Exercise 1: measurement, precision, uncertainty, graphing, & linear regression

Purpose: to become familiar with common data taking and analysis techniques.

Introduction

Every measurement is associated with some degree of uncertainty. Generally, Δq is used to indicate the numerical uncertainty of a physical quantity q, which can be either a measurement or a calculated quantity. Note that the symbol we are using for the uncertainty may also be used to signify a change in a quantity, which should not be confused with it being used as uncertainty. If we measure x to be 20.6 cm with an uncertainty $\Delta x = \pm 0.2$ cm, this means that the value lies in the range of 20.4 cm to 20.8 cm.

Errors in the data-collection process can be either *random* or *systematic*. Random errors arise due to slightly different measuring conditions and to different observer interpretations. Systematic errors occur when experimental parameters that effect the measurement are not taken into consideration. Systematic errors are reproducible, and are typically compensated in the experiment or analysis. One way to find the random error in a particular measurement is to measure the same quantity many times in the same manner. The higher the number of trials, the closer the average gets to the true value (assuming no systematic errors and a normal distribution of errors).

Uncertainty in measured quantities

A. Standard deviation of multiple measurements

The corrected standard deviation σ_x of a set of values x is a measure of the average variation of individual measurements from a mean value when assuming a normal distribution. The standard deviation of a physical parameter associates a single measurement's variation from the true value for a specific set of experimental conditions. The corrected standard deviation of a measured value x for a discrete set of N measurements is given by

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{N} (x_i - x_{\mathrm{avg}})^2}{N-1}}$$

where

$$x_{\mathrm{avg}} = \frac{1}{N} \sum_{i}^{N} x_i$$

As an activity, take a ball and raise it to eye level. Use the stop watch to measure the time it takes the ball to drop and ball. Do this measurement five times dropping from the same height for each drop. Measure the height at which you are consistently dropping the ball. Make a table similar to Table 1 to record the data. Also calculate the 1) average value and 2) corrected standard deviation and place them in the table.

Table 1: Measurements of the time it takes for the ball to hit the floor

Measurement	1	2	3	4	5	Average	Std. dev.
Time (s)							

$h = $ _____ m $\Delta h = $ _____ m

(height)   (uncertainty in height)

B. Estimated uncertainty of a measured value

In some instances, multiple measurements give the same value (or nearly the same value) for all measurements. This does not mean that the value is exact. Instead, the precision of the measuring apparatus is less than the accuracy of the values measured. Thus, the accuracy is greater than the precision, which means that the primary source of random error is due to the precision of the measuring apparatus. Note that parallax, placement of the instrument, and irregularities commonly add up to the estimated uncertainty. Note that systematic errors can occur for biased measurements. 1) Use the caliper to measure the diameter of the ball, and 2) estimate the uncertainty in the measurement. 3) Also, estimate the uncertainty of the height from when you dropped the ball.

Graphing

Two-dimensional graphs display information of a dependent variable based on the value of an independent variable. For a continuous function $y(x)$, y is called the dependent variable and x is called the independent variable. Thus, the value of y depends on the value of x. The dependent variable's axis is oriented in the vertical direction, and the independent variable's axis is oriented in the horizontal direction. If these variables have units, then they must also be clearly displayed after the axes labels.

When plotting data, the mean values should first be placed in columns with every dependent variable value corresponding to the adjacent independent variable value. For example, Microsoft Excel plots two column data by default when the x-data is in the first column and the y-data is in the second column as shown in Figure 1 using the data from excel table to the left of the graph.

The uncertainties of each variable can be inserted as "error bars" in the graph. First, we make a column of x uncertainties corresponding to the average x-data points, and a column of y uncertainties corresponding to the average y-data points as shown in Figure 1. To make the graph in excel, we select the x and y columns presented in the order shown in Figure 1. Then in the insert tab, select the "scatter plot" option. You can use the "+" symbol at the top right of the graph when the graph is highlighted. You should select "axis titles" to label your plot's axes.

For the uncertainties, click on the "+" symbol and select error bars. The error bars that show up are NOT the error that you have in you two uncertainty columns. Thus, you must also click the small arrow to the right of the "error bars" selection and click "more options." To the right of your screen there is a "format error bars" window that pops up. At the bottom, select the "custom" radio button. Then click on the specify value bottom and a small window pops up. Use this window to insert your custom error bars into the graph (They will be the same values for the positive and negative error bars). Do custom error bars for both the x-error and y-error. The end result will be the graph in Figure 1.

