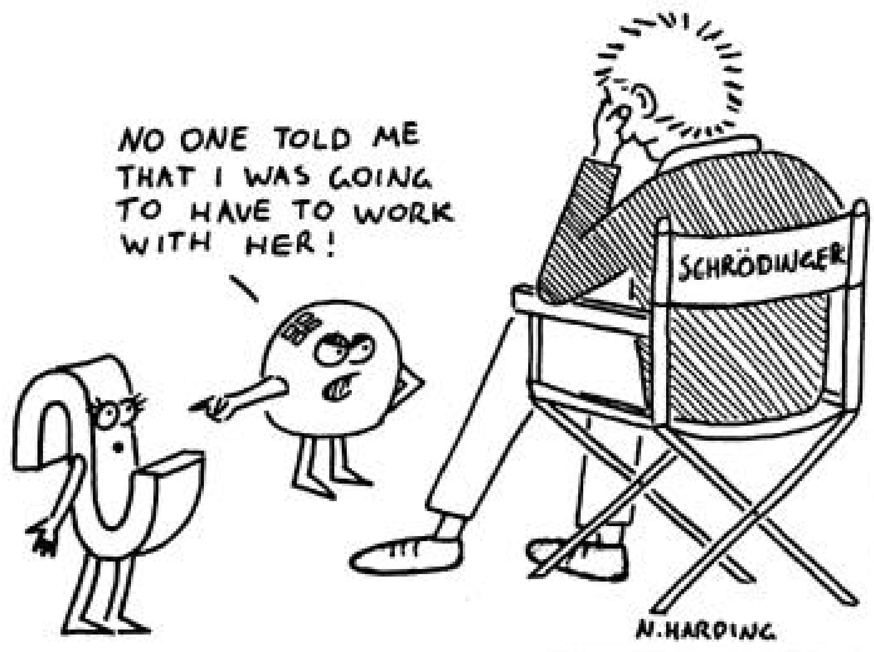
Chapter 38: Photons

- Light absorbed as photons
- X-ray production
- Light scattered as photons
- Wave-particle duality, probability, and uncertainty

Absorption of photons

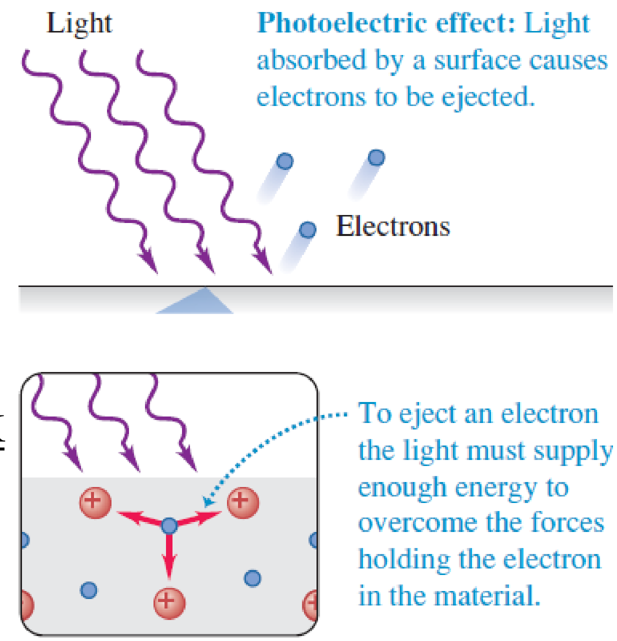
Light acts sometimes like waves and sometimes like particles. Interference and diffraction demonstrate wave behavior, while emission and absorption of photons demonstrate the particle behavior.

The energy of an electromagnetic wave is quantized, where it is absorbed and emitted in particle-like packets of definite energy, called photons.

