Exercise 3: Kirschoff's rules

Purpose: to investigate the current and voltage characteristics in linear circuits

Introduction

In a simple DC circuit containing a power supply (source of emf with known voltage (V)) and one resistor with known resistance (R), it is possible to calculate the expected current (i.e., analyze the circuit) from the applied voltage of the power supply and the known resistance using Ohm's Law.

$$V = IR$$

For circuits with multiple resistors all in series or all in parallel, we can analyze the circuit by first determining an equivalent resistance for the series combination

$$R_{\mathrm{eq}} = \sum_{i=1}^{N} R_i$$

or the parallel combination

$$\frac{1}{R_{\mathrm{eq}}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

More complicated circuits often cannot be reduced to simple combinations of series and parallel resistors, and we must use other techniques to analyze the circuits. The most common circuit analysis technique involves applying Kirchhoff's Rules.

Loop Rule: Around any closed loop in a circuit, the sum of all changes in potential (emfs and potential drops across resistors) must equal zero. As charge carriers move around a circuit loop, they experience changes in potential, and therefore changes in potential energy. When the charge carriers move completely around a loop in the circuit, they are at the same potential as when they started, and therefore there has been no change in potential energy. This first rule is simply an expression of conservation of electric potential energy as charge carriers move around a circuit loop.

Junction Rule: At any junction point in a circuit, the sum of all currents entering (+) and leaving (−) must be zero. Since charge does not accumulate anywhere in a circuit containing only sources of emf and resistors, the rate at which charge carriers enter a junction must equal the rate at which charge carriers leave the junction. This second rule is simply an expression of conservation of electric charge at places where the circuit branches.

<u>Procedure for using Kirchhoff's Rules to analyze a circuit</u>

1. Identify the current in every segment of the circuit between junction points (*i.e.*, every branch) and label them as unknown currents (I_1, I_2, I_3, …). The number of unknown currents represents the number of linearly independent equations that must be generated and solved to completely analyze the circuit.

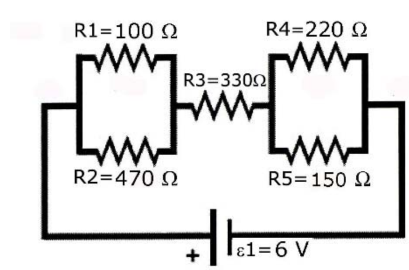
2. Assign a direction to each current. The choice of direction is arbitrary. Once you solve for all of the currents, a positive value indicates you chose the correct direction and a negative value indicates you chose the incorrect direction. The magnitudes of the currents will be correct.
3. Assign an arbitrary direction for a closed loop that traverses part of the circuit. Write an equation that adds the potential changes across every circuit element in the loop and sets the sum equal to zero. The potential change for a source of *emf* will be positive if you traverse the source from negative electrode to positive electrode, and negative if from positive to negative. The potential change associated with a linear resistor is described by Ohm's Law ($V = IR$). Assign a negative value to the potential if you traverse the resistor in the direction of the current and positive if opposite.
4. Write comparable loop equations for the other possible closed loops in the circuit. Every circuit element must appear in at least one equation, and every equation must have at least one new circuit element.
5. Write junction equations that express the fact that the net current flowing into a junction point must equal the net current leaving a junction point. Currents flowing into a junction point should be labeled positive and currents flowing out should be labeled negative. For a circuit with "n" junction points there are "$n - 1$" linearly independent equations that can be written. Writing an equation for the last junction point yields no new mathematical information.
6. Combine the loop equations with the junction equations. You should have "n" equations for "n" unknown currents. Solve the equations to obtain numerical values for all of the unknown currents.
7. For any negative currents, change the direction of the arrow in your drawing and change the sign of the current.
8. Based on the currents, calculate the potential drop across each resistor in the circuit.

Laboratory assignment

1. Record the color code for each of the five resistors, and interpret the code with respect to resistance value and precision.
2. Measure the resistance of each resistor using a multimeter. These measured values will be considered the "actual" values of resistance. Place these actual values in the tables you later use to record the currents and voltages (instead of R_1, R_2, etc.) for circuits #1 and #2.

Circuit #1

3. Sketch the following circuit in your laboratory book,



4. Use Kirschoff's rules to write down a set of linearly independent equations.
5. Solve the equations for the current through each resistor (using the actual resistances) and place the values in a table similar to Table I.

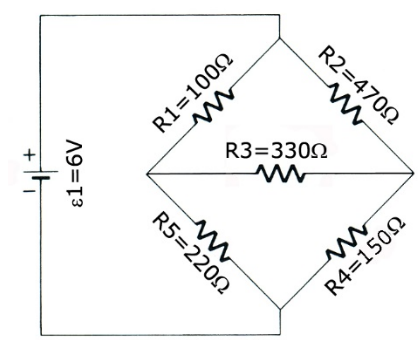
Table I: Current and voltage characteristics of circuit

	Calculated values		Experimental values		Error	
Resistor	I_{calc} (mA)	V_{calc} (V)	I_{exp} (mA)	V_{exp} (mA)	% diff (I)	% diff (V)
R_1						
R_2						
R_3						
R_4						
R_5						

6. Use these currents and the actual resistances to find the voltage drop across each resistor and place the values in the table.
7. Construct the circuit with your breadboard and use your DC voltage supply set to 6V as the battery shown in the circuit.
8. Measure the current through each resistor and the voltage drops across each resistor, and place the values in the table.
9. Determine the % difference between the calculated and experimental values for current and voltage.

Circuit #2

10. Sketch the following circuit in your laboratory book,



11. Use Kirschoff's rules to write down a set of linearly independent equations.
12. Solve the equations for the current through each resistor (using the actual resistances) and place the values in a table similar to Table I.
13. Use these currents and the actual resistances to find the voltage drop across each resistor and place the values in the table.
14. Construct the circuit with your breadboard and use your DC voltage supply set to 6V as the battery shown in the circuit.

15. Measure the current through each resistor and the voltage drops across each resistor, and place the values in the table.
16. Determine the % difference between the calculated and experimental values for current and voltage.

Equipment list: 100Ω, 150Ω, 220Ω, 330Ω, and 470Ω resistors, multimeter, DC power supply, banana-aligator cords, wires, breadboard.