Exercise 7: Conservation of mechanical energy

Purpose: investigate the conservation of mechanical energy using a spring loaded cart.

Introduction

External work performed on a system causes a change in that system's energy,

$$\Delta K + \Delta U_g + \Delta U_s + \Delta U_{\text{int}} + \cdots = W_{\text{ext}} ,$$

where ΔK is the change in the kinetic energy, ΔU_g change in gravitational potential energy, ΔU_g is the change in a spring's potential energy, and ΔU_{int} is the change in the internal energy of the system.

When there is no external work done on the system,

$$\Delta K + \Delta U_{\text{mech}} = 0 ,$$

For the case of spring loaded cart on a level track, the only change in mechanical potential energy is from the spring. Therefore,

$$\Delta U_{\text{mech}} = \Delta U_s .$$

If we assume that the cart starts from rest with the spring compressed, and the spring is released to the equilibrium position when the cart moves along the track with a final velocity, then the conservation of mechanical energy for this system becomes

$$\frac{1}{2}k\left(0 - x_i^2\right) + \frac{1}{2}M(v^2 - 0) = 0 .$$

Laboratory assignment

1. Level the aluminum track.
2. Measure the length of the card ℓ to be used as a beam block and attach it to the cart. Assume negligible error.
3. Determine the mass of the cart with a balance.
4. Measure the displacement of the spring extending from the end of the cart. Its initial position from equilibrium x_i will be this value (treat the measurement uncertainty as negligible).
5. Load the spring into the cart and place it on the track with the spring facing the track bumper.
6. Set the photogate approximately 10 cm away from the front edge of the cart's beam block.
7. Release the spring so that the cart gains velocity down the track.
8. Record the time in Table 1.
9. Repeat steps 4-7 until you record 5 values for the time. Calculate the average and standard deviation.

Table 1: data for conservation of energy lab.

M (kg)	t_1 (s)	t_2 (s)	t_3 (s)	t_4 (s)	t_5 (s)	t_{avg} (s)	σ_t (s)	$v_{\text{avg}} \pm \Delta v$ (m/s)

10. Add 250 g to the cart and repeat steps 4-9 and fill in the next row of time data in Table 1.

11. Add another 250 g to the cart for a total of 500 g riding in the cart and repeat steps 4-9, and fill in the final row of time data in Table 1.
12. Solve for the average velocity and propagate the error from the time measurements to find the experimental range of the velocity, $v = v_{\text{avg}} \pm \Delta v$. Fill in the final column of Table 1.
13. Knowing the conservation of mechanical energy equation in the preliminaries, graph v^2 vs. $\frac{x_i^2}{M}$ in the space provided below. Include error bars in the velocity squared. Note that $\Delta(v^2) \neq (\Delta v)^2$.

14. Use you program to determine the slope and the uncertainty in the slope, which will be the spring constant k. Place it in the form $k \pm \Delta k$ and include your units.

$k =$ ______________________.

15. How close to zero is the intercept? Is it zero within uncertainty?

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Equipment list: aluminum track, PAScar, photogate, 250 gram masses (2), cards, scotch tape.