Online laboratory assignment 7 – Inductor-resistor-capacitor (LRC) circuits

Purpose: to study the relationship of the phase and resonance on capacitor and inductor reactance in a circuit driven by an AC signal.

Introduction

By definition, alternating current in a circuit involves periodic reversal of current. Most commonly, the current reversals in ac circuits are sinusoidal with angular frequency ω (given in rad/s), and can be described by

$$i(t) = I_{peak} \sin \omega t$$

where I_{peak} is the maximum value of the current and t is the time.

Linear resistors in AC circuits behave the same as they do in DC circuits, where they follow Ohm's law

$$v_R = iR$$

$$v_R = I_{peak} R \sin \omega t$$

Capacitors in DC circuits allow a transient current as they charge or discharge, but they do not allow a steady current to flow, since they function as an open switch when fully charged. Because AC current described by Eq. 1 is alternating sinusoidally, a capacitor in an AC circuit is continuously charging and discharging, and thereby allowing a current to oscillate in the circuit. Because voltage across a capacitor depends on the charge on the capacitor, and charge represents the time integration of the current, the voltage across a capacitor is not in phase with the current. The voltage across the capacitor lags the current by 90°, where

$$v_C = \frac{1}{C} \int I_{peak} \sin \omega t \; dt$$

$$\Rightarrow \; v_C = -\frac{I_{peak}}{\omega C} \cos \omega t$$

The ideal capacitive reactance X_C is given by

$$X_C = \frac{1}{\omega C}$$

which is a proportionality constant between voltage amplitude and current amplitude for a capacitor in an AC circuit, much like resistance is a proportionality constant between voltage amplitude and current amplitude for a resistor.

Inductors in DC circuits create a resisting emf and impact a circuit only when the current is changing. With steady current, a perfect inductor (i.e., no resistance) functions as a connecting wire. Because current is continuously changing, an inductor in an AC circuit is continuously creating a resisting emf. Because the voltage across an inductor V depends on the rate of change of current di/dt, it leads the current by a 90° phase shift,

$$v_L = L \frac{d}{dt} I_{peak} \sin \omega t$$

$$\Rightarrow \; v_L = L I_{peak} \cos \omega t$$

The inductive reactance X_L is given by

$$X_L = \omega L$$

Inductive reactance is a proportionality constant between voltage amplitude and current amplitude for an inductor in an AC circuit, much like capacitive reactance is a proportionality constant between voltage amplitude and current amplitude for a capacitor.

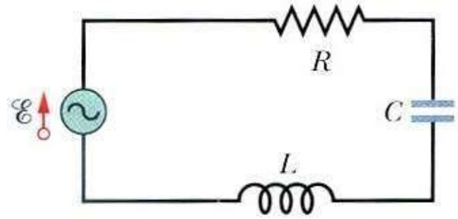


Fig. 1: a series inductor-resistor-capacitor circuit.

A series LRC circuit is shown in Fig. 1. The angular frequency of the current and the total voltage across all three components $\mathcal{E}$ is driven by the AC power source. According to Kirchhoff's loop rule, the sum of the instantaneous voltages across all circuit elements in the loop must be zero,

$$\mathcal{E} = v_R + v_L + v_C$$

$$\Longrightarrow\ \mathcal{E} = I_{peak} R \sin \omega t + I_{peak}\left(\omega L - \frac{1}{\omega C}\right)\cos \omega t$$

Because of the phase differences among the three voltages, the relationships can best be viewed using a phasor diagram like the one shown in Fig. 2.