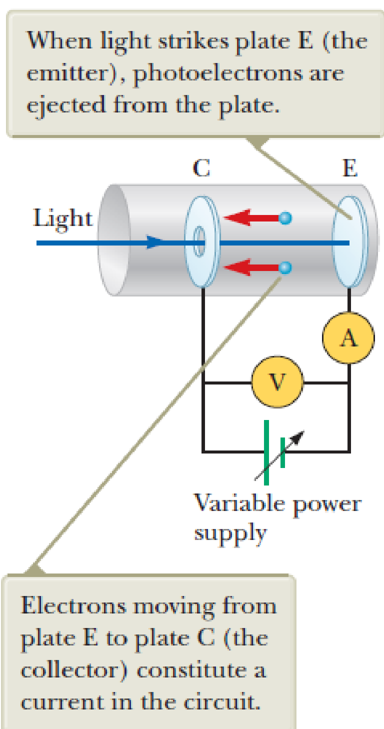


The photoelectric effect

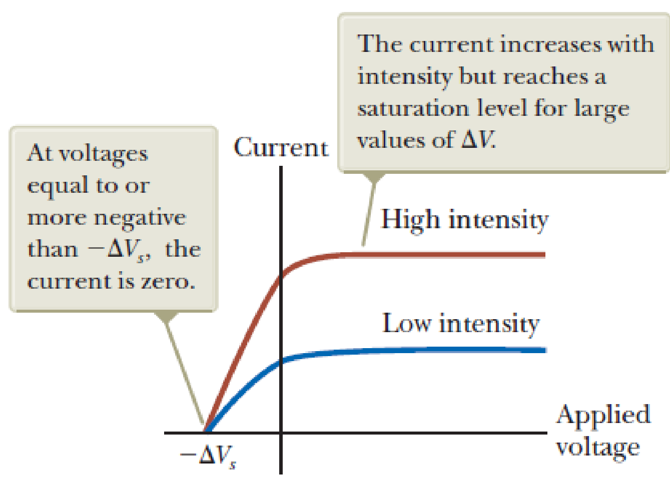
At high enough frequencies (energies) materials emit electrons from their surface when illuminated. To escape from the surface, an electron must absorb enough energy from light to overcome the attraction of positive ions (potential wells) in the material.



It takes a minimum amount of energy, called the <u>work function</u> ϕ, to eject a single electron from a surface.



When ΔV is equal to or more negative than $2\Delta V_s$, where ΔV_s is the <u>stopping potential</u>, no photoelectrons reach C and the current is zero.



The photoelectric effect (continued)

A purely wave-like picture does not properly address the experimental results showing the photoelectric effect.

Time interval between incidence of light and ejection of photoelectrons

Wave-like prediction

At low light intensities, a measurable time interval should pass between the instant the light is turned on and the time an electron is ejected from the material. This time interval is required for the electron to absorb the incident radiation before it acquires enough energy to escape from the material.

Experimental result

Electrons are emitted from the surface of the material almost *instantaneously* (less than 10^{-9} s after the surface is illuminated), even at very low light intensities.

The photoelectric effect (continued)

Dependence of ejection of electrons on light frequency

Wave-like prediction

Electrons should be ejected from the metal at any incident light frequency, as long as the light intensity is high enough, because energy is transferred to the metal regardless of the incident light frequency.

Experimental result

No electrons are emitted if the incident light frequency falls below some cutoff frequency f_c, whose value is characteristic of the material being illuminated. No electrons are ejected below f_c *regardless* of the light intensity.

Dependence of photoelectron kinetic energy on light frequency

Wave-like prediction

There should be no relationship between the frequency of the light and the electron kinetic energy. It should be related to the intensity of the light.

Experimental result

The maximum kinetic energy of the photoelectrons increases with increasing light frequency.

Energy of a photon

Einstein postulated that a beam of light consists of small packages of energy called photons, which is an extension of Max Planck's idea to explain the properties of blackbody radiation.

