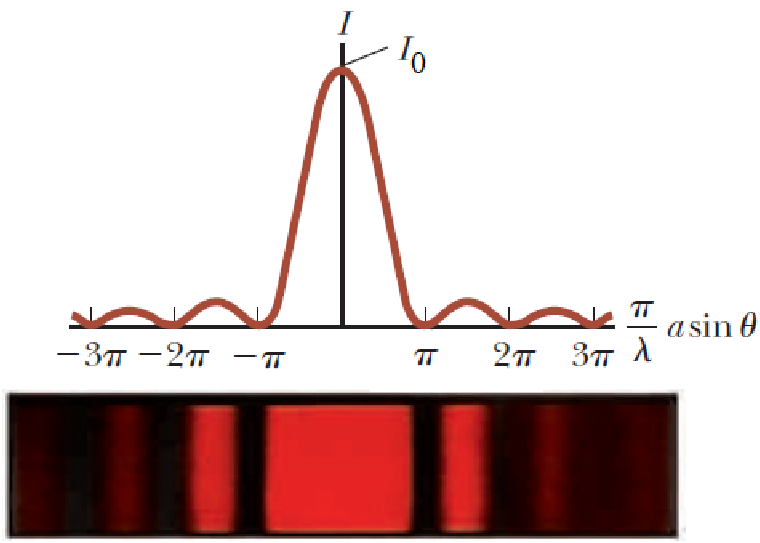
C. It will double.

D. It will become four times as large.

Intensity of a single slit diffraction pattern

The intensity profile of the single slit diffraction pattern is given by

$$I = I_0 \left[\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi a}{\lambda}\sin\theta} \right]^2$$



Interference fringes are the succession of bright and dark bands. For *small angles only*, we can write down the position of the bright and dark fringes y_m as a function of the parameters shown in the above diagram, where

Maxima of a single slit diffraction pattern

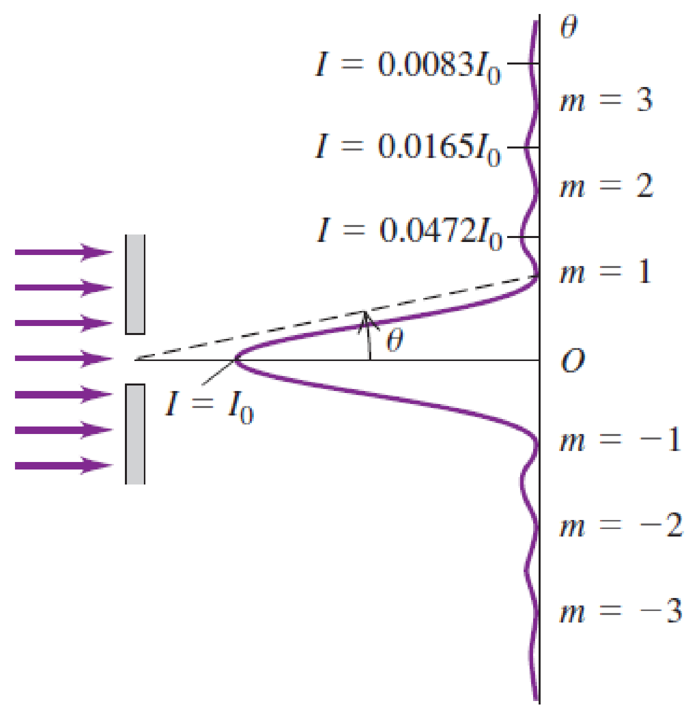
The maxima (excluding the center maximum) for a single slit can be approximated to be at the positions given by the equation

$$\pm(2m+1) = \frac{2a}{\lambda}\sin\theta \qquad \text{for} \qquad m = 1, 2, \ldots$$

The global maximum located at $\theta = 0$, completes the set of all maxima for the single slit diffraction pattern.

The intensity at each maxima (excluding the center maximum) is given by the equation

$$I_m = \frac{I_0}{\left(m+\frac{1}{2}\right)^2 \pi^2}$$



Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit.

(a) Find the positions of the first dark fringes.

(b) Find the width of the central bright fringe.

In a single-slit diffraction experiment, the width of the slit through which light passes is reduced. What happens to the width of the central bright fringe?

