Exercise 10: Imaging with thin lenses

Purpose: to understand the projection and magnification using converging lenses.

Introduction

Projector

The design of high quality optical instruments can he quite complex, involving compound lenses and intricate lens coatings. But the complexity arises primarily from the need to reduce the effects of spherical and chromatic aberration. Understanding the basic principles of standard optical instruments is not complex. It requires only an understanding of the fundamental lens equation,

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

The magnification of the image is given by

$$m = -\frac{d_i}{d_o}$$

When an object is located between the focal point and twice the focal point, $2f$, of a converging lens, a real, inverted, magnified image is formed as shown in the diagram of Fig. 1. If a viewing screen is placed at the location of the image, the image will be focused onto the screen. In this case the lens functions as a projector.

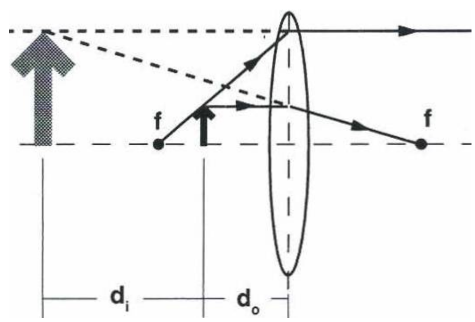

Fig. 1: Ray diagram of lens projection.

Magnifier

When an object is located between a converging lens and its focal point, a virtual, magnified, uninverted image is formed. Since the image is not real, it cannot be focused onto a screen; however, it can be viewed directly by an observer. When a converging lens is used as a magnifying glass, the magnification provided by a converging lens is not unlimited. This does not mean that the magnification equation $m = -d/d_o$ is in error.

This equation does give the correct ratio between the image size and the object size. However, image size is not the only important variable in determining the magnification of an optical system, such as a magnifier. Equally important is the distance between the observer and the image he is looking at. Just as a distant object appears smaller than the same object up close, an image viewed through an optical system appears larger if the image is close than if it is farther away.

Figure 2 shows an object of height h_o, a distance d_o from the observer. The size of the image on the retina of the observer is proportional to the angle θ_{eye}. For small angles,

$$\theta_{\text{eye}} \approx h_o/d_o$$

There is an important litation to the magnitude of θ_{eye}. There is a distance (called the near point) at which the image begins to blur, because the rays entering your eye from the object are too divergent for your eye to focus. The near point differs for different people, but the average is approximately 25 cm. Therefore $\theta_{\text{eye-max}} \approx h_o/(25\text{ cm})$, where $\theta_{\text{eye-max}}$ is the maximum value of θ_{eye} for which the eye can focus an image.

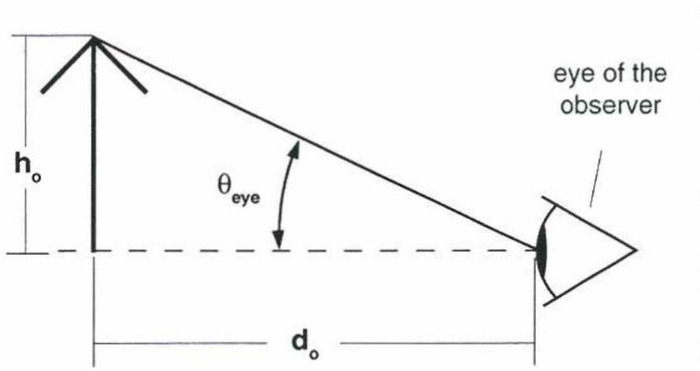

Fig. 2: Diagram for angular magnification.

When using a magnifier, or any optical system for that matter, the apparent size of the image depends on the size and location of the image rather than on the size and location of the object, so that θ_{mag}, the angular magnification for the magnifier, is equal to h_i/d_i. From the fundamental lens equation, $h_i = mh_o = (-d_i/d_o)h_o$. Therefore, ignoring the minus sign, $\theta_{\text{mag}} = h_o/d_o$, the same as without the magnifier.

This result seems to imply that a magnifier doesn't produce any magnification. Using a magnifier, the object can be brought closer to the eye than the near point, and yet still be focused by the eye. If the object is placed at the focal point of the magnifier for example, the equation $\theta_{\text{mag}} = h_o/d_o$ becomes $\theta_{\text{mag}} = h_o/f$. Therefore, the magnifying power of a magnifier is a function of how much closer it allows the observer to be to the object. This, in turn, is a function of the focal length of the magnifying lens.

