Exercise 4: Capacitors

Purpose: to investigate the charging of capacitors, and their series or parallel relationship.

Introduction

A capacitor is an energy storage device commonly used in electric circuits. When an emf source is connected to a capacitor, the two separated conductive surfaces become oppositely charged with equal magnitudes of charge q. As the plates charge an electric field grows between the plates, as does a voltage difference V across the plates. The energy provided by the source of emf is stored in the electric field between the capacitor's charged conductors.

When connected in a single loop, the series resistance R determines the maximum amount of current $I = dq/dt$ that can travel through the circuit, where the resistance affects the charging and discharging time of a capacitor. The capacitance is defined as the ratio of the charge held on a capacitor plate to the voltage across the plates, or the parallel combination

$$C = \frac{q}{V_c}$$

where the capacitance depends on the capacitor's geometry and dimensions.

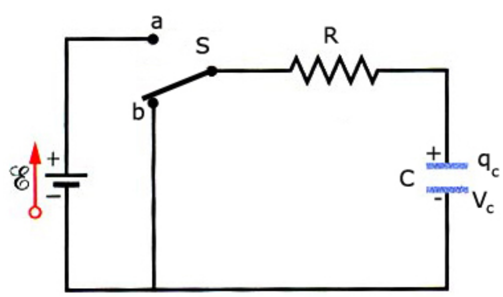

Fig. 1: Series circuit for charging/discharging a capacitor.

Figure 1 shows a resistor-capacitor series circuit (RC circuit). If the switch is flipped to the "a" position, then the capacitor begins to charge. The equation describing such a circuit is given by the emf and voltages associated with the resistor and capacitor components,

$$\mathcal{E} - IR - V_c = 0$$

Substituting our expression for the current and the voltage across the resistor gives

$$\mathcal{E} - R\frac{dq}{dt} - \frac{q}{C} = 0$$

For a charging capacitor, $q(t = 0) = 0$. We can then rewrite the above equation as an integral by separating the dq and dt terms with an initial integral limit of zero charge at zero time,

$$\int_0^q \frac{dq'}{(q' - C\mathcal{E})} = \int_0^t \frac{dt'}{RC}$$

The charge as a function of time from this equation follows as

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

Thus, the voltage across the capacitor plates while charging can be written as

$$V_c(t) = \mathcal{E}\left(1 - e^{-t/RC}\right)$$

As the capacitor begins to charge, the voltage drop across the resistor diminishes while the switch is in position "a", where

$$V_r(t) = \mathcal{E}e^{-t/RC}$$

After the capacitor is fully charged, the switch can be flipped back to the "b" position to discharge the capacitor. The voltage across the capacitor plates while discharging can be derived in a similar way,

$$V_c(t) = V_{c0}e^{-t/RC}$$

where V_{c0} is the initial voltage across the capacitor plates. The voltage drop across the resistor also diminishes when the voltage drops across the capacitor plates while the switch is in the "b" position.

Laboratory assignment

1. Measure the capacitance of each capacitor with the multimeter. These values will be the actual capacitances you will use to calculate the voltmeter's internal resistance from the RC times.
2. Set the DC voltage source to 6 V and turn the current knob to the ¾ position (3 o'clock position). Read the voltage of the source with the multimeter to get a precise value.
3. Set up the circuit shown in Fig. 1 with a 5 μF capacitor and use the multimeter's internal voltmeter's resistance as the resistor. (Be sure to have the voltage reading set to 20 V (or 40 V reading for some voltmeters) before placing in the circuit, and DO NOT change the multimeter to read current while connected to the circuit in this manner!)
4. Start with the switch in the "b" position.

Charging time of the capacitor

5. Allow the circuit to sit for a few moments to be fully discharged. Log in the data for t = 0 in Table I, where we begin with $V_r = 6$ V or whatever the voltage between the positive and ground terminal initially read before connecting the circuit.

Table I: Voltage as a function of time

Time (s)	Voltage (V)
0	
10	
20	
30	
⋮	⋮

6. Change the circuit so that the switch is in the "a" position. Write down the multimeter reading after 10 seconds of charging.
7. Continue to take data every 10 seconds until the voltmeter reads less than 1.5 V.
8. Make a plot of V_r vs. t.
9. To get a linear line graph, we make a second plot of t vs. $-\ln\left(\frac{V_r}{\varepsilon}\right)$.
10. Find the slope and uncertainty of the slope for the second plot.
11. Use the slope and uncertainty in the slope, and the value of the capacitor (assume 5% uncertainty in the capacitance) to find the internal voltmeter resistance, $R \pm \Delta R$.

Two capacitors in series

12. Set up the circuit as in step 3, but this time place two 5 μF capacitors in series.
13. Repeat steps 3-6.
14. Make a linear line graph of t vs. $-\ln\left(\frac{V_r}{\varepsilon}\right)$.
15. Use this time constant and the value of the resistance found in step 10 to determine $C_{\text{eq}} \pm \Delta C_{\text{eq}}$.
16. Is this value for the capacitance the same as you expect? Explain with your equations for capacitors in series.

Two capacitors in parallel

17. Set up the circuit as in step 3, but this time place two 5 μF capacitors in parallel.
18. Repeat steps 3-6.
19. Make a linear line graph of t vs. $-\ln\left(\frac{V_r}{\varepsilon}\right)$.
20. Use this time constant and the value of the resistance found in step 10 to determine $C_{\text{eq}} \pm \Delta C_{\text{eq}}$.
21. Is this value for the capacitance the same as you expect? Explain with your equations for capacitors in parallel.

Equipment list: 5 μF capacitors (2), multimeter, DC voltage supply, wires, banana-alligator cords (2), breadboard, wires, switch or additional wires to switch breadboard locations.