A. Its becomes more intense.

B. It becomes narrower.

C. It becomes wider.

D. It stays the same.

A single slit forms a diffraction pattern, with the first minimum at an angle of 40.0° from central maximum, when monochromatic light of 540-nm wavelength is used. The same slit, illuminated by a new monochromatic light source, produces a diffraction pattern with the second minimum at a 60.0° angle from the central maximum. What is the wavelength of this new light?

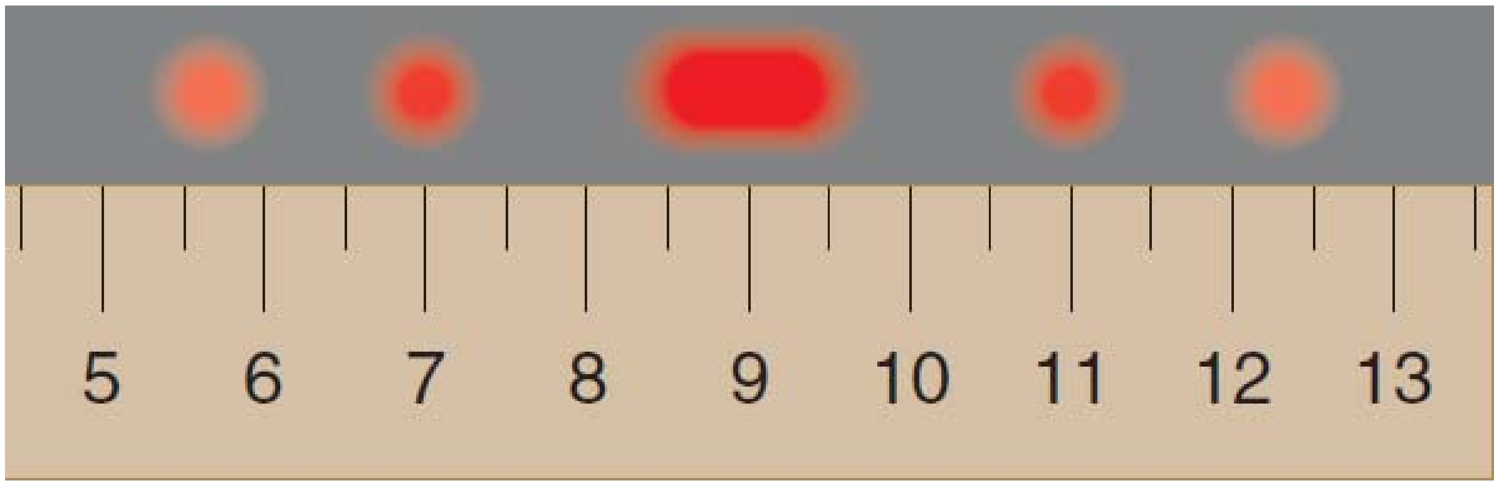
A. 429 nm

B. 407 nm

C. 386 nm

D. 364 nm

Laser light with a wavelength of 632.8 nm is directed through one slit or two slits and allowed to fall on a screen 2.60 m beyond. The figure shows the pattern on the screen, with a centimeter ruler below it.



(a) Did the light pass through one slit or two slits? Explain how you can determine the answer.

(b) If one slit, find its width. If two slits, find the distance between their centers.

If the 5th order minimum in the diffraction pattern due to a thin slit is at 40° from the central maximum, at what angle does the 1st order minimum occur?

A. 3.4°

B. 4.0°

C. 7.4°

D. 8.0°

A laser beam passes through a thin slit. When the pattern is viewed on a screen 1.25 m past the slit, you observe that the 5^{th}-order dark fringes occur at ±2.41 cm from the central bright fringe. The entire experiment is now performed within a liquid, and you observe that each of the 5^{th}-order dark fringes is 0.790 cm closer to the central fringe than it was in air. What is the index of refraction of this liquid?

A. 1.33

B. 1.40

C. 1.49

D. 1.62

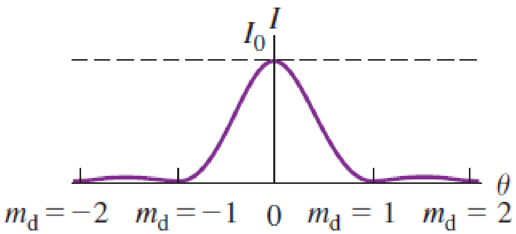
Finite width slit effects in double slit diffraction

For a double slit experiment, we assumed an infinitesimal width; however, the slits have some small width in real experiments. Thus, the intensity is the double slit diffraction pattern enveloped by the single slit diffraction pattern.