The energy E of an individual photon is equal to a constant h (called the Planck constant) multiplied by the photon frequency f,

$$E = hf$$

Using the wave relationship $c = \lambda f$ we may also relate the photon energy to its wavelength in free space,

$$E = h\frac{c}{\lambda}$$

The value for Planck's constant has been measured to the accuracy of

$$h = 6.62606896(33) \times 10^{-34}\ \mathrm{J\,s}$$

Photon energy and the photoelectric effect

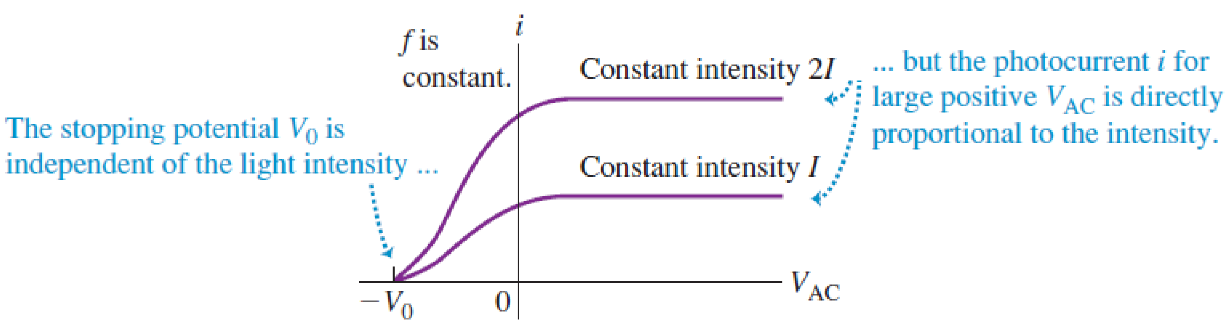
An individual photon arriving at a material's surface is absorbed, where this energy transfer is an *all-or-nothing* process. This picture contradicts the continuous transfer of energy in a strictly wave theory.

Applying the conservation of energy to find that the maximum kinetic energy $K_{\max} = \frac{1}{2}mv_{\max}^2$ for an emitted electron is the energy gained from a photon minus the work function ϕ,

$$\frac{1}{2}mv_{\max}^2 = hf - \phi$$

We also know that the work required to bring the charge of magnitude e out of the potential V_0 to be $W = eV_0$. Using the work-kinetic energy theorem, we may write the equation as

$$eV_0 = hf - \phi$$



Monochromatic light strikes a metal surface and electrons are ejected from the metal. If the intensity of the light is increased, what will happen to the ejection rate and maximum energy of the electrons?

A. Greater ejection rate; greater maximum energy

B. Same ejection rate; same maximum energy

C. Same ejection rate; greater maximum energy

D. Greater ejection rate; same maximum energy

A beam of red light and a beam of violet light each deliver the same power on a surface. For which beam is the number of photons hitting the surface per second the greatest?

A. The red beam.

B. The violet beam.

C. It is the same for both beams.

A sodium surface is illuminated with light having a wavelength of 300 nm. The work function for sodium metal is 2.46 eV.

(a) Find the maximum kinetic energy of the ejected photoelectrons.

(b) Find the maximum wavelength (minimum energy photon) that can eject a sodium electron.

Light of wavelength 400 nm falls on a metal surface having a work function 1.70 eV. What is the maximum kinetic energy of the photoelectrons emitted from the metal? ($c = 3.00\times10^{8}$ m/s; $h = 6.626\times10^{-34}$ Js $= 4.141 \times 10^{-15}$ eVs; 1 eV $= 1.60 \times 10^{-19}$ J)

A. 1.41 eV

B. 1.70 eV

C. 2.82 eV

D. 3.11 eV

Momentum of a photon

Every particle that has energy must also have momentum according to special relativity, even if it has no rest mass.

A photon (as with any particle that has zero rest mass) has an energy E given by

$$E = pc$$

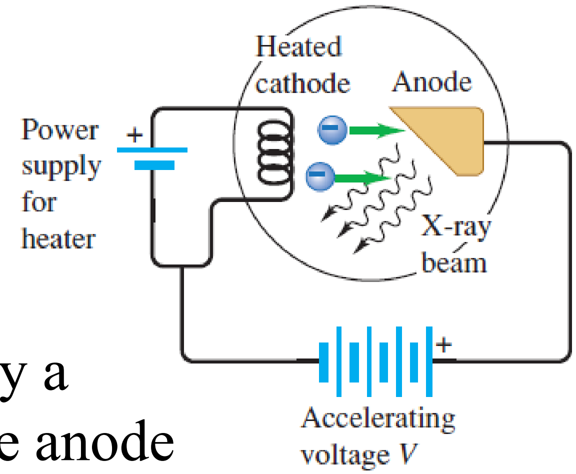
Solving for the magnitude of the momentum p, we find

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

The direction of the photon's momentum is the direction in which the electromagnetic wave is moving.