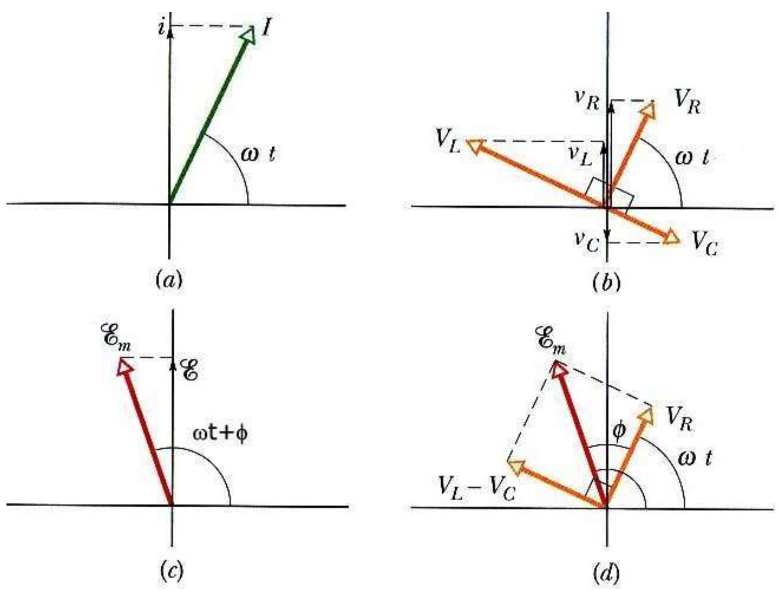


Fig. 2: Phasor diagrams of (a) current, (b) voltages across circuit elements, (c) circuit voltage, and (d) voltage components from the circuit elements.

Because the phase relationships among current and voltage amplitudes are always the same, we can generalize their relationship for an ideal circuit,

$$\mathcal{E}^2 = V_R^2 + (V_L - V_C)^2$$

The above <u>ideal</u> equation can be written in terms of current amplitude, resistance, and reactance,

$$\mathcal{E}^2 = I_{peak}^2[R^2 + (X_L - X_C)^2]$$

$$\mathcal{E} = I_{peak}\sqrt{R^2 + (X_L - X_C)^2}$$

The total impedance of an <u>ideal</u> circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The current will have a maximum amplitude when the impedance has a minimum value, which occurs when $X = X$, which happens at the resonant frequency. From the phasor diagram, the phase angle ϕ between emf across the power supply and current depends on the voltage amplitudes,

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

The term $\cos\phi$ is called the power factor because it determines the average power dissipated in the circuit by the resistance,

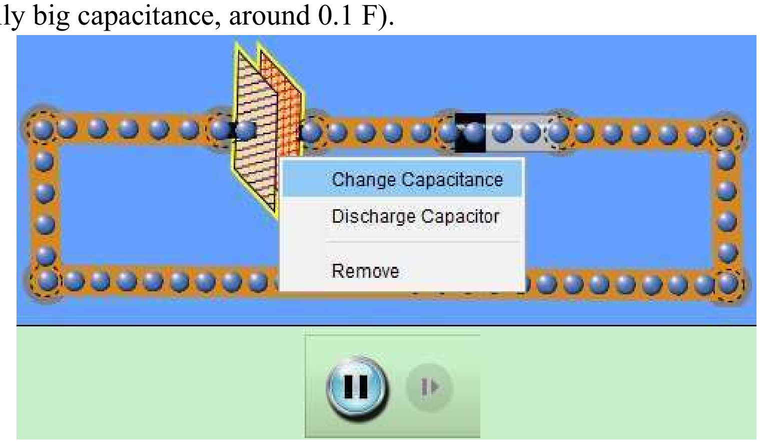
$$\langle P \rangle = \frac{1}{2} I_{peak}\mathcal{E}\cos\phi$$

For a real circuit, the inductor and capacitor carry a real resistance along with their ideal reactance. Therefore, the phase shifts of both <u>real</u> reactances are not ±90°.

Laboratory assignment

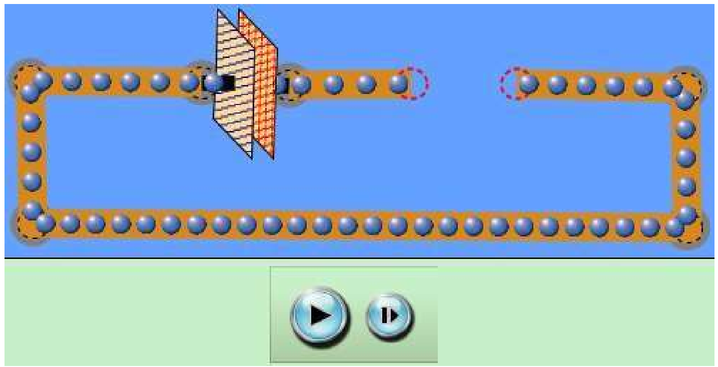
Part I: LC circuit

1. Run the "Circuit construction kit AC virtual lab" PhET simulation.
2. Create a circuit with a battery and a capacitor in series to charge the capacitor similar to that shown below. The capacitor plates will turn red and blue when the circuit is connected.
3. Right click the capacitor, click "Change Capacitance" and note down the capacitance (it will likely be a really big capacitance, around 0.1 F).

