Exercise 10: Conservation of Angular Momentum

Purpose: to study the conservation of angular momentum in a system where a stationary object collides with a rotating object.

Introduction

The angular momentum of a single particle is given by, $\vec{L} = \vec{r} \times \vec{p}$.

For rigid objects, the angular momentum is the product of the moment of inertia and the angular velocity. For a simple object rotating about a principal axis, the magnitude of the angular momentum is given by

$$L = I\omega$$

In a closed system, the angular momentum is conserved, $\Delta\vec{L} = 0$.

For a disk (or cylinder) with moment of inertia $I_{cyl} = \frac{1}{2} m_{cyl} R_{cyl}^2$ rotating with angular velocity ω, the initial angular momentum would be $L_i = I_{cyl}\omega_i$. Suppose we drop a stationary rectangle on a rotating disk. The rectangle of width a and length b has a moment of inertia $I_{rec} = \frac{1}{12} m_{rec}(a^2 + b^2)$. The final angular momentum of the disk and rectangle system rotating together about their center of masses is given by $L_f = (I_{cyl} + I_{rec})\omega_f$. Using the law of conservation of angular momentum, the resultant expression becomes

$$I_{cyl}\omega_i = (I_{cyl} + I_{rec})\omega_f$$

$$\Rightarrow \frac{1}{2} m_{cyl} R_{cyl}^2 \omega_i = \left[\frac{1}{2} m_{cyl} R_{cyl}^2 + \frac{1}{12} m_{rec}(a^2 + b^2)\right]\omega_f$$

Laboratory assignment

1. Measure the radius of the disk R_{cyl} and the width a and length b of the rectangle and record it in Table 1.
2. Measure the mass of the disk m_{cyl} and the mass of the rectangle m_{rec} and record it in Table 1.
3. Set up the photogate and attach the beam block to the edge of the rotating wheel so that it can be detected as it passes through the photogate.
4. The success of this lab depends on how quickly the angular speed can be determined before and after the collision. Spin the disk and hold the rectangle above the disk without touching the disk.
5. Take the initial time measurement of the rotating disk and immediately write it down.

$t_i =$ ____________________.

6. *Immediately* after step 5 is completed, drop the rectangle so that it collides with the disk and rotates about its center of mass on top of the disk.
7. *Immediately* after the rectangle and disk start rotating together, take the final time measurement and write it down.

$t_f =$ ____________________.

8. Determine the initial and final angular velocities from the length of the strip, distance from the axis of rotation, and the recorded times. Place the values in Table 1.
9. Assume the objects are an ideal disk and an ideal rectangle. Also assume that the system is perfectly closed. Using the experimental initial angular velocity and the final equation in the introduction to calculate the theoretical final angular velocity. Place the value in Table 1.
10. Calculate the percent difference between the experimental final angular velocity and the theoretical final angular velocity and write it down in Table 1.
11. Determine the percent of lost rotational kinetic energy due to the inelastic rotational collision and place it in Table 1.

Table 1: data for the disk-rectangle collision

Mass of the disk (kg)	
Mass of the rectangle (kg)	
Radius of the disk (m)	
Width of the rectangle (m)	
Length of the rectangle (m)	
Experimental initial angular velocity (rad/s)	
Experimental final angular velocity (rad/s)	
Theoretical final angular velocity (rad/s)	
% difference for final angular velocities (%)	
% of lost rotational kinetic energy (%)	

12. Was angular momentum expected to be conserved in the collision and do your results agree? Explain.

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13. Was kinetic energy expected to be conserved in the collision and do your results agree? Explain.

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Equipment list: rotation stage, photogate, string, scissors, flash card, hanging masses.