Photon with x-ray energies

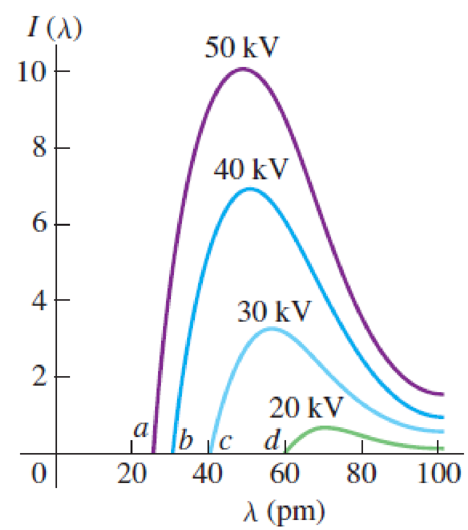
X-rays are produced by heating a cathode like the one in the diagram to a very high temperature. Electrons are then released from the cathode via thermionic emission



The electrons are then accelerated toward the anode by a potential difference V. These electrons collide with the anode and energy is released from the collision in the form of x-rays.

The radiation caused by the fast deceleration of the electrons is called *bremsstrahlung* ("braking radiation").

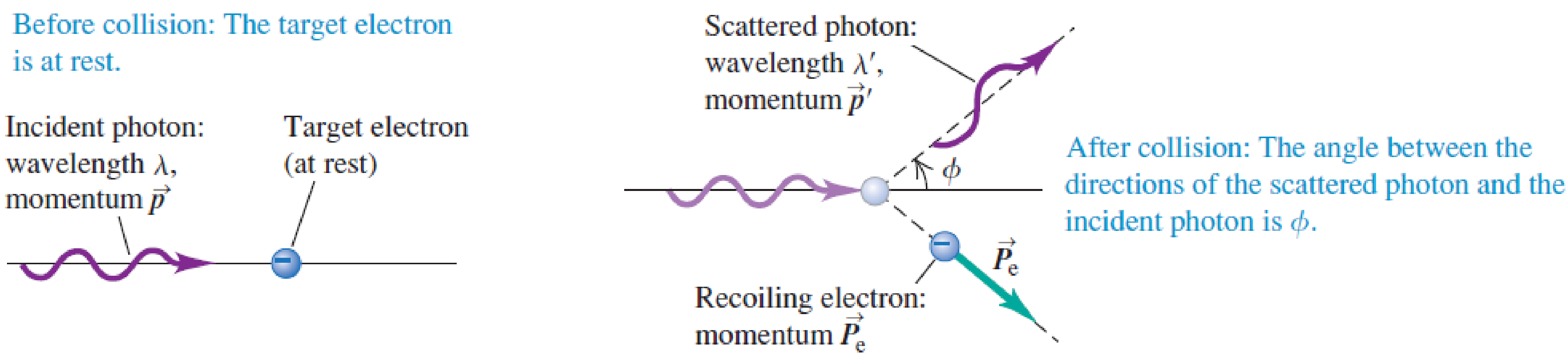
For a simple model, let us neglect the initial kinetic energy of the electrons after being released by the cathode as well as the work function. If the electrons are perfectly stopped at the electrode, all energy is transferred into a photon. This maximum energy (minimum wavelength) is