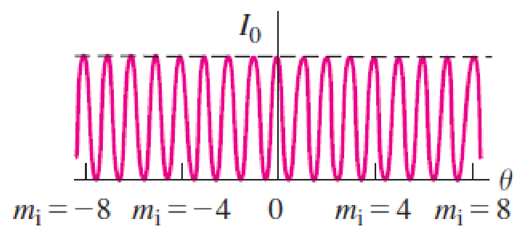
The intensity for the single slit and double slits are respectively

$$I_{\text{single}} = I_0 \left[\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi a}{\lambda}\sin\theta} \right]^2 \quad \text{and} \quad I_{\text{double}} = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

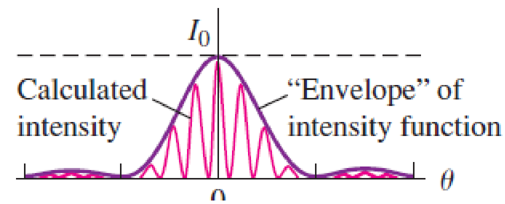
Single-slit diffraction pattern for a slit width a



Two-slit interference pattern for narrow slits whose separation d is four times the width of the slit in (a)



Calculated intensity pattern for two slits of width a and separation $d = 4a$, including both interference and diffraction effects

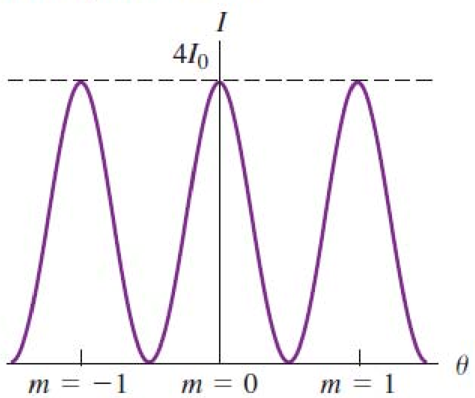


The intensity of a double slit experiment with finite sized slits follows as

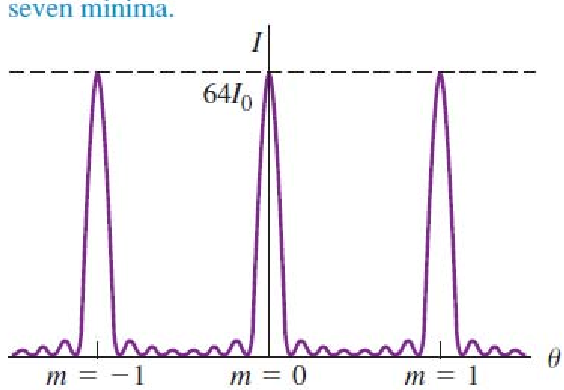
$$I = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) \left[\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi a}{\lambda}\sin\theta} \right]^2$$

Diffraction gratings (multi-slit diffraction)

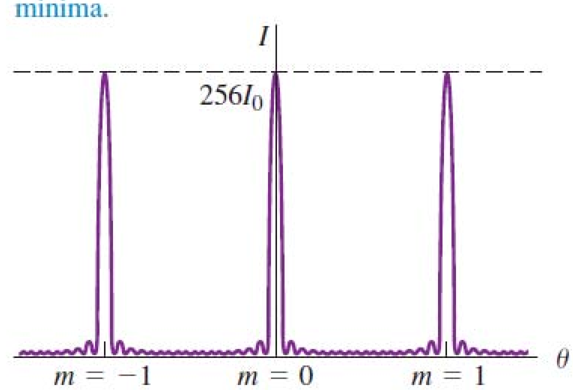
The number of slits in an interference experiment (while keeping the spacing of adjacent slits constant) gives interference patterns in which the maxima are in the same positions, but progressively narrower, than with two slits.



N = 2: two slits produce one minimum between adjacent maxima.



N = 8: eight slits produce taller, narrower maxima in the same locations, separated by seven minima.



N = 16: with 16 slits, the maxima are even taller and narrower, with more intervening minima.