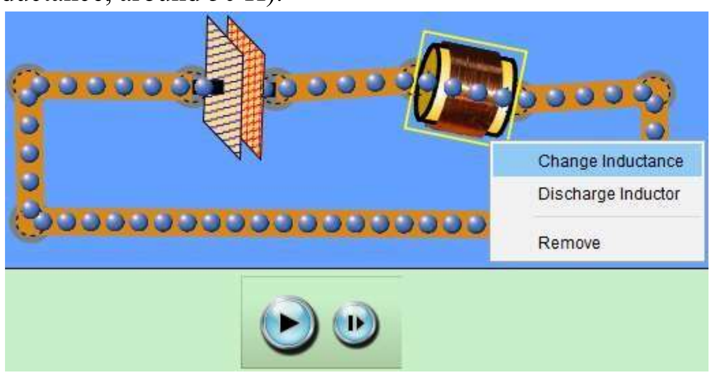


4. Press pause at the bottom of the screen. Make sure to do this step before disconnecting the circuit!

5. Disconnect the battery and remove it from the simulation by right-clicking the battery and selecting “Remove.” The circuit diagram should look similar to the one shown below.



6. Connect an inductor in series with the capacitor where the battery used to be and make sure that the circuit is closed.
7. Right click the inductor, click “Change Inductance” and write down the inductance (it will likely be a really big inductance, around 50 H).



8. Press play arrow.
9. What do you observe while the simulation is playing?

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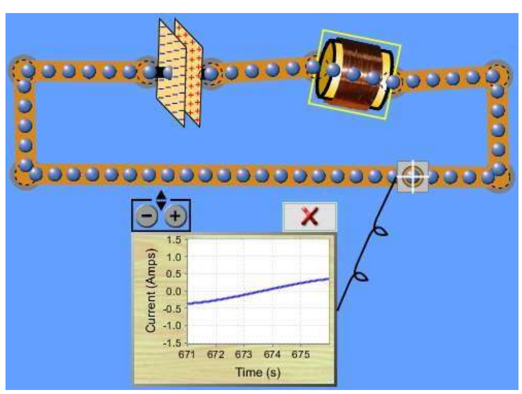
10. Click on the “Current Chart” and place the probe over the wire as shown below. You can adjust the scale as needed.



11. Record a time and a subsequent time when the current and slope of the current are the exact same, e.g. recorded times at two adjacent maxima. Use these values to determine the period. The period T was found to be

$$T = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

12. Using the inductance and capacitance values you wrote down when adding the elements, calculate the resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

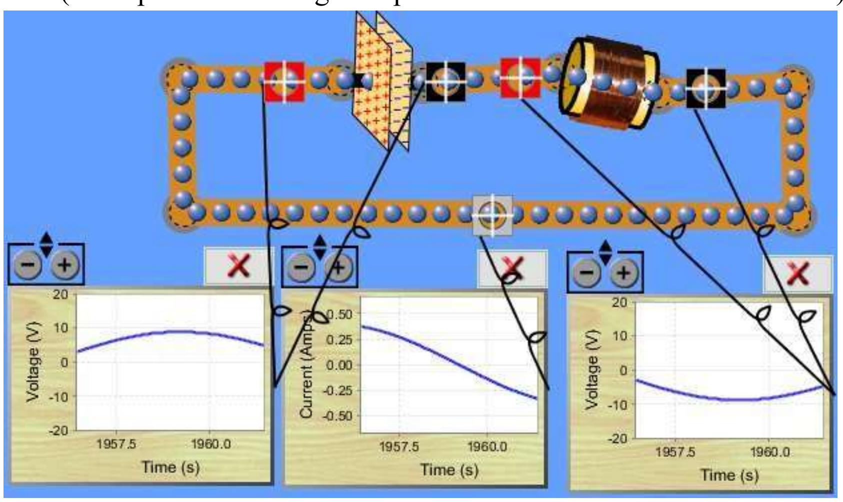
13. Covert the period you measured in step 11 to an angular frequency.

$$\omega_{meas} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

14. Calculate the % difference between the measured and calculated angular frequencies.

$$\% \text{ difference} = (100\%) \times \left|\frac{\omega_{meas} - \omega_0}{\omega_0}\right| = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

15. Click Voltage Chart and place the detectors on either side of the capacitor. Add a second chart on both sides of the inductor as shown below. Note the polarity of the voltage measurements needs to be the same (same positive and negative probe orientation across each element).



