Problem 1

Consider a time-independent wave function given by $\psi(x) = A\sin kx$ where $k = 2\pi/\lambda$ and A is a real constant.
(a) For what values (include form for all integer values) of x is there the highest probability of finding the particle described by this wave function? Explain.
(b) For which values (include form for all integer values) of x is the probability zero? Explain.

Problem 2

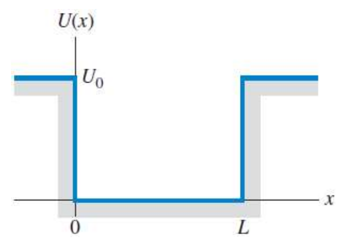
Find the width of a one-dimensional box (infinite square-well potential) for which the ground-state energy of an electron in the box equals the absolute value of the ground state of a hydrogen atom, −13.6 eV.

Problem 3

A certain atom requires 3.0 eV of energy to excite an electron from the ground level to the first excited level. Model the atom as an electron in a box and find the width of the box.

Problem 4

An electron is moving past the square well shown in the figure. The electron has energy $3U_0$. What is the ratio of the de Broglie wavelength of the electron in the region $x < 0$ to the wavelength for $0 < x < L$?



Problem 5

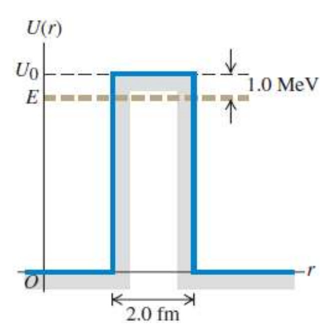
An electron is bound in a square well with a depth equal to six times the ground-level energy $E_{1\text{IDW}}$ of an infinite well of the same width. The longest-wavelength photon that is absorbed by the electron has a wavelength of 400.0 nm. Determine the width of the well.

Problem 6

Calculate $\frac{d^2\psi}{dx^2}$ for the wave function of $\psi(x) = A\sin\left(\frac{\sqrt{2mE}}{\hbar}x\right) + B\cos\left(\frac{\sqrt{2mE}}{\hbar}x\right)$, and show that the function is a solution of $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) = E\psi(x)$.

Problem 7

In a simple model for a radioactive nucleus, an α particle ($m = 6.64\times10^{-27}$ kg) is trapped by a square barrier that has width 2.0 fm and height 30.0 MeV as shown in the figure.
(a) What is the tunneling probability when the alpha particle encounters the barrier if its kinetic energy is 1.0 MeV below the top of the barrier?
(b) What is the tunneling probability if the energy of the alpha particle is 10.0 MeV below the top of the barrier?



Problem 8

Chemists use infrared absorption spectra to identify chemicals in a sample. In one sample, a chemist finds that light of wavelength 5.80 μm is absorbed when a molecule makes a transition from its ground harmonic oscillator level to its first excited level.
(a) Find the energy of this transition.
(b) If the molecule can be treated as a harmonic oscillator with mass 5.6×10^{-26} kg, find the force constant.

Problem 9

Using the integral $\psi(x) = \int_{-\infty}^{\infty} B(k)\cos(kx)\,dk$, determine the wave function $\psi(x)$ for a function $B(k)$ given by

$$B(k) = \begin{cases} 0 & \text{for } k < 0 \\ \dfrac{1}{k_0} & \text{for } 0 \le k \le k_0 \\ 0 & \text{for } k_0 \le k \end{cases}$$

Problem 10

A quantum particle is described by the wave function

$$\psi(x) = \begin{cases} A\cos\left(\dfrac{2\pi x}{L}\right) & \text{for } -\dfrac{L}{4} \le x \le \dfrac{L}{4} \\ 0 & \text{elsewhere} \end{cases}$$

(a) Determine the normalization constant A.
(b) What is the probability that the particle will be found between $x = 0$ and $x = \frac{L}{8}$ if its position is measured?

Problem 11

Numerical. (a) Run the program "Schrodinger1D" with the example potential (QHO). Are the energy states separated as you would expect?
(b) Now change the left and right boundaries "xlft" and "xrght" to be -0.2 and +0.2, respectively. Are the energy levels still representative of a QHO? Do they look more like the energies for a one-dimensional particle in a box? Explain.
(c) Change the left and right boundaries "xlft" and "xrght" back to -4 and +4. These time change the number of steps to "N = 17." How do your energies compare to part (a). Describe the appearance of the wavefunctions. Explain what happened.
(d) Change the number of steps back to "N=10001" and also change the boundaries "xlft" and "xrght" to be zero and +6, respectively. This will calculate the "clipped" quantum harmonic oscillator. Compare the energy and wavefunction of the ground state (1st state) to the 2nd state in part (a). Compare the energy and wavefunction of the 2nd state to 4th state in part (a). Describe your findings and explain.

Problem 12

Numerical. Make a potential that is quartic in the form $U = d + cx + bx^2 + ax^4$.
(a) Set the parameters to be $d = 5$, $c = 0$, $b = -3$, and $a = 0.2$. Note that you must set boundaries and step size appropriate to calculate the potential with correct results as we learned from problem 11 parts (b) and (c). You may also adjust the "Vdispamp" parameter to change how the wave function features are

presented. Find the energy levels and print the wavefunction graph. Also, state any degeneracies if present.
(b) Now set the parameters to be $d = 5$, $c = 0.05$, $b = -3$, and $a = 0.2$ while always keeping in mind the requirements on boundaries and step size for a good finite difference approximation. Find the energy levels and print the wavefunction graph.
(c) Look at the differences between wavefunctions from the potentials in parts (a) and (b). What happened to the degeneracies? Describe the differences between the two potentials on the graphs (small but powerful differences). Explain why the wavefunctions now look as they do making note of the preferred location for finding the particle in the ground state relative to the preferred location for finding the particle in the first excited state.