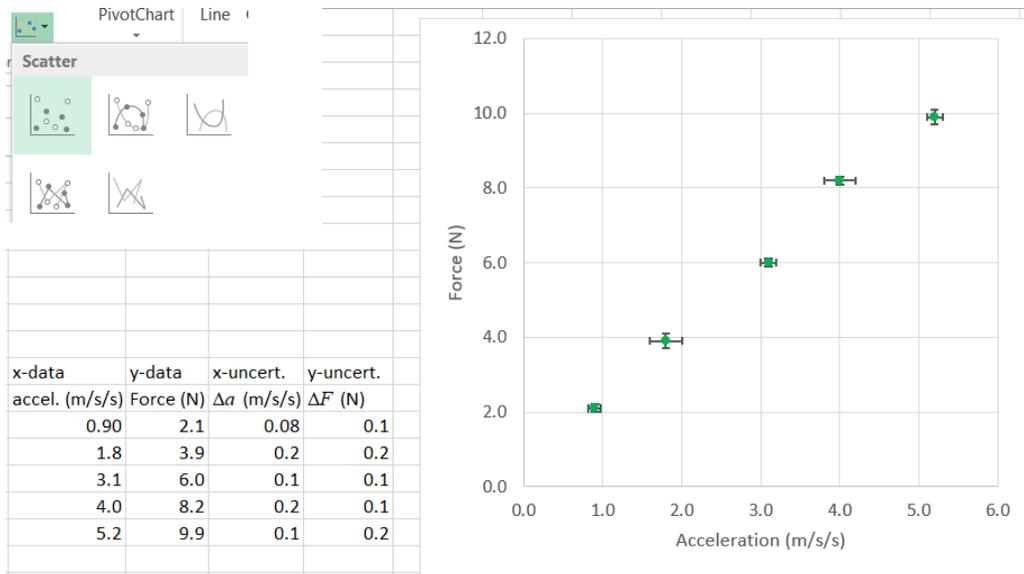| x-data	y-data	x-uncert.	y-uncert.
accel. (m/s/s)	Force (N)	Δa (m/s/s)	ΔF (N)
0.90	2.1	0.08	0.1
1.8	3.9	0.2	0.2
3.1	6.0	0.1	0.1
4.0	8.2	0.2	0.1
5.2	9.9	0.1	0.2

Figure 1. Excel graph from the data given in Table 2 which includes axes labels and error bar representations of the data point uncertainties.

Take the ball and drop it from chest level 5 times recording the times and the height and fill in another table similar to Table 1 as you did from eye level. Drop the ball again from waist level 5 times and fill in another table. Do this same procedure once again for knee level. You should now have four data points for your height and four data points for the time, each with uncertainty estimates.

Table 2: Drop data

	$\frac{1}{2}t^2$	h
Knee-level		
Waist-level		
Chest-level		
Eye-level		

We know that the time it takes the ball to drop from rest is given by $x = (g/2)t^2$. Thus, to get a linear curve we should take the height as our dependent variable, and one-half the time squared as our independent variable. You should fill in a table similar to that shown in Table 2. Make sure that you propagated the error in time squared by using the formula specific to the error of the squared value

$$\Delta\left(\frac{t^2}{2}\right) = t\,\Delta t\ .$$

Graph h vs. $t^2/2$ making sure to add the error bars along both axes. Then, find the slope and intercept by adding a trend line in Excel and check the "Display Equation on chart" box in the "More options" menu.

Finding the uncertainties of the slope and intercept using Excel

Usually a fitted line isn't perfect. Thus, the slope and intercept calculated from the trend line is the mean value. We must perform further analysis to find the range of possibilities for the slope and intercept values. The LINEST function in Excel performs a linear regression to find these values.

a) Start by entering your x and y values into columns in Excel similar to the example data shown in Figure 1.
b) In a box below, Type in "=LINEST" with y-data in the first entry and x-data in the second comma-delimited entry. Type "TRUE" for the second two comma-delimited entries. An example is shown in Figure 2.

	A	B	C	D
1	x-data	y-data	x-uncert.	y-uncert.
2	accel. (m/s/s)	Force (N)	Δa (m/s/s)	ΔF (N)
3	0.90	2.1	0.08	0.1
4	1.8	3.9	0.2	0.2
5	3.1	6	0.1	0.1
6	4.0	8.2	0.2	0.1
7	5.2	9.9	0.1	0.2
8				
9				
10	LINEST function example			
11	=LINEST(B3:B7,A3:A7,TRUE,TRUE)			
12				

Figure 2. Example of using the LINEST function in Excel.

c) You should see the slope in the box after entering the LINEST function into the box. Next, highlight 2 columns across and 5 rows down as shown in Figure 3.

11	1.83675214	
12		
13		
14		
15		

Figure 3. Highlight boxes in excel after using the LINEST function

d) After the boxes have been highlighted, press f2 on the keyboard (The text showing the LINEST function should reappear similar to the diagram shown for step b). Then press ctrl+shft+enter. You should see values filled into all locations of the highlighted boxes. The two uncertainties are labeled Figure 4.

11	1.83675214	0.50974
12	0.07476394	0.25177
13	0.99505403	0.25573
14	603.554912	3
15	39.4718034	0.1962

— Intercept uncertainty

Slope uncertainty

Figure 4. The data given by the LINEST function after following steps a) through d).

Use the steps above to have Excel's LINEST function find the 1) slope, 2) uncertainty in the slope, 3) intercept, and 4) uncertainty in the intercept. The slope and intercept should match the trend line from earlier.