16. Adjust the vertical axis using the +/- arrows so that you can see the full sinusoidal curve and maximum voltage on each graph.
17. Qualitatively note the similarities and differences between the curves, and not just the differences in absolute voltages between the two voltage graphs.

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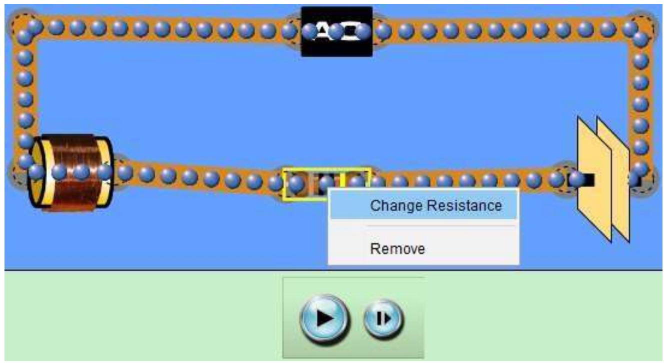
Part II: RLC circuit

18. Exit the application and restart it.
19. Create a circuit with an AC Voltage in series with a resistor, inductor, and capacitor as shown below. Note also the capacitance, inductance, and resistance.



20. Write down the values for each of the circuit elements in SI units.

$R =$__________________. $L =$___________________. $C =$___________________.

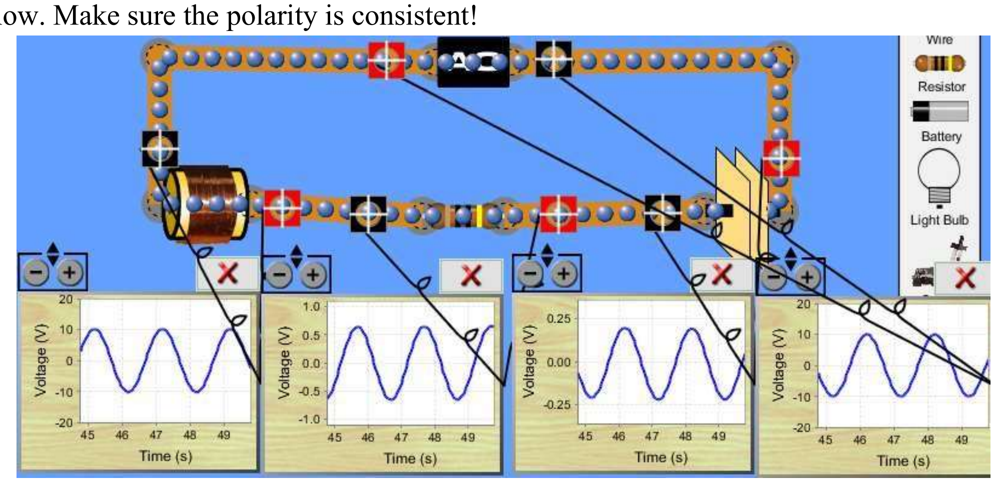
21. Right click the AC Voltage and click Change Voltage and note the current value of the voltage. Then calculate the rms voltage of the source.

$$V_{rms}^{source} = \frac{V_{peak}^{source}}{\sqrt{2}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

22. Right click the AC voltage and click Change Frequency and note the value of frequency. Then calculate the angular frequency.

$$\omega = 2\pi f = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

23. Place Voltage Chart around each element in the circuit including the AC Voltage as shown below. Make sure the polarity is consistent!



24. Which two graphs are perfectly 180° out of phase with one another?

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25. Measure the peak voltages across each circuit element and calculate their rms values.

$V_{R,rms} =$_________________. $V_{L,rms} =$_________________. $V_{C,rms} =$_________________.