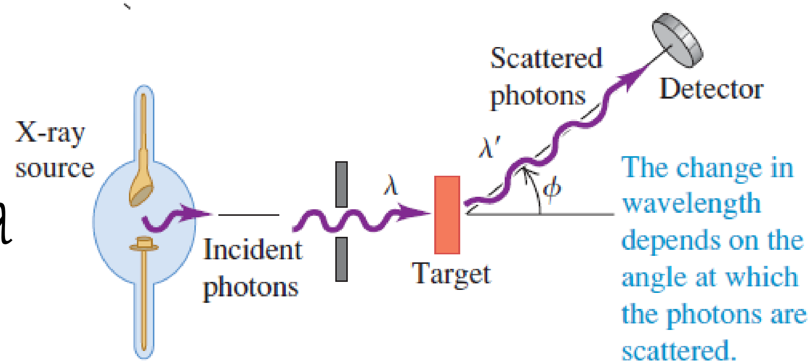
$$eV = E_{\max} = hf_{\max} = \frac{hc}{\lambda_{\min}}$$

Compton scattering

The Compton effect is a phenomenon that deals with photons behaving like particles and scattering from fundamental particles.



The Compton scattering experiment showed results in which the wavelength change $\lambda' - \lambda$ is a function of the deflection angle ϕ.



When the photon collides with the electron, it transfers some of its momentum (and energy) to the electron. Thus, the scattered photon should have a lower energy (or lower frequency; longer wavelength). The Compton scattering equation is given by

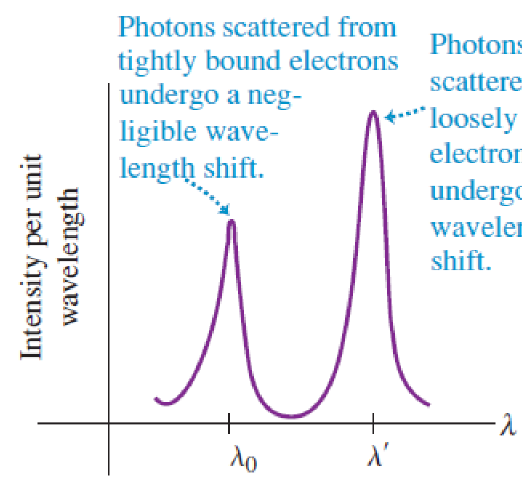
$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

Compton scattering from different materials

Free electrons (or nearly free electrons such as the conduction band electrons in a metal) show a large wavelength shift in the Compton scattering experiment.

When electrons are tightly bound, such as the core electrons in a molecule, the there is a finite number of possible energies that an electron can have unless ionization occurs (free electrons can have a continuous spectrum of energies).

For tightly bound systems such as the hydrogen atom, unless there is some quanta of energy absorbed, the scattering does not affect the electron's momentum relative to the nucleus, and therefore, the photon scatters of the system and loses significantly less energy.



X-rays of wavelength $\lambda = 0.200000$ nm are scattered from a block of material. The scattered x-rays are observed at an angle of 45.0° to the incident beam. Calculate their wavelength. ($m_{el} = 9.11\times10^{-31}$ kg, $h = 6.626\times10^{-34}$ Js, $c = 3.00\times10^{8}$ m/s)

In a particular case of Compton scattering, a photon collides with a free electron and scatters backwards. The wavelength after the collision is exactly double the wavelength before the collision. What is the wavelength of the incident photon? ($m_{el} = 9.11\times10^{-31}$ kg, $h = 6.626\times10^{-34}$ Js, $c = 3.00\times10^{8}$ m/s)

A. 2.4 pm

B. 3.6 pm

C. 4.8 pm

D. 6.0 pm

Particle pair production

The particle pair production phenomenon can only be explained with the photon picture.