The magnifying power of a lens (angular magnification) is calculated as

$$\frac{\theta_{\text{mag}}}{\theta_{\text{eye-max}}} = (25\text{ cm})/f$$

Laboratory assignment

I. Projector

1. Set up a projection experiment as shown in Fig. 3 using the 75 mm lens.

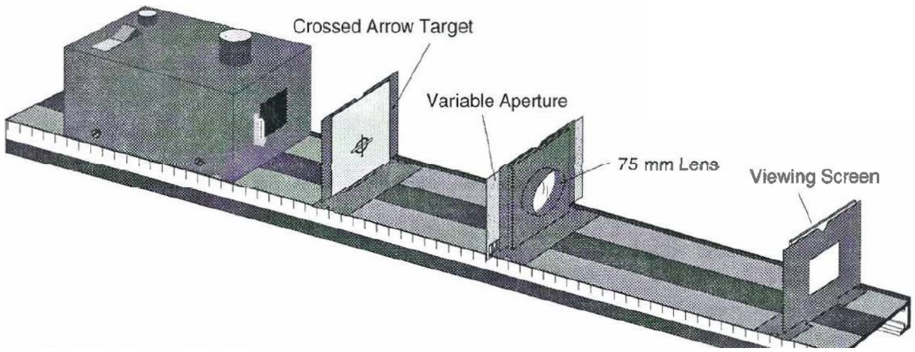

Fig. 3: Diagram for projector experiment.

2. What happens to the image if d_o is less than f. Can the image still be focused onto the viewing screen? Explain.
3. What happens to the image if do is greater than $2f$? Can the image now be focused onto the viewing screen? Explain.
4. Are there practical limits to the degree of magnification of the image? If so, what are they?
5. Pick a location d_o between f and $2f$. Adjust the viewing screen to form an image. Calculate the magnification of the image.
6. Repeat for the 150 mm lens.
7. Is it possible, using a single lens, to project an image that is non-inverted? Explain.

II. Magnifier

1. Set up a magnifier as shown in Fig. 4 using the 75 mm lens.

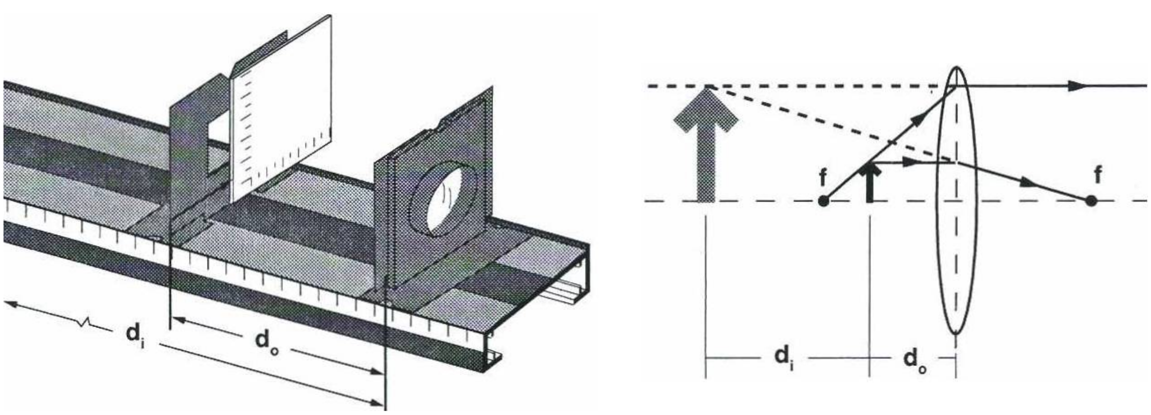

Fig. 4: Diagram for angular magnification.

2. Adjust the distance between the object (the viewing screen) and the lens so the magnification is a maximum and the image is clearly focused. Record this distance.
3. Repeat for the 150 mm lens.
4. Does the fundamental lens equation place any limit on the magnification m that a lens can produce?
5. Calculate the angular magnification for the 75 mm and 150 mm focal length lenses.

6. Which lens seems to provide the greater magnification?
7. Would a converging lens with a 50 cm focal length be useful as a magnifier? Explain.

Equipment list: Pasco optics kit.