26. Calculate the rms voltage of the source using the rms voltage across each element.

$$V_{rms}^{calculated} = \sqrt{V_{R,rms}^2 + \left(V_{L,rms} - V_{C,rms}\right)^2} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

27. Calculate the percent difference between the values of step 26 and step 21.

$$\%\ \text{difference} = (100\%) \times \left|\frac{V_{rms}^{calculated} - V_{rms}^{source}}{V_{rms}^{source}}\right| = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

28. Using the resistance, capacitance, inductance, and the frequency of the AC Voltage calculate the impedance.

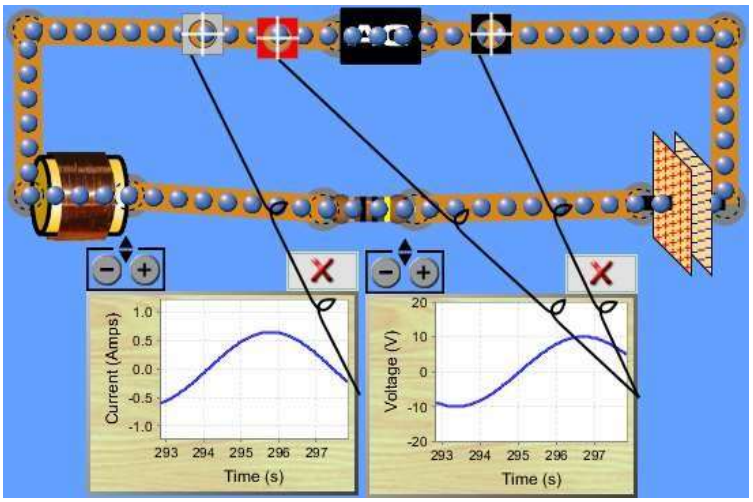
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

29. Calculate the rms current, $I_{rms} = I_{peak}/\sqrt{2}$, in the RLC circuit.

$$I_{rms} = \frac{V_{rms}^{source}}{Z} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

Part III: Resonance of an RLC circuit

30. Remove the voltage measurements from the resistor, capacitor, inductor, and the AC source.
31. Add a current measurement and place the sensor over the wire so that your circuit looks similar to the one shown below.

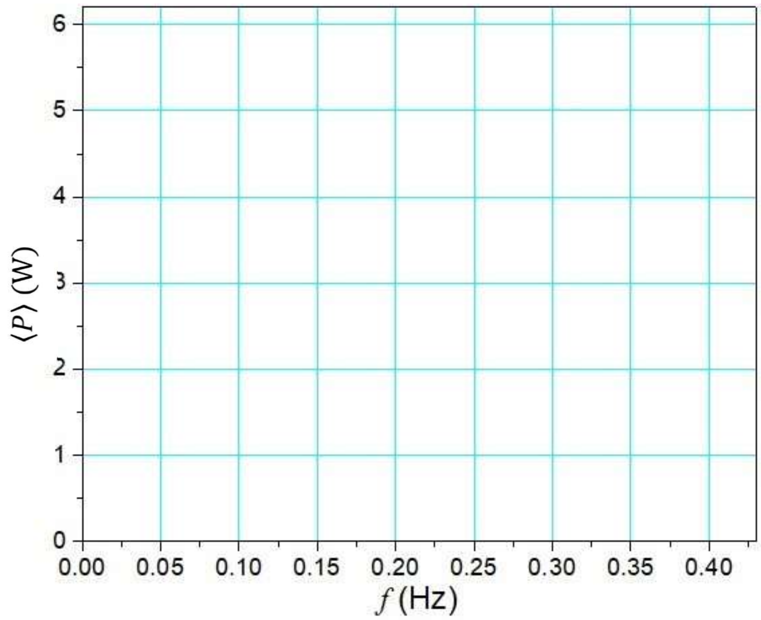


32. Change the inductor value to L = 10.0 H, the capacitor value to C = 0.05 F, and the resistor value to R = 10.0 Ω. The AC source peak voltage should be set to 10.0 V.
33. Start with an AC source frequency of f = 0.05 Hz. Then increase the frequency in increments of 0.05 Hz until you have reached 0.40 Hz. You must wait longer than 10 seconds after changing the frequency for the circuit to come into equilibrium before recording the new maximum current. The peak voltage across the source should remain constant.
34. Fill in the values from Table I after your measurement of the peak current after each change in the frequency. For the calculated power remember that $\langle P \rangle = I_{rms} V_{rms}^{source} \cos\phi$ where $\cos\phi = \frac{R}{Z}$.

Table I: Resonance in an RLC circuit (R = 10.0 Ω and V_{rms}^{source} = 7.071 V)

f (Hz)	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.4
ω (rad/s)								
I_{rms} (A)								
Z (Ω)								
$\langle P \rangle$ (W)								

35. Plot your values of the power on the below graph and draw a smooth curve connecting the points.



36. Write a short conclusion to the laboratory assignment.