Exercise 10: Torque and moment of inertia

Purpose: to the torque on a wheel from a hanging mass and to determine the rotational inertia of a rotating wheel

Introduction

A force applied to an object that is allowed to rotate around a fixed axis causes a torque,

$$\vec{\tau} = \vec{r} \times \vec{F} .$$

The magnitude of the torque is related to the angular acceleration and the object's moment of inertia from the following relationship,

$$\tau = \alpha I .$$

In this experiment, we will place a mass on a hanger which causes a tension,

$$T = m(g - a) = m(g - \alpha r),$$

in a string wrapped around a grooved cylinder on the object. The wheel will begin to rotate as the hanging mass falls. If we start the wheel and hanger from rest and record the time taken for two complete rotations, $\theta - \theta_0 = 4\pi$, then we may use our rotational kinematics equation,

$$\theta - \theta_0 = \frac{1}{2}\alpha t^2 ,$$

To solve for the angular acceleration. We may then determine the moment of inertia of the disk from our knowledge of the force and torque.

Laboratory assignment

1. Measure and record the radius of the wheel R and the radius of the largest of the grooves on top of the wheel r that the string will wrap around. Estimate the uncertainty in your measurements of the radii.
2. Measure and record the mass of the wheel M and place it on the rotating stage.
3. Set up the photogate and attach the beam block to the edge of the rotating wheel so that it can be detected as it passes through the photogate.
4. Place the string over the pulley so that the mass hanger is over the edge of the table. Set the total mass of the hanger, m, to 50 g.
5. Wind the wheel so that the string wraps around the groove with the largest radius. Stop when the mass hanger is near the pulley, and when the beam block moves just behind the photogate beam path. (Make sure that the pulley's height is close to the same height as the groove.)
6. Turn on the photogate by setting it to the triple pulse "pendulum" position.
7. While in the position where the beam block is in the photogate beam path, release the hanger and let the wheel rotate. The beam block should enter the photogate immediately after you release the hanger.
8. After the beam block has passed the photogate three times (two complete rotations), stop the rotation and record the time, t.
9. Repeat steps 5-8 for a total hanging mass of 100 g, 150 g, and 200 g.
10. Make a table similar to Table I and fill in the data.

Table I: data from torque experiment

m (kg)	t (s)	grt^2 (m^2)	$\frac{8\pi}{m}$ (kg^{-1})
0.050			
0.100			
0.150			
0.200			

11. Draw two free body diagrams (one of the disk and one of the hanging mass) and derive the relationship, $m(g - \alpha r)r = \alpha I$.
12. Solve the kinematic equation for the angular acceleration after two full rotations and substitute the expression into your derived equation in step 11 to get the equation, $grt^2 = \frac{8\pi}{m} I + 8\pi r^2$.
13. Make a graph of grt^2 vs $\frac{8\pi}{m}$ and include error bars along the vertical axis based on your source of uncertainty in your measurement of r.
14. Use your program to find the slope and the uncertainty in the slope, $I_{\text{measured}} \pm \Delta I_{\text{measured}}$.
15. Assume that the wheel is a rotating disk and calculate the moment of inertia $I_{\text{disk}} \pm \Delta I_{\text{disk}}$ using your value of R and the mass of the disk M. $\left(I_{\text{disk}} = \frac{1}{2} M R^2\right)$
16. Compare the two values for the moment of inertia. Is a "disk" given by the equation in step 15 a good approximation for the rotating wheel?

Equipment list: rotation stage, photogate, string, scissors, flash card, hanging masses.