Exercise 12: Standing sound waves and resonance

Purpose: to investigate the resonance in a tube and the speed of sound.

Introduction

When the diaphragm of a speaker vibrates, a sound wave is produced that propagates through the air. The sound wave consists of small motions of the air molecules toward and away from the speaker. If you were able to look at a small volume of air near the speaker, you would find that the volume of air does not move far, but rather it vibrates toward and away from the speaker at the frequency of the speaker vibrations. This motion is analogous to waves propagating on a string. An important difference is that, if you watch a small portion of the string, its vibrational motion is transverse to the direction of propagation of the wave on the string. The motion of a small volume of air in a sound wave is parallel to the direction of propagation. A sound wave is a type of longitudinal wave.

Another way of conceptualizing a sound wave is as a series of compressions and rarefactions. When the diaphragm of a speaker moves outward, the air near the diaphragm is com-pressed, creating a small volume of relatively high air pressure, a compression. This small high-pressure volume of air compresses the air adjacent to it, which in turn compresses the air adjacent to it, so the high pressure propagates away from the speaker. When the diaphragm of the speaker moves inward, a low-pressure volume of air, a rarefaction, is created near the diaphragm and propagates away from the speaker.

In general, a sound wave propagates out in all directions from the source of the wave. However, the study of sound waves can be simplified by restricting the motion of propagation to one dimension, as is done with the resonance air tube. Reflection of the sound wave occurs at both open and closed tube ends. If the end of the tube is closed, the air has nowhere to go, so a displacement node (a pressure antinode) must exist at a closed end. If the end of the tube is open, the pressure stays very nearly at room pressure, so a pressure node (a displacement antinode) exists at an open end.

As described above, a standing wave occurs when a wave is reflected from the end of the tube and the return wave interferes with the original wave. However, the sound wave will actually be reflected many times back and forth between the ends of the tube, and all these multiple reflections will interfere together. In general, the multiply reflected waves will not all be in phase, and the amplitude of the wave pattern will be small. At certain frequencies, all the reflected waves are in phase, resulting in a very high amplitude standing wave. These 'loudest' frequencies are called resonant frequencies.

In investigating the relationship between the length of the tube and the frequencies at which resonance occurs, the conditions for resonance can be understood in terms of the wavelength of the wave pattern, rather than in terms of the frequency. The resonance modes also depend on whether the ends of the tube are open or closed. For an open tube (a tube open at both ends), resonance occurs when the wavelength of the wave, λ, satisfies the condition,

$$L = \frac{n\lambda}{2} \quad \text{for} \quad n = 1,2,3 \ldots \qquad (1)$$

where L is the length of the tube.

These wavelengths allow a standing wave pattern such that a pressure node (displacement antinode) of the wave pattern exists naturally at each end of the tube. Another way to characterize the resonance states is to say that an integral number of half wavelengths fits between the ends of the tube.

For a closed tube, resonance occurs when the wavelength of the wave satisfies the condition,

$$L = \frac{n\lambda}{4} \quad \text{for} \quad n = 1,3,5 \ldots \qquad (2)$$

It is a useful experiment to investigate the wave behavior at the ends of the tube. Due to the behavior, the effective length of the tube is slightly longer than the measured length. The following empirical formulas give an approximate description of the resonance requirements for standing waves in a tube. For an open tube,

$$L + qd = \frac{n\lambda}{2} \quad \text{for} \quad n = 1,2,3 \ldots \qquad (3)$$

and for a closed tube,

$$L + qd = \frac{n\lambda}{4} \quad \text{for} \quad n = 1,3,5 \ldots \qquad (4)$$

where d is the diameter of the tube and q is a factor between 1 and 2 which includes other resonance modes and imperfections with the boundaries.

The resonance modes of the air column can be investigated by adjusting the frequency,

$$f = \frac{v}{\lambda}, \qquad (5)$$

of sound waves (traveling at the speed of sound, v) from the speaker or by adjusting the length of the air column in the tube.

