Exercise 7: inductor-resistor-capacitor (LRC) circuits

Purpose: to study the relationship of the phase and resonance on capacitor and inductor reactance in a circuit driven by an AC signal.

Introduction

By definition, alternating current in a circuit involves periodic reversal of current. Most commonly, the current reversals in ac circuits are sinusoidal with angular frequency ω (given in rad/s), and can be described by

$$i(t) = I_{\text{peak}} \sin \omega t$$

where I_{peak} is the maximum value of the current and t is the time.

Linear resistors in AC circuits behave the same as they do in DC circuits, where they follow Ohm's law

$$v_R = iR$$

$$v_R = I_{\text{peak}} R \sin \omega t$$

Capacitors in DC circuits allow a transient current as they charge or discharge, but they do not allow a steady current to flow, since they function as an open switch when fully charged. Because AC current described by Eq. 1 is alternating sinusoidally, a capacitor in an AC circuit is continuously charging and discharging, and thereby allowing a current to oscillate in the circuit. Because voltage across a capacitor V_C depends on the charge on the capacitor, and charge represents the current in an integral of time, the voltage across a capacitor is not in phase with the current. The voltage across the capacitor actually lags behind the current by 90°, where

$$v_C = \frac{1}{C} \int I_{\text{peak}} \sin \omega t \; dt$$

$$= -\frac{I_{\text{peak}}}{\omega C} \cos \omega t$$

The ideal capacitive reactance X_C is given by

$$X_C = \frac{1}{\omega C}$$

which is a proportionality constant between voltage amplitude and current amplitude for a capacitor in an AC circuit, much like resistance is a proportionality constant between voltage amplitude and current amplitude for a resistor.

Inductors in DC circuits create a resisting emf and impact a circuit only when the current is changing. With steady current, a perfect inductor (i.e., no resistance) functions as a connecting wire. Because current is continuously changing, an inductor in an AC circuit is continuously creating a resisting emf. Because the voltage across an inductor V_L depends on the rate of change of current di/dt, it leads the current by a 90° phase shift,

$$v_L = L\frac{d}{dt}I_{\text{peak}}\sin\omega t$$

$$= LI_{\text{peak}}\cos\omega t$$

The inductive reactance X_L is given by

$$X_L = \omega L$$

Inductive reactance is a proportionality constant between voltage amplitude and current amplitude for an inductor in an AC circuit, much like capacitive reactance is a proportionality constant between voltage amplitude and current amplitude for a capacitor.

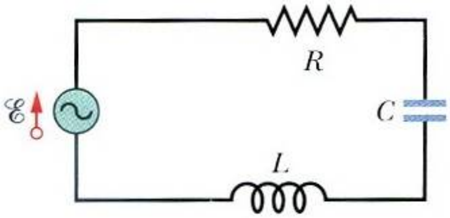

Fig. 1: a series inductor-resistor-capacitor circuit.

A series LRC circuit is shown in Fig. 1. The angular frequency of the current and the total voltage across all three components $\mathcal{E}$ is driven by the AC power source. According to Kirchhoff's loop rule, the sum of the instantaneous voltages across all circuit elements in the loop must be zero,

$$\mathcal{E} = v_L + v_R + v_C = I_{\text{peak}}\left(L\cos\omega t + R\sin\omega t - \frac{1}{\omega C}\cos\omega t\right)$$

Because of the phase differences among the three voltages, the relationships can best be viewed using a phasor diagram like the one shown in Fig. 2.

