Exercise 11: Spring-mass oscillations

Purpose: to investigate the oscillations of a vertical spring-mass system.

Introduction

The force of an ideal spring stretched in one dimension is characterized by Hooke's law,

$$F_s = -k(x - x_{\text{eq}}) ,$$

where x_{eq} is the equilibrium position. Setting the equilibrium position to zero, $x_{\text{eq}} = 0$, the force of a spring act

$$F_s = -kx .$$

Suppose a mass is attached to a single spring, and it is stretched away from the equilibrium position, $x_{\text{eq}} = 0$. If the mass attached to an ideal spring is released at its initially stretched position, x_{max}, then the spring will oscillate between the minimum and maximum positions with simple harmonic motion. This motion is described using mathematical shorthand as

$$x(t) = x_{\text{max}} \cos(\omega t - \varphi) ,$$

where φ is the phase associated with the time we begin observing this harmonic motion.

Using the definition of one-dimensional acceleration $a_x = d^2x/dt^2$, we may use the above equation to derive the acceleration of the mass caused by the spring

$$a_x(t) = -x_{\text{max}}\, \omega^2 \cos(\omega t - \varphi) .$$

The force of the spring acting on the mass as a function of time follows as

$$F_s(t) = -x_{\text{max}}\, m\omega^2 \cos(\omega t - \varphi) ,$$

which is also a simple harmonic function. Note that a vertical spring-mass system is subject to a gravitational force; however, this force is a constant, $F_g = mg$, and only changes the equilibrium position. Also note that dissipative forces will be present in *real* springs and the amplitude will reduce over long time scales.

Laboratory assignment

There will be two components to this laboratory assignment. The first part is concerned with the properties of a spring that is stretched by a hanging mass and allowed to hang in static equilibrium. The second part concerns oscillations of the spring-mass system.

Part 1: stretched spring in static equilibrium

1. Hang the spring on the force sensor attached to the vertical stand.
2. Measure and record the vertical distance, h, from the end of the spring to the floor.
3. Attach the 50 g hanger and add an additional 100 g mass for a total hanging mass of 150 g.
4. Measure and record the height at this new static position.
5. Calculate the magnitude of the displacement from equilibrium, $d = |h - h_0|$.
6. Repeat this measurement for additional hanging masses totaling 300 g, 450 g, and 600 g.
7. Fill in Table 1 including your uncertainty estimate, Δd.

Table 1: data for a vertical spring in static equilibrium.

mg (N)	d (m)	Δd (m)

8. Make a graph of mg vs. d and place it in the space below. Be sure to include x-axis error bars using Δd.

9. Find the spring constant by using your program to calculate the slope and uncertainty in the slope. Use our standard notation for uncertain quantities, $(k \pm \Delta k)$ multiplied by the units.

$k =$ ______________________.

Part 2: oscillating spring-mass system

10. Start with a total of 600 g for the hanging mass.
11. Tare the force sensor after making the oscillations as small as possible while in equilibrium.
12. Stretch the spring a few centimeters and let the system oscillate.
13. Run the force sensor shortly after the system begins to oscillate and view the graph.
14. Measure and record the period of oscillation T from the graph and estimate the uncertainty ΔT.
15. Repeat this measurement for a total hanging mass of 450 g, 300 g, and 150 g.
16. Fill in Table 2.

Table 2: data for an oscillating vertical spring-mass system.

m (kg)	T (s)	ΔT (s)	ω (rad/s)	$\Delta\omega$ (rad/s)	$1/\omega^2$ (s^2/rad^2)	$\Delta(1/\omega^2)$ (s^2/rad^2)

17. Knowing that $\omega = \sqrt{k/m}$, graph m vs. $1/\omega^2$ in the space below. Be sure to include x-axis error bars using $\Delta(1/\omega^2)$.

18. Use your program to determine the slope and the uncertainty in the slope to get the spring constant. $(k \pm \Delta k)$ multiplied by the units.

$k =$ ______________________.

Part 3: additional analysis

19. Compare your values for the spring constant k. Do the ranges overlap? Explain.

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20. A spring that follows Hooke's law will have an intercept of $m/3$ for your graph of m vs. $1/\omega^2$. Check the intercept and the uncertainty of your intercept for your m vs. $1/\omega^2$ graph. Does this value of $m/3$ fall within the range of your intercept? Explain.

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Equipment list: vertical stand, tapered Hooke's spring, mass hanger, slotted weights, force sensor, laptop.