Laboratory Assignment

Part 1: constant frequency, variable length

1. Measure the diameter of the tube and write it down. Assume negligible uncertainty

   $d =$ ________________.

2. Put the piston into the resonance air tube and move the piston near the speaker.
3. Set the signal generator to produce a frequency of 800 Hz.
4. Begin with the amplitude at the lowest counter-clockwise setting. Do not turn beyond the one-half turn setting!!!
5. Connect the microphone to the channel 1 connector on the oscilloscope and turn the microphone on (make sure a battery is inside the microphone).
6. Turn on the signal generator and oscilloscope.
7. Set the oscilloscope time/division to be 0.1 ms.
8. Set the oscilloscope channel 1 to 10 mV/division.
9. Trigger the signal to show a static wave with the trigger tick mark at the 12 o'clock position.
10. Slowly increase the length of the air column by moving the piston away from the speaker until the oscilloscope amplitude is at a maximum. Place the first position value in Table 1.

Table 1: Resonance and length

Resonance found	L (m)
1	
2	
3	
4	

11. Continue to increase the length of the air tube by moving the piston. Place each position in Table 1 that corresponds to a maximum amplitude (resonance) up to the 4th resonance.
12. Plot a figure of L as a function of the number of resonances found (1,2,3,4) in the space below. Note that the number of resonances found does not necessarily correspond to "n" for all instruments as seen for the differences in open vs. closed pipe equations.

13. Determine the slope and the uncertainty in the slope using your linear regression program (include units).

    Slope = ___________________.   Uncertainty in slope = __________________.

14. Using the slope and uncertainty, and the knowledge that $\lambda_1 = 2(\text{Slope})$, determine the fundamental resonance wavelength using the standard notation $(\lambda \pm \Delta\lambda)$ multiplied by the units.

    $\lambda =$ ______________________.

15. Determine the intercept and uncertainty in the intercept.

    Intercept = ___________________.   Uncertainty in intercept = __________________.

16. Calculate the range for $qd = (1.5d \pm 0.5d)$ and make sure to include proper units.

    $qd =$ ____________________.

17. Does the range for qd fall within the range of the intercept? Explain.

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Part 2: constant length, variable frequency

18. Move the piston so that the length of the air tube is 40 centimeters (0.40 m).
19. Set the signal generator to produce a frequency f of 200 Hz (2k button with dial at 0.2).
20. Begin with the amplitude at the lowest counter-clockwise setting. Do not exceed one-half turn!
21. Connect the microphone to the channel 1 connector on the oscilloscope and turn the microphone on (make sure a battery is inside the microphone).
22. Turn on the signal generator and oscilloscope.
23. Set the oscilloscope time/division to be 0.5 ms.
24. Set the oscilloscope channel 1 to 10 mV/division.
25. Trigger the signal to show a static wave with the trigger tick mark at the 12 o'clock position.
26. Slowly increase the frequency on the signal generator until the oscilloscope amplitude is at a maximum. Place the first position value in Table 2.
27. Continue to increase the frequency of the signal generator. Place each position in Table 2 that corresponds to a maximum amplitude (each resonance) up to the 4^{th} resonance.

Table 2: Resonance and frequency

Resonance found	f (Hz)
1	
2	
3	
4	

28. Plot a figure of f as a function of the resonance found in the space below.

29. Determine the slope and the uncertainty in the slope using your linear regression program. (include units)

    Slope = ___________________.   Uncertainty in slope = __________________.

30. Using the slope and the uncertainty, and the knowledge that $v = 2L(\text{Slope})$, determine the speed of sound in the resonance tube using the notation $(v \pm \Delta v)$ multiplied by the units.

$$v = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

31. Does the accepted value for the speed of sound (343 m/s) fall within the range of calculated velocities? Explain your result.

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Equipment list: Resonance tube apparatus with speaker and microphone (needs batteries), banana cords, bnc connectors, oscilloscope, signal generator.