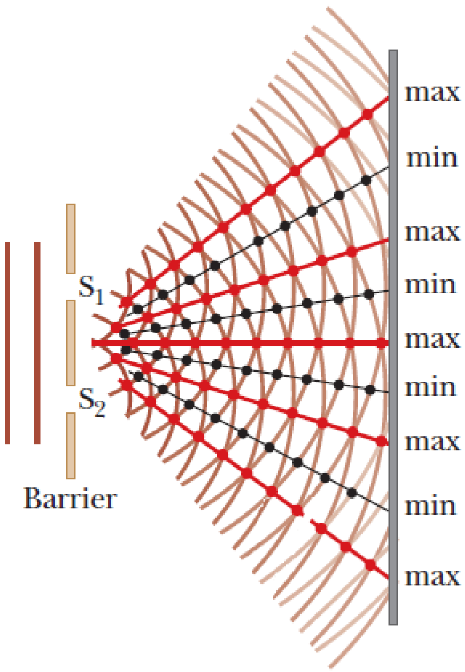
If a gamma-ray photon (classification for highest energy photons) of sufficiently high frequency (short wavelength) strikes a target, it may not scatter at all! Instead it may be annihilated (within experimental precision of detectors) and we detect by two new particles being created of equal energy (again, within experimental uncertainty).

This particle pair must conserve many of the particle physics quantities such as charge and lepton number. A commonly observed pair production is that of an electron ($q/e = -1$ & $L_e = +1$) and a positron ($q/e = +1$ & $L_e = -1$).

Enough energy must be available to account for the rest energy $2mc^2$ of the two particles. To four significant figures, this minimum energy required to produce an electron and a positron is

$$E_{\min} = 2mc^2 = 2\left(9.109 \times 10^{-31}\,\mathrm{kg}\right)\left(2.998 \times 10^{8}\,m/s\right)^2$$
$$= 1.637 \times 10^{-13}\,\mathrm{J} = 1.022 \times 10^{6}\mathrm{eV}$$

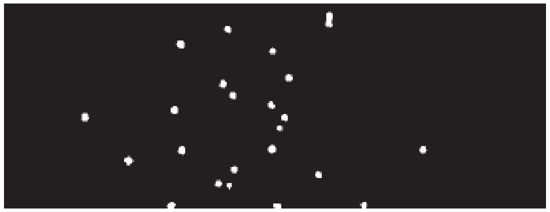
Diffraction and interference with photons



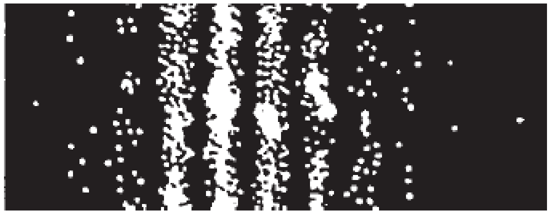
Current technology allows us to detect individual photons (photomultiplier tubes & avalanche photodiodes)

Suppose we send single photons (as particles) through a double-slit pattern. Is there still going to be a diffraction pattern?

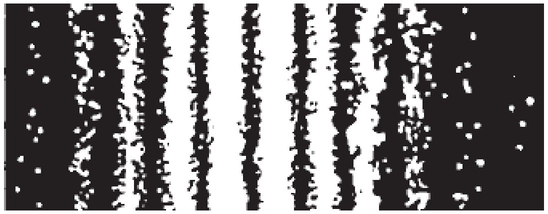
After 21 photons reach the screen



After 1000 photons reach the screen



After 10,000 photons reach the screen



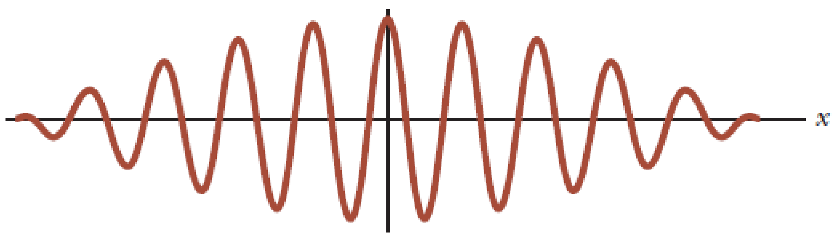
As we begin to count photons that hit the screen and mark their locations, the distribution of photons agrees with the wave predictions!

The pattern tells us the probability density of an individual photon's target location.