The positions of the maxima for a diffraction grating are the same as for the double slit experiment for the broad peaks, which are located at

$$d \sin\theta = m\lambda \qquad \text{for} \qquad m = 0, \pm 1, \pm 2, \ldots$$

Consider two diffraction gratings. One grating has 3000 lines per cm, and the other one has 6000 lines per cm. Both gratings are illuminated with a beam of the same monochromatic light. Which grating produces the greater dispersion?

A. Both gratings produce the same dispersion.
B. The grating with 6000 lines produces the greater dispersion.
C. The grating with 3000 lines produces the greater dispersion.
D. The dispersion cannot be determined without additional information.

Consider two diffraction gratings with the same slit separation. The only difference between the two gratings is that one grating has 5 slits and the other 10 slits. Both gratings are illuminated with a beam of the same monochromatic light. Which of the following statements is true?

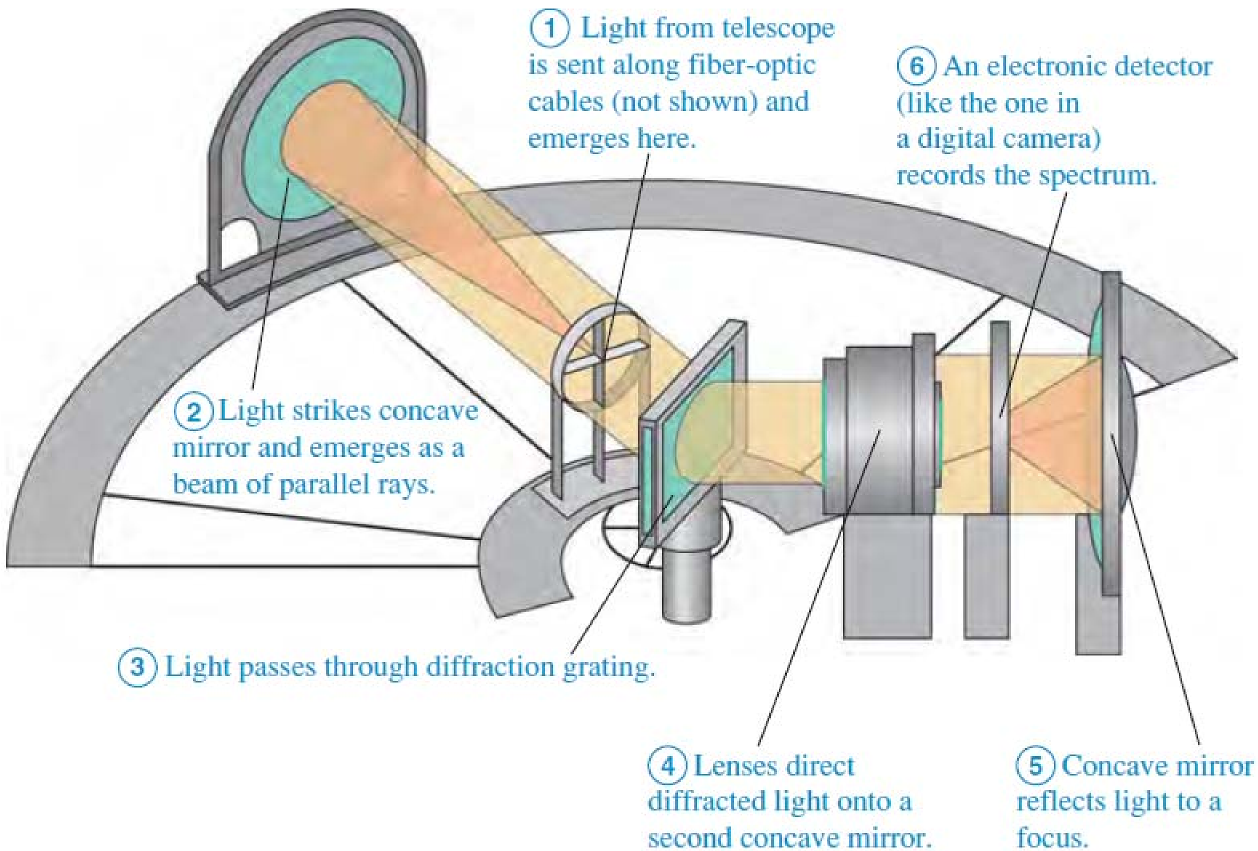
A. The grating with 5 slits produces the greater separation between peaks.

