

## Exercise 4: Projectile motion

Purpose: discover the relationship between the parameters in two-dimensional projectile motion.

### Introduction

When objects undergo motion in two (or even three) dimensions rather than in just one, the overall motion can be analyzed by looking at the motion in any two (or three) mutually perpendicular directions and then putting the motions “back together.” In the case of projectiles, the obvious horizontal direction and vertical direction are usually chosen. Ignoring the effects of air resistance, an object moving vertically near the surface of Earth experiences a constant acceleration. We know this by experiment. Likewise an object moving horizontally experiences zero acceleration. Any other choice of perpendicular directions would have nonzero, constant values of acceleration in both directions. When we write the descriptions of the motion in mathematical terms, the horizontal/vertical choice of directions results in the simplest description.

Under what conditions can the effects of air resistance be ignored? One condition is that the object’s speed is not too high, since the effect of the air resistance increases with speed. If two objects are the same size and shape, the lighter one of the two will experience the larger effect on its motion due to the air. Care has been taken to ensure that air resistance is negligible for this laboratory. When conditions are such that air resistance cannot be ignored, the motion is more complicated.

Neglecting air resistance, and assuming that up is the positive  $y$  direction, we have the following equations,

#### Horizontal direction

$$a_x = 0$$

$$v_x = v_{0x}$$

$$x = x_0 + v_{0x}t$$

#### Vertical direction

$$a_y = -g$$

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

where  $v_{0x} = v_0 \cos\theta$  and  $v_{0y} = v_0 \sin\theta$ . These equations are the same as the ones you learned in the classroom, and we will use these to analyze our projectile motion laboratory results.

### Laboratory assignment

A few notes before we begin.

- Make sure the launcher is at the proper angle by using the built-in plumb bob on the side of the launcher. Note that the angle measured by this plumb bob is the angle between the end of the launcher's “barrel” and the horizontal.
- Since the projectiles will be hitting the floor, use a second plumb bob to locate and mark the position on the floor with some tape directly below the launch point of the projectile. This indicates the initial horizontal position of the ball at floor level so the range (horizontal distance traveled by the ball) can be measured. You will have to measure the height to get the vertical distance.
- To launch the projectile, load the ball into the projectile launcher. Use the rod when pushing the ball into the launch tube to the first of the three preset launch positions. We will only use the first setting due to space requirements, where you should hear a click and see the launcher's trigger rise and fall once you reach the first position.

- Do a test launch before taking data every time you change angles so that your group members recording the hit position can tape a piece of paper to the floor in that general region to mark each impact position.

### Determine the initial launch speed

1. Derive an expression for the range  $R = x - x_0$  for the case when  $\theta = 0$  (horizontal launch). Then solve for  $v_0$  in terms of  $R$ ,  $h$ , and  $g$ . Note that the height above the floor is the initial vertical position  $h = y_0$  when we take the floor as  $y = 0$ .
2. Set the launcher to fire horizontally, that is, to launch at an angle of zero degrees. Care with this angle setting can significantly improve your results later in the lab.
3. Measure the initial height (from bottom of the ball diagram on the launcher, NOT from the center of the diagram) relative to the floor and estimate the uncertainty from sources of error that you identify.
4. Fire the projectile on the shortest launcher setting for a test run noting where the projectile lands. Center the paper target as best you can where the ball will land and tape the paper to the floor. Launch the projectile five times using the same initial conditions marking the landing spot after each launch. Record this data in a table similar to Table I.
5. From your data determine the average range of the ball and estimate the uncertainty by calculating the standard deviation, i.e., find  $R \pm \Delta R$ .
6. Use your tabulated data to find the initial velocity and be sure to estimate the uncertainty by propagating the error from your uncertainty estimates of  $R$  and  $h$ , i.e., find  $v_0 \pm \Delta v_0$ .

Table I: Range data horizontal launch.

| $h$ (m) | $\Delta h$ (m) | $R_1$ (m) | $R_2$ (m) | $R_3$ (m) | $R_4$ (m) | $R_5$ (m) | $R_{\text{avg}}$ (m) | $\sigma_R$ (m) |
|---------|----------------|-----------|-----------|-----------|-----------|-----------|----------------------|----------------|
|         |                |           |           |           |           |           |                      |                |

### The range at a nonzero launch angle

7. Derive a general expression for the range when the launch angle is zero. The expression should be in terms of  $v_0$ ,  $R$ ,  $h$ ,  $\theta$ , and  $g$ .
8. Set the launch angle to an angle between  $30^\circ$  and  $40^\circ$ , and record this angle in your notes.
9. Perform a test launch. Tape the sheet of blank paper where the ball impacted the ground.
10. Then launch the ball five times marking the impact location after each launch. Record this data in a table similar to table II.

Table II: Range data for angled launch.

| $h$ (m) | $\Delta h$ (m) | $\theta$ ( $^\circ$ ) | $R_1$ (m) | $R_2$ (m) | $R_3$ (m) | $R_4$ (m) | $R_5$ (m) | $R_{\text{avg}}$ (m) | $\sigma_R$ (m) |
|---------|----------------|-----------------------|-----------|-----------|-----------|-----------|-----------|----------------------|----------------|
|         |                |                       |           |           |           |           |           |                      |                |

11. From your data determine the average range of the ball and use the standard deviation as the uncertainty, i.e., find  $R_{\text{exp}} \pm \Delta R_{\text{exp}}$ .
12. Using your equation from step 7, calculate the theoretical range  $R_{\text{calc}}$  you would get from your known or measured values of  $v_0$ ,  $R$ ,  $h$ ,  $\theta$ , and  $g$ .

13. Calculate the uncertainty in the theoretical range  $\Delta R_{\text{calc}}$  by propagating the error from  $v_0$  and  $h$ . Although the uncertainty in the angle does contribute to the uncertainty in the theoretical range, you may ignore the angle error contributions for this exercise.
14. Compare the two “fuzzy” values for the range,  $R_{\text{exp}} \pm \Delta R_{\text{exp}}$  and  $R_{\text{calc}} \pm \Delta R_{\text{calc}}$ . Do they overlap?

*Equipment list: launcher, plastic ball, tape, blank printer paper, plumb bob, ramrod, meter stick, two meter stick.*