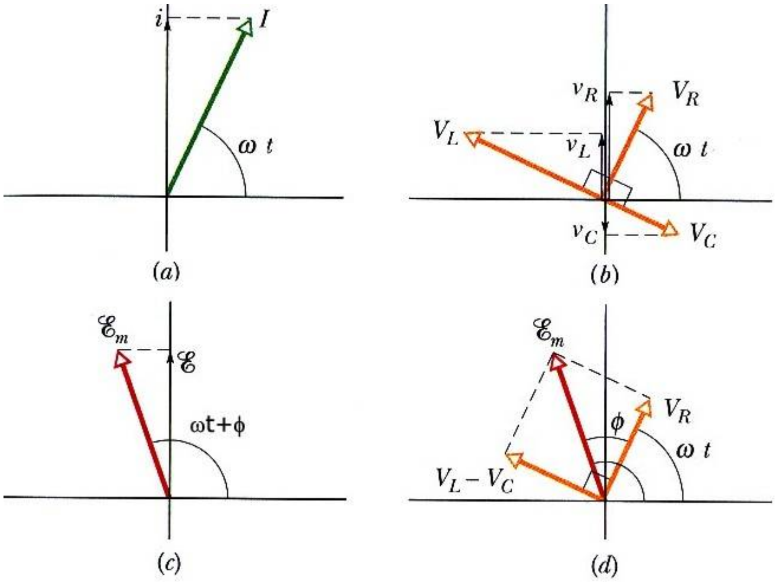

Fig. 2: Phasor diagrams of (a) current, (b) voltages across circuit elements, (c) circuit voltage, and (d) voltage components from the circuit elements.

Because the phase relationships among current and voltage amplitudes are always the same, we can generalize their relationship for an ideal circuit,

$$\mathcal{E}^2 = V_R^2 + (V_L - V_C)^2$$

The above ideal equation can be written in terms of current amplitude, resistance, and reactance,

$$\mathcal{E}^2 = I_{\text{peak}}^2[R^2 + (X_L - X_C)^2]$$

$$\mathcal{E} = I_{\text{peak}}\sqrt{R^2 + (X_L - X_C)^2}$$

The total impedance of an ideal circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The current will have a maximum amplitude when the impedance has a minimum value, which occurs when $X_L = X_C$, which happens at the resonant frequency. From the phasor diagram, the phase angle ϕ between emf across the power supply and current depends on the voltage amplitudes,

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

The term $\cos\phi$ is called the power factor because it determines the average power dissipated in the circuit by the resistance,

$$\langle P \rangle = \frac{1}{2}\mathcal{E}I_{\text{peak}}\cos\phi$$

For a real circuit, the inductor and capacitor carry a real resistance along with their ideal reactance. Therefore, the phase shifts of both real reactances are not ±90°.

Laboratory assignment

Single resistor test circuit

1. Turn on the frequency generator and set it to 10 Hz.
2. Use the multimeter (kHz) to measure the output frequency ad see if it matches the setting on the frequency generator.
3. Place a (100±10)Ω resistor on the breadboard and measure the resistance, then make a simple circuit by connecting the ends to the frequency generator.
4. Start data studio (*use Pasco capstone shortcut on desktop*) and select the scope display by double clicking the “scope” icon on right. Make sure and set the sampling rate to 1000 Hz.
5. Start data recording and adjust the amplitude setting on the frequency generator to give you a 4 volt amplitude on the voltage sensor (this is equivalent to 8 volts peak-to-peak).
6. Select the “trigger” button to stabilize the signal. Adjust the volts-per-division to give you a reasonable peak height. Adjust the milliseconds-per-division to give you a reasonable peak width.
7. Add the current sensor reading to the scope (use right vertical axis) as a second trace on the voltage scope. Adjust the amps-per-division to give you a reasonable peak height. There current and voltage traces should have no phase shift, and they should both be positive at the same time. If the two signals are out of phase, exchange the voltage sensor leads.

8. Adjust the frequency from 10 Hz to 100 Hz in 10 Hz increments. At each step, record in a table similar to Table I the frequency, voltage amplitude, and current amplitude measured with the smart tool.

Table I: Series resistor circuit values

Freq (Hz)	10	20	30	40	50	60	70	80	90	100
V_R (V)										
I (A)										

9. When finished, return the frequency setting to 10 Hz and turn off the frequency generator, and remove the resistor. The two signals should remain in phase as you change the frequency.

Inductor circuit

10. Use the LRC meter to measure the inductance of the coil (with iron bar inserted).
11. Also measure the resistance of the coil. The coil will function like an inductor in series with a small resistor that cannot be separated from the coil because the small resistance is from the wire in the coil itself.
12. Remove the resistor and connect the coil to the circuit.
13. Connect the voltmeter sensor with positive lead to the positive side of the inductor, just as with the resistor. Turn on the power supply and adjust the amplitude to 4 volts. The inductor voltage peak should be shifted relative to the current.
14. Adjust the frequency from 10 Hz to 100 Hz in 10 Hz increments. At each step, record in a table similar to Table II the frequency, voltage amplitude, current amplitude, time period between two adjacent current peaks, and the time increment between the current maximum and the voltage maximum (include positive or negative time).