B. The grating with 10 slits produces the greater separation between peaks.

C. The grating with 5 slits produces better-defined (narrower) peaks.

D. Both gratings produce the same separation between peaks.

Grating spectrographs



The minimum wavelength difference $\Delta\lambda$ that can be distinguished by a spectrograph is described by the chromatic resolving power R,

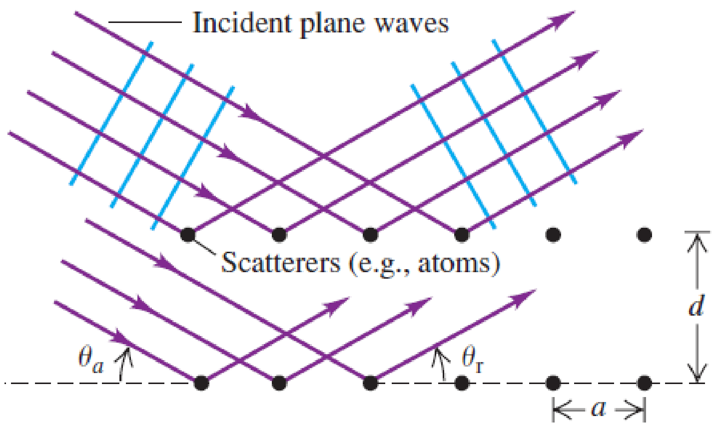
$$R = \frac{\lambda}{\Delta\lambda}$$

X-ray diffraction

The Bragg condition for constructive interference from an array of atoms in a crystal is given by

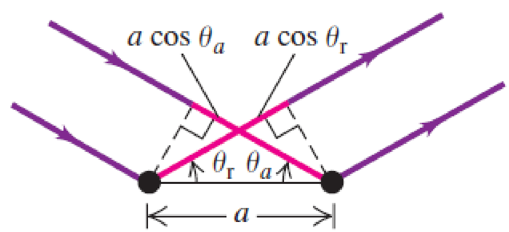
$$2d\sin\theta = m\lambda \qquad \text{for} \qquad m = 0, \pm 1, \pm 2, \ldots$$

Scattering of waves from a rectangular array



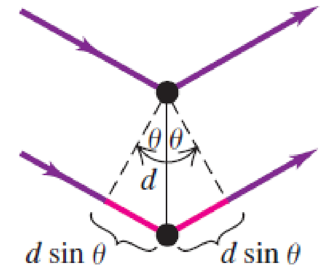
Scattering from adjacent atoms in a row

Interference from adjacent atoms in a row is constructive when the path lengths $a\cos\theta_a$ and $a\cos\theta_r$ are equal, so that the angle of incidence θ_a equals the angle of reflection (scattering) θ_r.



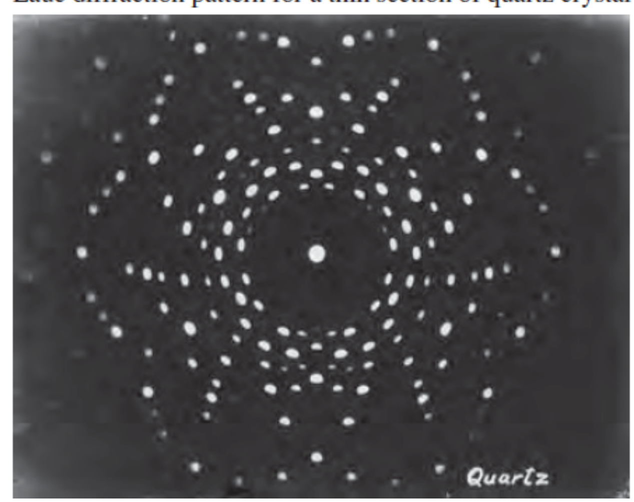
Scattering from atoms in adjacent rows

Interference from atoms in adjacent rows is constructive when the path difference $2d\sin\theta$ is an integral number of wavelengths, as in Eq. (36.16).