Position of a photon in space

An ideal wave has a single frequency and is infinitely long, and is therefore completely delocalized in space. An ideal particle has zero size, and is therefore completely localized in space.

Neither the wave model nor the particle model solely describe the photon. It is somewhat localized like a particle and somewhat delocalized like a wave. This is called the particle/wave duality.

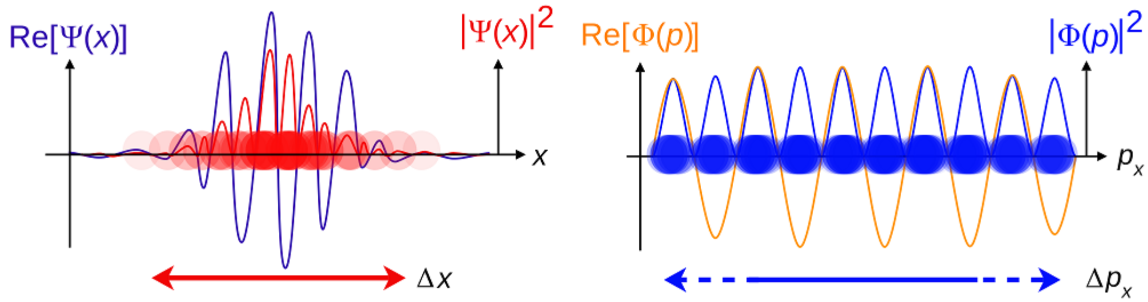


Note that the amplitude is damped on both sides. This damping causes the frequency to be slightly different along the x-direction. Because the momentum is related to the frequency of the photon, $p = hf/c$, the momentum is also different along the x-direction. Thus there is some uncertainty in both the position and momentum of the photon.

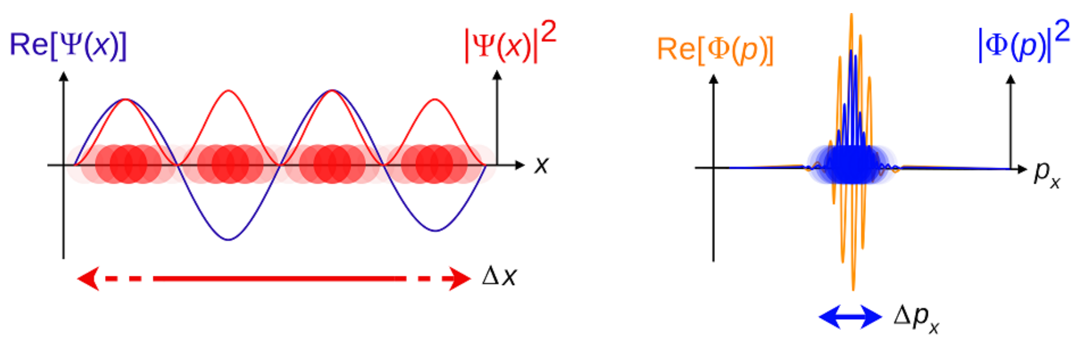
Probability and uncertainty

Let us assume that we know to a great degree of certainty that a photon traveling along the x-direction is at some location. Then, the probability density of finding that photon in space $|\psi(x)|^2$ is fairly narrow.

This narrow (small uncertainty) wave packet has steep slopes from damping, which corresponds to a broad range of possible frequencies. This broad frequency range corresponds to a probability density of momentum $|\psi(p)|^2$.



Likewise, a particle with a broad probability density of the position can have a narrow probability density of the momentum.