Table II: Series inductor circuit values

Freq (Hz)	10	20	30	40	50	60	70	80	90	100
V_L (V)										
I (A)										
T (ms)										
Δt (ms)										
ϕ_L (rad)										

15. Calculate the phase shift at each frequency step and include these in your table.
16. Return the frequency setting to 10 Hz, turn off the frequency generator, and remove the connections to the coil.

Capacitor circuit

17. Make an equivalent capacitor by creating an array of capacitors that consists of 4 polarized 100 μF capacitors, two connected positive to positive in series, in parallel with a second pair just like the first.
18. Measure and record the equivalent capacitance of the array.

19. Connect the equivalent capacitor created from the capacitor array in series to the supply.
20. Connect the voltmeter sensor to the capacitor array. Turn on the power supply and adjust the amplitude to 4 volts. The inductor voltage peak should be shifted relative to the current.
21. Adjust the frequency from 10 Hz to 100 Hz in 10 Hz increments. At each step, record in a table similar to Table III the frequency, voltage amplitude, current amplitude, time period between two adjacent current peaks, and the time increment between the current maximum and the voltage maximum (include positive or negative time).

Table III: Series capacitor circuit values

Freq (Hz)	10	20	30	40	50	60	70	80	90	100
V_C (V)										
I (A)										
T (ms)										
Δt (ms)										
ϕ_c (rad)										

22. Calculate the phase shift at each frequency step and include these in your table.
23. Return the frequency setting to 10 Hz, turn off the frequency generator, and remove the connections to the capacitor array.

LRC series circuit

24. Connect the inductor coil, capacitor array, and resistor in series with the frequency generator and current sensor. Place the current sensor to measure the current coming from the AC power supply, with the positive sensor lead on the positive jack.
25. Measure the output voltage with the multimeter and adjust the amplitude to 4 volts (or $\sqrt{2}V_{\text{rms}}$).
26. Now place a voltage sensor on each of the three components (resistor, inductor, capacitor). Make sure you have four traces on the scope window, the current and the three voltages.
27. Adjust the frequency from 10 Hz to 100 Hz in 10 Hz increments. At each step, record in a table similar to Table IV the frequency, three voltage amplitudes, current amplitude, time period between two adjacent current peaks, and the time increment between the current maximum and the voltage maximum (include positive or negative time).
28. Place your phase data from Tables II and III into Table IV.
29. Determine impedance in the circuit from real components as a function of frequency and place in Table IV by splitting the out-of-phase reactance and in-phase resistance of each component and using Pythagorean's theorem using the following equation (note the sign of the phase from either leading or lagging)

$$Z = \sqrt{\left(\frac{V_L}{I_{\text{peak}}}\sin\phi_L + \frac{V_C}{I_{\text{peak}}}\sin\phi_C\right)^2 + \left(R + \frac{V_L}{I_{\text{peak}}}\cos\phi_L + \frac{V_C}{I_{\text{peak}}}\cos\phi_C\right)^2}$$

30. Plot the voltage across the resistor V_R as a function of the frequency.
31. Is there are resonance in the circuit, or is the frequency above or below resonance?

Table IV: RLC circuit values

Freq (Hz)	10	20	30	40	50	60	70	80	90	100
V_R (V)										
V_L (V)										
V_C (V)										
I (A)										
ϕ_L (rad)										
ϕ_c (rad)										
Z (Ω)										

Equipment list: Laptop computer with Data Studio software, Pasco PasPort Powerlink, Pasco PasPort voltage/current sensor (1 current / 3 voltage), RLC meter w/short leads, Multimeter w/ probes, banana/banana patch cord (4 black / 4 red), banana/alligator adapters (4 black / 4 red), Pasco Frequency Generator, Pasco EM8656 electronics laboratory board, Pasco wire kit for electronics laboratory board, iron core for coil on electronics board, 100 μF polarized capacitors(4), 100Ω resistor.