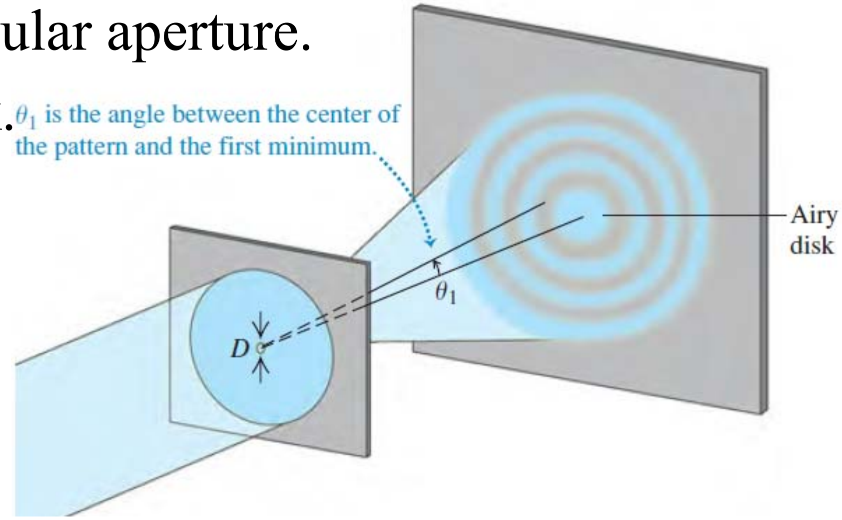
In x-ray diffraction there is nearly complete cancellation (similar to multiple slits) in all but certain very specific directions in which constructive interference occurs and forms bright spots.

Laue diffraction pattern for a thin section of quartz crystal



Circular apertures

A diffraction pattern is also formed by a circular aperture. The central bright spot is called an Airy disk.



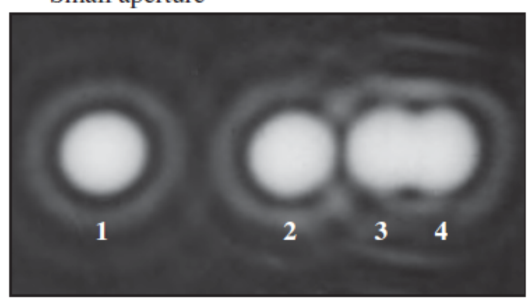
If the aperture diameter is D and the wavelength is λ, then the angular radius θ_1 of the first few dark (minima) rings have the conditions

$$\sin\theta_1 \approx 1.22\frac{\lambda}{D}$$

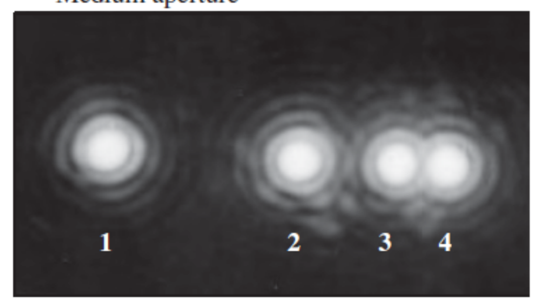
$$\sin\theta_2 \approx 2.23\frac{\lambda}{D}$$

$$\sin\theta_3 \approx 3.24\frac{\lambda}{D}$$

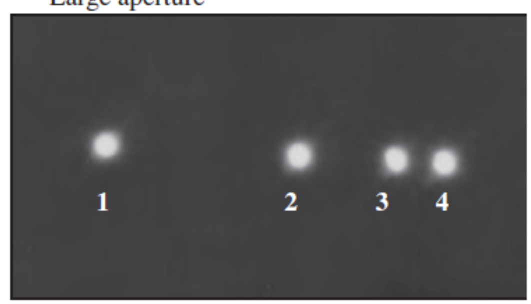
Small aperture



Medium aperture



Large aperture



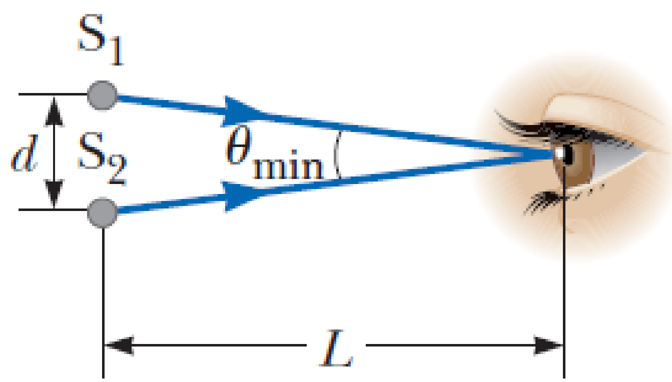
The minimum separation of two objects that can barely be resolved by an optical instrument is the limit of resolution. The smaller the limit of resolution, the greater the resolution (resolving power).