Heisenberg's uncertainty principle

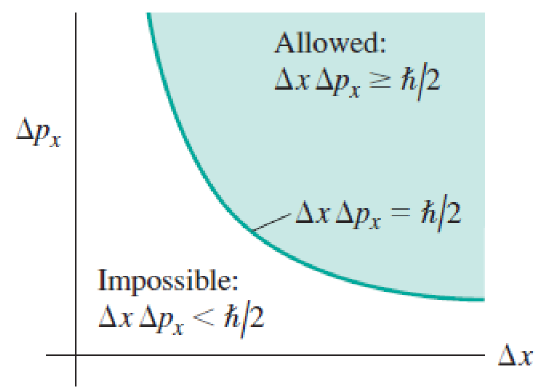
If a coordinate x has an uncertainty Δx and if the corresponding momentum component p_x has an uncertainty Δp_x then those standard-deviation uncertainties are found to be related, in general, by the inequality

$$\Delta x \, \Delta p_x \geq \frac{\hbar}{2}$$

where $\hbar$ is called the "reduced" Planck constant,

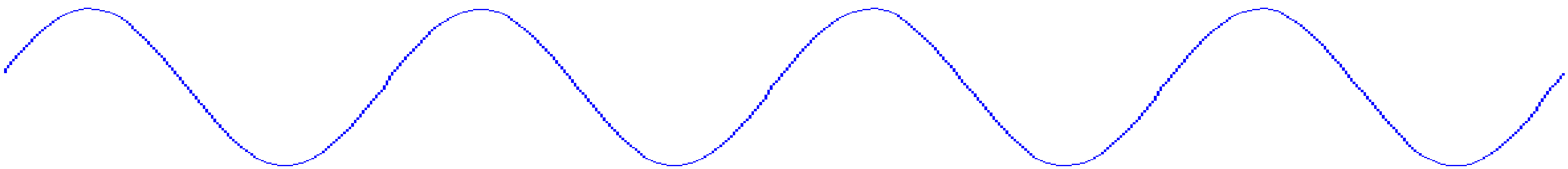
$$\hbar = \frac{h}{2\pi}$$

In other words, it is impossible for the product $\Delta x \, \Delta p_x$ to be less than $\frac{\hbar}{2} = \frac{h}{4\pi}$.



The equation can also be rewritten for energy and time, $\Delta E \, \Delta t \geq \frac{\hbar}{2}$.

Uncertainties of plane waves



If a plane wave has wavelength λ, frequency f, and amplitude A, we can write the wave function as

$$E_y(x,t) = A\sin(kx - \omega t)$$

We can express the momentum and energy of the photon as

$$p_x = \frac{h}{\lambda} = (2\pi\hbar)\left(\frac{k}{2\pi}\right) = \hbar k \quad \text{and} \quad E = hf = (2\pi\hbar)\left(\frac{\omega}{2\pi}\right) = \hbar\omega$$

Thus, we can rewrite the wave equation as

$$E_y(x,t) = A\sin\left(\frac{p_x}{\hbar}x - \frac{E}{\hbar}t\right)$$

Because this plane wave function has a definite value of momentum p_x, then there is no uncertainty in the value of this quantity, $\Delta p_x = 0$. Therefore, the uncertainty in the position Δx must be infinite, which is true because the plane wave equation never attenuates and stretches to infinity in either direction!!!

If the accuracy in measuring the position of a particle increases, the accuracy in measuring its momentum will

A. decrease.

B. remain the same.

C. increase.

D. It is impossible to say since the two measurements are independent and do not affect each other.

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A. decrease.

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The excited state of a certain atom is 3.4 eV ± 0.23 eV.
($\hbar$ = 1.055×10^{-34} Js = 6.582×10^{-16} eVs, 1 eV = 1.60×10^{-19} J)

(a) What is the average lifetime of this state? (ignore fractions and assume that $\Delta t \, \Delta E \approx \hbar$.)

(b) If the excited energy were doubled to 6.8 eV ± 0.23 eV, how would the lifetime be affected?

A. Decreased

B. Increased

C. Unchanged

A 440-nm spectral line is produced by a transition from an excited state to the ground state. Half of the natural line width, $\Delta\lambda$, of the spectral line is 0.020 pm. The average time the atom spends in the excited state is closest to which of the following? ($\hbar = 1.055\times10^{-34}$ Js $= 6.582\times10^{-16}$ eVs; ignore fractions and assume that $\Delta t\, \Delta E \approx \hbar$.) [

A. 5.14×10−11 s

B. 5.14×10−10 s

C. 5.14×10−9 s

D. 5.14×10−8 s