Resolution improves with larger diameter, and it also improves with shorter wavelengths!

Light of wavelength 500 nm, near the center of the visible spectrum, enters a human eye. Although pupil diameter varies from person to person, let's estimate a daytime diameter of 2 mm.

(a) Estimate the limiting angle of resolution for this eye, assuming its resolution is limited only by diffraction.

(b) Determine the minimum separation distance *d* between two point sources that the eye can distinguish if the point sources are a distance $L = 25$ cm from the observer.



A lens is designed to work in the near-ultraviolet, visible, and near-infrared. The best resolution of this lens from a diffraction standpoint is

A. in the near-ultraviolet.

B. in the visible.

C. in the near-infrared.

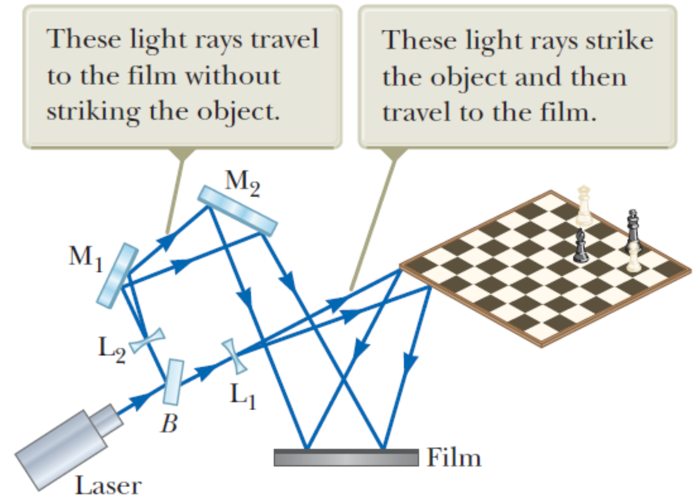
D. the same for all wavelengths.

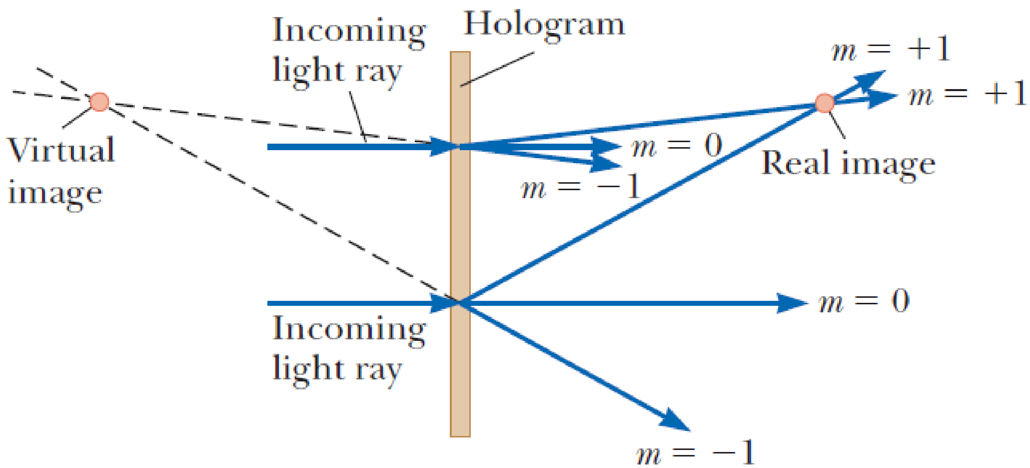
If the diameter of a radar dish is doubled, what happens to its resolving power assuming that all other factors remain unchanged?

A. The resolving power is reduced to 1/4 of its original value.
B. The resolving power is reduced to 1/2 of its original value.
C. The resolving power doubles.
D. The resolving power quadruples.

Holography

Holography is a technique for recording and reproducing an image of an object through the use of interference effects.





The $m = +1$ rays converge to form a real image of the scene, which is not the image that is normally viewed.

By extending the light rays corresponding to $m = -1$ behind the film, we see that there is a virtual image located there, with light coming from it in exactly the same way that light came from the actual object when the film was exposed.